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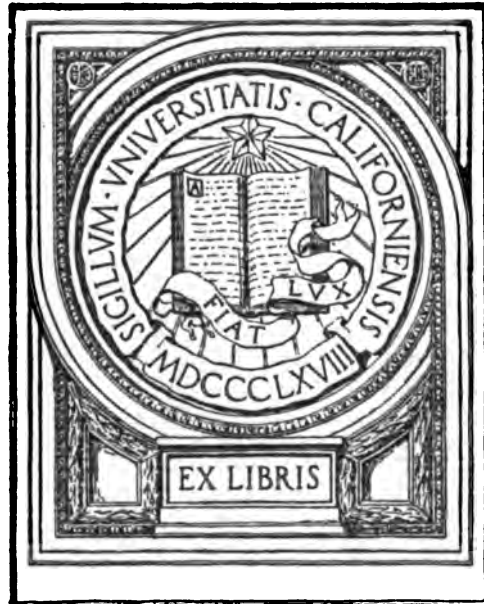
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DEPARTMENT OF COMMERCE AND LABOR

BULLETIN
OF THE
BUREAU OF STANDARDS

S. W. STRATTON, DIRECTOR

VOLUME 4

(Nos. 1, 2, 3, 4)

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CLARK AND WESTON STANDARD CELLS.

By F. A. Wolff and C. E. Waters.

INTRODUCTION.

The important rôle now played by the standard cell in both technical and scientific work, and the possibility of its adoption as a *primary* standard of electromotive force, have led in recent years to a considerable number of investigations concerning its reproducibility and constancy. The character of these investigations and the results obtained will, however, be better understood after a brief review of the previous work on the subject.

The need of a definite and universal system of electrical units was early recognized. Owing to its preponderating importance in the earlier applications of electricity, the unit of resistance naturally received first attention. The committee on electrical standards appointed by the British Association in 1861 recommended the adoption of the C. G. S. electromagnetic system together with a practical system defined as decimal multiples and submultiples of the C. G. S. units. In addition, its labors led to the construction of concrete standards of resistance in the form of coils of platinum-silver of special design adjusted to represent 10^9 C. G. S. units as determined by a series of absolute measurements.

The definition of unit current and electromotive force in terms of the C. G. S. units long met every requirement, particularly as currents were generally measured by the aid of the tangent galvanometer, while electromotive forces were generally measured in terms of the electromotive force of the Daniell cell.

In 1872 Latimer Clark called to the attention of the British Association committee the superiority of the cell which now bears his

name, recommending it as a suitable standard of electromotive force. In its original form¹ it consisted of a mercury electrode covered with a paste made of mercurous sulphate, zinc sulphate crystals and solution, while a rod of zinc in a saturated zinc sulphate solution above the mercury and paste and in contact with an excess of zinc sulphate crystals formed the second electrode. It will thus be seen that Clark fully appreciated the importance of reversibility, a fundamental condition to be satisfied by the standard cell. Clark also foresaw nearly all of the modern applications of the standard cell, as for example in the calibration of the ballistic galvanometer, in maintaining a current at a constant value, in the measurement of currents and electromotive forces, etc. No action was, however, taken on the suggestion by the committee.

The discovery of considerable error in the British Association ohm determination, the rapid development of electrotechnics, and the resulting demand for increased accuracy induced the French Government to issue a call for an international electrical congress to meet in Paris in 1881. This was followed by an international commission, recommended by the congress, which met in 1882 and 1884, and which defined the ohm in terms of the Siemens mercury unit, the ampere as one-tenth of a C. G. S. unit of current and the volt in terms of the ohm and ampere.

While the demands for accuracy of reproduction were fully met by the mercury ohm, the definitions of the ampere and the volt were found to be far from satisfactory. During the next few years attention was therefore directed to the investigation of the silver coulometer and the Clark cell. The former was studied by Mascart⁽²⁾, Rayleigh and Sidgwick⁽³⁾, Kohlrausch⁽⁴⁾, and others.

Rayleigh's⁵ investigations on the Clark cell showed that the main variations could be traced to impurities in the materials employed and to a faulty construction, the zinc not being completely covered with zinc sulphate crystals, thus retarding the attainment of saturation equilibrium for the temperature of observation. To remedy

¹ Phil. Trans. Roy. Soc., 164, p. 1; 1874.

² Jour. de Phys. (2), 1, p. 109; 1882. Jour. de Phys. (2), 3, p. 283; 1884.

³ Phil. Trans. Roy. Soc., 175, p. 411; 1884.

⁴ Wied. Ann., 27, p. 1; 1886.

⁵ Phil. Trans. Roy. Soc., 176, p. 781; 1885.

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S. W. STRATTON, DIRECTOR

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1907

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the latter defect Rayleigh proposed the H cell in which the zinc rod of the older type was replaced by zinc amalgam. Another cause of wide variations in Clark cells was found by Rayleigh to be a transformation of zinc sulphate, stable at ordinary temperatures as heptahydrate, into the hexahydrate at temperatures above 40°. Rayleigh showed that the inverted form persisted in a metastable condition for a long period after cooling, thus explaining the abnormal values which were frequently observed, and which were probably the rule when the older specifications which prescribed heating the constituents to boiling were followed.

Little further work was done on the Clark cell until shortly before the Chicago International Electrical Congress. In England the subject was taken up in 1891 by Glazebrook and Skinner,⁶ in connection with a committee appointed by the Board of Trade on Standards of Electrical Measurement. They studied the reproducibility of Clark cells constructed by themselves and others, determined the limits of error to be expected, and redetermined the electromotive force in terms of the electrochemical equivalent of silver. The cells set up by them were all of the Board of Trade type and an accuracy of only 0.01 per cent was sought.

The subject was taken up about the same time at the Reichsanstalt by Kahle,⁷ who used specially purified materials and made a careful study of the influence of added impurities. Practically all of his work was done with cells of the H type, with which trustworthy measurements could be made to a few hundred thousandths of a volt. Unfortunately, he employed only one sample of mercurous sulphate, the constituent which has since been found to be the main source of variation. Later he also made a redetermination of the temperature coefficient and the absolute value.⁸

From the results obtained it was found that there was little choice between the standard cell and coulometer from the standpoint of accuracy; indeed, both the coulometer and the cell were adopted in the definitions of the ampere and volt by the Chicago congress.

⁶ Phil. Trans. Roy. Soc., 188, p. 567; 1892.

⁷ Zs. für Instrk., 12, p. 117; 1892.

Zs. für Instrk., 18, p. 293; 1893.

Wied. Ann., 51, p. 174; 1894.

⁸ Zs. für Instrk., 18, p. 230; 1898.

Wied. Ann., 67, p. 1; 1899.

Subsequent to the Chicago congress the temperature coefficient of the Clark cell was redetermined by Jaeger and Kahle and also by Callendar and Barnes, with close agreement between the two.

Further studies on the hysteresis,⁹ polarization,¹⁰ and the inversion of zinc sulphate¹¹ were also made by various investigators.

In 1892 Edward Weston¹² described a cell in which the zinc and zinc sulphate of the Clark cell are replaced by cadmium and cadmium sulphate, respectively. It was discovered by Weston that a cadmium cell set up with a cadmium sulphate solution saturated at 4° had a negligible temperature coefficient. A portable type was also designed and has since been on the market.

The elaborate studies of the saturated type made at the Reichsanstalt by Jaeger and Wachsmuth,¹³ Jaeger,¹⁴ Jaeger and Kahle,¹⁵ and Jaeger and Lindeck,¹⁶ indicated that the Weston cell possessed all the merits of the Clark cell, besides having a much smaller temperature coefficient, showing no tendency to crack at the amalgam limb, so frequently observed with Clark cells, and not forming gas at the amalgam electrode.

In the specifications first proposed by Jaeger for the cadmium cell a 1:6 cadmium amalgam was recommended, the employment of which led to considerable mistrust, as irregularities of behavior were

⁹ Ayrton and Cooper: *Proc. Roy. Soc.*, **59**, p. 368; 1896.

Spiers, Twyman and Waters: *Phil. Mag.* (5), **45**, p. 285; 1896.

Callendar and Barnes: *Proc. Roy. Soc.*, **62**, p. 117; 1897.

¹⁰ Skinner: *Phil. Mag.* (5), **38**, p. 271; 1894.

Wulf: *Wien. Sitz-Ber.* II a, July, 1897.

Jaeger: *Ann. d. Phys.* (4), **14**, p. 726; 1904.

¹¹ Jaeger: *Wied. Ann.*, **63**, p. 354; 1897.

Callendar and Barnes: *Proc. Roy. Soc.*, **62**, p. 117; 1897.

Cohen: *Zs. f. phys. Chem.*, **25**, p. 300; 1898.

Cohen: *Zs. f. phys. Chem.*, **31**, p. 164; 1899.

Cohen: *Zs. f. phys. Chem.*, **34**, p. 62; 612; 1900.

Barnes: *Jour. of Phys. Chem.*, **4**, p. 1; 1900.

¹² *Electrician*, Lond., **30**, p. 741; 1893.

¹³ *Electrotech. Zs.*, **15**, p. 507; 1894.

¹⁴ *Electrotech. Zs.*, **18**, p. 647; 1897.

¹⁵ *Zs. für Instrk.*, **18**, p. 161; 1898.

Wied. Ann., **65**, p. 926; 1898.

¹⁶ *Zs. für Instrk.*, **21**, 33; 1901.

Ann. d. Phys. (4), **5**, p. 1; 1901.

observed by Callendar and Barnes and confirmed by others¹⁷ at temperatures below 15°. These were first attributed to an inversion of the cadmium sulphate similar to that of zinc sulphate referred to above, but later investigations¹⁸ showed that they were undoubtedly due to the cadmium amalgam and were entirely eliminated by the employment of 12–13 per cent amalgams.

Further work by Bijl¹⁹ and Puschin²⁰ has shown that cadmium amalgams consist within certain temperature limits of a liquid phase and a solid phase, the composition of the latter depending on the temperature, and that the mixed crystal is therefore an isomorphous mixture of cadmium and mercury. With both phases coexistent, the electromotive force toward cadmium sulphate at a given temperature is independent of the relative amounts of the metals present, as found by Jaeger,²¹ while in the absence of the liquid phase irregularities in behavior are found. The composition of the amalgam to be employed in the standard cell must therefore be such that both phases will be present within the usual temperature range 0° to 40°, a condition satisfied by the 12.5 per cent amalgam, now in general use.

Previously unsuspected irregularities due to the mercurous sulphate were observed at the Reichsanstalt toward the end of 1900,²² when a sample of mercurous sulphate, obtained from a different source, was employed. Even after the cells had reached constant values they were still several tenths of a millivolt higher than the older cells all made from another sample of mercurous sulphate.

In 1904 an investigation of the subject was begun in temporary quarters at the Bureau of Standards by one of the authors and Dr.

¹⁷ Callendar and Barnes: *Electrician*, **39**, p. 638; 1897.

Cohen: *Wied. Ann.*, **65**, p. 344; 1898.

Cohen: *Zs. f. phys. Chem.*, **84**, p. 621; 1900.

¹⁸ Jaeger: *Wied. Ann.*, **65**, p. 106; 1898.

Zs. für Instrk., **20**, p. 317; 1900.

Ann. d. Phys. (4), **8**, p. 366; 1900.

Ann. d. Phys. (4), **4**, p. 123; 1901.

von Steinwehr: *Ann. d. Phys.* (4), **9**, p. 1046; 1902.

¹⁹ *Zs. für phys. Chem.*, **41**, p. 641; 1902.

²⁰ *Jr. Russ. phys. Chem. Soc.*, **34**, p. 856; 1902.

Zs. für anorg. Chem., **36**, p. 201; 1903.

²¹ *Wied. Ann.*, **65**, p. 106; 1898.

²² Jaeger and Lindeck: *Zs. für Instrk.*, **21**, p. 73; 1901.

H. N. Stokes. Some preliminary work was done on the purification of materials, but efforts were mainly directed to the preparation of mercurous sulphate of uniform electromotive properties, an electrolytic method being devised. This method was described by one of the authors³³ at the Washington meeting of the American Electrochemical Society, April 7, 1904, at which a paper by Carhart and Hulett,³⁴ giving essentially the same method, was also read. A number of cells, set up with samples of mercurous sulphate prepared by this method, showed excellent agreement among themselves.³⁵ The work at the Bureau was interrupted by the establishment of the branch laboratory at St. Louis and by the extra work incident to moving into the new laboratories at Washington.

Considerable further work has been done by Hulett³⁶ on the reproducibility of the Weston cell, in connection with which a study was made of the influence of hydrolysis of the mercurous sulphate. It was shown by Gouy³⁷ that pure mercurous sulphate is hydrolyzed by water with the formation of a yellowish, difficultly soluble, basic product, which he stated is easily transformed into neutral mercurous sulphate by 0.1 normal sulphuric acid. He also stated that mercurous sulphate is not hydrolyzed by saturated solutions of zinc or cadmium sulphate.

Hulett, on the other hand, by rotating a sample of mercurous sulphate with mercury and successive portions of water at constant temperature, found that the electrical conductivity of the solution remained constant until all the mercurous sulphate was hydrolyzed, after which a different, fixed conductivity was observed. He found that the completely hydrolyzed product was gray, and not yellow. He also concluded that a soluble acid sulphate and a slightly soluble basic sulphate are formed, and that the hydrolyzed salt does not dissolve as such, but is further decomposed by water. He also determined the solubility curves of mercurous sulphate, in the pres-

³³ Wolff: *Trans., Am. Electrochem. Soc.*, 5, p. 49; 1904.

³⁴ *Ibid.*, 5, p. 59; 1904.

³⁵ These were ruined by exposure to light after removal to the new laboratory and during Doctor Wolff's absence on duty at the St. Louis Exposition.

³⁶ *Zs. für phys. Chem.*, 49, p. 483; 1904.

Phys. Rev., 22, p. 321; 1906.

Phys. Rev., 28, p. 166; 1906.

³⁷ *Compt. rend.*, 180, p. 1399; 1900.

ence of mercury, in sulphuric acid of varying concentrations, and found a maximum solubility at $V=1$ (V denoting the number of liters containing one mole sulphuric acid) and a break in the curve at $V=4$, from which he concluded that hydrolysis apparently begins at this dilution.

Hulett set up a considerable number of cells with samples of mercurous sulphate prepared by the electrolytic method, varying the current density and strength of acid between wide limits, and also with samples prepared by chemical methods. Where the concentration of the acid was less than molecular, the electrolytic samples, which were white even with the greatest current density, gave higher results than the gray samples obtained when stronger acid was used, and which showed excellent agreement among themselves. Less satisfactory results were obtained with samples prepared by chemical methods.

He also found that cells in which the depolarizer was a mixture of neutral and completely hydrolyzed mercurous sulphate gave values irregularly increasing with the percentage of basic sulphate employed. For other cells the mixture was prepared by rotating mercurous sulphate with definite amounts of water, thus giving a definite percentage of basic salt. These cells had, however, considerably higher values, for the same percentage of basic sulphate, than those mentioned above. The above results, unexpected in the light of the phase rule, are attributed to the possible formation of new phases. According to the above there should be a condition of unstable equilibrium to which Hulett attributes the slow decrease of e. m. f. observed by him in older cells. This was further tested by single electrodes charged with a mixture of mercury, mercurous sulphate and cadmium sulphate, the materials being rotated for a number of days in a thermostat. When measured against an amalgam electrode abnormally high values were observed, which decreased on stopping the rotation. These cells were not kept under observation long enough to determine whether normal values would be reached. It is to be noted that this effect will not account for the decrease in the e. m. f., observed by Hulett in the case of old cells, except on the assumption that further reaction takes place between the products of hydrolysis and the ingredients of the cell. This will be discussed more fully in a later paper.

At the Reichsanstalt a special study has been made of the influence of size of grain on the electromotive properties of mercurous sulphate. According to v. Steinwehr²⁸, this is mainly responsible for variations noted in the case of commercial samples, fine-grained samples giving, in accordance with theory, much higher values than coarse-grained ones. On grinding the latter he found a considerable increase in the e. m. f. and a very slow recovery.

Work on the standard cell has been in progress for several years at the English National Physical Laboratory,²⁹ but no detailed report of the results has been published. According to preliminary reports considerable progress has been made. Samples of mercurous sulphate prepared by three chemical methods agree in e. m. f. with samples prepared by the electrolytic method.

In October, 1905, the work was resumed by the present authors with the object of investigating the purification and preparation of materials for Clark and Weston cells, the influence of impurities, and any other sources of variation, such as size of grain, etc., influencing the electromotive force.

A systematic study of reproducibility can of course be made only by comparing the various materials in actual cells in which only one ingredient is varied at a time. It was therefore necessary to set up a large number, and in order to employ identical materials a considerable supply had to be accumulated, which, in the case of cadmium sulphate, required a long time, as our object was, at first, to employ only clear crystals, though it has since been found that cloudy crystals from a purified solution give the same electromotive force.

The details and results of the work done at the Bureau of Standards, a brief account of which was given at the New York meeting of the American Physical Society, December, 1906,³⁰ are submitted below under the following heads:

²⁸ *Zs. für Instrk.*, **25**, p. 205; 1905.

Zs. für Elektrochem., **12**, p. 578; 1906.

²⁹ Smith: B. A. Reports 1904, 1905, and 1906.

Electrician (Lond.), **55**, p. 857; 1905.

³⁰ Wolff and Waters: *El. World*, **49**, p. 100; 1907.

Phys. Rev., **24**, p. 252; 1907.

Electrician (Lond.), **58**, p. 692; 1907.

Preparation and purification of materials.

The cells.

The comparing baths.

The electrical measurements.

Tabulation of results.

Discussion of results.

General conclusions.

PREPARATION AND PURIFICATION OF THE MATERIALS.

Mercury.—All the mercury used in this investigation was purified by distilling it at least twice in a current of air under greatly reduced pressure, according to Hulett and Minchin.²¹

The essential feature of this method is the oxidation of any metals that may distil with the mercury by means of a slow current of air, admitted through a tube drawn out at one end to a very fine capillary extending almost to the bottom of the distilling bulb. This tube passes through a tightly fitting cork in the neck of the bulb, and the flow of air is regulated by means of a pinchcock and a short piece of rubber tubing, the free end of which is plugged with cotton wool to filter out dust. To prevent charring the cork a bulb with the side tube as low as possible on the neck should be selected.

An ordinary distilling bulb, of about 500 cc capacity, is sealed to one end of a slightly inclined, wide tube, 2 cm in diameter and 50 cm long, which serves as the condenser. At the lower end of the condenser is sealed a vertical tube of 3 to 4 mm bore and 80 to 90 cm in length, and bent into S form at the lower end, which thus serves as a trap for the mercury, which runs out, as rapidly as it distils, into a beaker or small dish. A side tube must be sealed to the upper, slightly widened part of the vertical tube for making connection with an aspirator. On commencing the distillation the vertical tube is filled by placing under the lower end a small beaker of mercury and then starting the aspirator. Of course somewhat more of the first part of the distillate than is needed to fill the vertical tube must be rejected.

As an additional precaution against mercury being carried over by bumping, a Claisen distilling bulb, with two necks, instead of

²¹ Phys. Rev., 21, p. 388; 1905.

one of the ordinary form, was employed by the authors, a thermometer being inserted in the second neck. The flask was heated in a hemispherical iron dish, the bulb being surrounded by a loose cylindrical sheath of asbestos paper and a cover of asbestos board through which the necks of the flask passed. By using iron turnings in the bath, instead of sand, which might scratch the bulb, and allowing at least an hour for cooling before refilling with mercury, it was possible to distil a large quantity of mercury without any apparent damage to the bulb.

From time to time the still was cut apart, cleaned with nitric acid, washed and dried. Usually about four-fifths of the mercury was distilled over, and when it was not very impure, the residue was simply sucked from the flask by means of a pipette.

The surface of the distilled mercury was slightly coated with oxides of the metals originally contained in it. These were removed by passing it through a pinhole in a filter paper.

In most of the work commercial mercury was distilled at least twice by this method. Some which had been used in the laboratory and was badly contaminated was, however, first subjected to a preliminary purification by electrolysis. This was found to be simpler and more efficient than the usual treatment with dilute nitric acid or mercurous nitrate, the action of which is necessarily superficial and therefore slow.

By making the mercury the anode, a piece of platinum foil the cathode, and using 2 per cent nitric acid as the electrolyte, the more positive metals go almost completely into solution by electrolysis, leaving in the mercury the less positive metals which exert only a minor influence on the e. m. f. and which may with the others be afterwards removed by distillation as described above.

With a current density of about 0.5 ampere per square decimeter, the electrolysis was continued, constantly stirring the mercury, for some hours after it no longer tailed, the time required depending on the amount and the original condition of the sample. The mercury deposited on the cathode, possibly containing some of the electro-positive impurities, was prevented from dropping back into the anode mercury by suspending under the cathode a small beaker hung from the side of the battery jar by means of a support made of glass rod. The combination anode and stirrer described below

was used. The depth of the mercury was great enough to completely cover the blades of the stirrer when rotating, and the volume of dilute acid employed was about four times that of the mercury.

Twenty kilos of mercury, very impure from use in the laboratory, were subjected to this purification using a current of 1 ampere for seventeen hours, and then distilled in three successive portions, leaving the residue in the bulb each time. Then 7 kilos more of mercury, also purified electrolytically, were introduced and distilled. The final residue, amounting to about 1 kilo, was analyzed. Traces of zinc, iron, and copper, and a small amount of lead were found. In the mercury before distillation no lead was found and only a doubtful reaction for zinc was obtained, while copper and iron were detected.

Zinc.—To remove small quantities of cadmium, lead, iron, and arsenic ordinarily contained in the commercial chemically pure metal, a considerable quantity of Kahlbaum's best zinc was repeatedly distilled under diminished pressure according to the method of Morse and Burton.³² A large tube of Jena combustion glass was closed at one end, the zinc introduced, and the tube drawn out at equal intervals, thus forming three reservoirs. The narrow portions were arched somewhat to prevent the melted metal from flowing from one section to another. The tube was placed in a combustion furnace, connected by means of a rubber stopper and heavy tubing to a Geryk pump and exhausted, the pump being kept running, not only during the distillation but afterwards, until the tube cooled. The latter was first heated at the closed end and by smaller flames under the second section, care being taken not to allow the tube to soften and collapse. The flames were regulated so that nearly all the metal condensed in the second section. When about three-fourths of the zinc had distilled off, the second section was more strongly heated and the zinc distilled into the third section, thus effecting a double distillation in practically one operation. The flames under the closed end of the tube were turned low, but not extinguished, so that the metal would not distil back again into the first section. The flames were then extinguished, but air was not admitted until the tube was cold. The distilled metal adhered

³²Am. Chem. Jour., 10, p. 311; 1888.

strongly to the glass, which was removed by hammering with a porcelain pestle.

In some of the work, zinc, which had been distilled in this manner three times, was used, but in most of the cells Kahlbaum's best zinc was employed without further treatment, as it was found to give identical results.

Cadmium.—Kahlbaum's best cadmium, prepared electrolytically, was distilled five times in the same manner as the zinc.³³ In most of the cells, however, the metal was used without further treatment. Although considerable platinum was found in the residues and a trace of arsenic in the portions of the distillate which condensed beyond the third section of the tube, the original metal was found to give the same results.

Amalgams.—*Zinc amalgam*, containing 10 per cent by weight of zinc, was prepared by dissolving a weighed amount of pure zinc in nine times its weight of pure mercury. The latter was heated gently in a porcelain dish on a sand bath. The zinc, previously treated with very dilute sulphuric acid to remove the film of oxide, then washed with water and dried, was placed upon the hot mercury and frequently stirred to hasten solution, the heat being increased whenever the amalgam showed a tendency to solidify.

Cadmium amalgam, containing 12.5 per cent by weight of cadmium, was prepared in the same manner. On account of its relatively low melting point it was, however, prepared on the steam bath.

Oxidation of the amalgams.—On exposure to the air the surfaces of the amalgams are slowly tarnished by oxidation; but as considerable changes in composition have no appreciable influence on the electromotive force, this is of no practical importance. The oxidation is lessened by keeping the amalgam under a solution of zinc or cadmium sulphate; but in the course of time a deposit of basic salt is formed. Apart from the fact that the use of these solutions does not completely prevent oxidation is the further objection that the pipette used in introducing the amalgam into the cells may become coated with basic sulphate. In addition, the zinc amalgam must be heated so high that there is violent bumping. Accordingly this method of removing the oxide was abandoned, and the melted

³³ Morse and Jones: Am. Chem. Jour., 14, p. 261; 1892.

amalgam was simply strained, when necessary, through a test tube drawn out at the bottom to a small opening and heated gently from time to time to keep the amalgam melted.

Zinc sulphate.—The salt is apt to contain sulphates of cadmium, iron, lead, etc., and free sulphuric acid. The last of these has the greatest effect upon the e. m. f., and promotes the formation of gas in the amalgam limb. Although the effect of such small quantities of the other impurities as are apt to be contained in the chemically pure salt is practically negligible, the following methods of purification, both giving the same results with zinc sulphate from Kahlbaum and from the J. T. Baker Company, were used:

The chemically pure salt, as purchased, was dissolved in hot water, an excess of pure zinc oxide and sufficient pure hydrogen peroxide to effect the oxidation of any ferrous iron added and the solution kept at nearly the boiling point for several hours to throw down iron as completely as possible. It was then filtered, acidified slightly, and evaporated until the zinc sulphate began to crystallize out, then cooled with ice (5° or lower), stirring frequently meanwhile so as to obtain small crystals. These were filtered off, using a platinum cone, washed once or twice with very little ice-cold water, redissolved in a little hot water, and recrystallized as before. Further crops of crystals were obtained from the first and second mother-liquors, those from the first being recrystallized once, those from the second mother-liquor twice. The three lots of crystals were then combined and dissolved in sufficient warm water to form a saturated solution. The temperature was not allowed to exceed 35° to avoid the formation of the hexahydrate, which is stable above 39° . The solution was filtered, and with continuous stirring, cooled by surrounding the beaker with ice. The crystals were filtered off, using a platinum cone, and washed two or three times with a little ice-cold water. The air-dried crystals were preserved in a well stoppered bottle.

The zinc oxide employed was prepared by adding ammonia to a solution of zinc sulphate until the precipitate dissolved. It was then filtered into a large volume of water, allowed to settle, the supernatant liquid decanted, and the precipitate thoroughly washed on a Büchner funnel (using a hardened filter paper), removed from

the filter paper and ignited in a platinum crucible inclosed in one of porcelain to prevent access of reducing gases.

The zinc sulphate was also purified by electrolysis.³⁴ After removal of iron, as described above, a weak current (about 0.1 ampere per square decimeter) was passed through a nearly saturated solution containing suspended zinc oxide to keep it slightly basic. Platinum electrodes were used and the solution stirred continuously. The electrolysis was continued for several days until a clean anode no longer became coated with lead peroxide. The solution was sufficiently pure, though it still contained traces of other metals. It was filtered, acidified slightly, and the salt twice crystallized by evaporating to a small volume. The last crop of crystals was dissolved in a small quantity of water at room temperature and the solution allowed to evaporate spontaneously. Large, perfect crystals were obtained.

Cadmium sulphate.—The commercial chemically pure salt may contain zinc, lead, ferrous and ferric iron, and occasionally nickel. Several lots of the salt were purified by dissolving in an excess of water at about 70°, filtering when necessary, adding an excess of basic cadmium sulphate and a few cubic centimeters of hydrogen peroxide to oxidize any ferrous iron present and heating for some hours. The solution was then filtered, acidified slightly, and evaporated nearly at its boiling point in a large porcelain dish resting on a pipestem triangle on a hot plate or supported some distance above the flame of a gas stove. Even then the flakes of the lower hydrate of cadmium sulphate which were formed collected on the bottom and caused violent bumping unless they were frequently removed. The crystals were allowed to drain in a funnel with a platinum cone. When the solution had been evaporated to a small volume it was poured, while still hot, through the funnel, the crystalline flakes packed down with a pestle, allowed to cool, and washed twice with a little cold water, using suction. They were then redissolved and the solution evaporated down to a small volume, as before. After this operation the crystals were dissolved in a slight excess of water at room temperature. The solution was filtered when necessary, and then set aside in shallow layers (2 to 3 cm) in

³⁴ Mylius and Fromm: *Zs. f. anorg. Chem.*, 9, p. 144; 1895.

crystallizing dishes covered with filter paper. The solution should not be saturated when placed in the dishes, especially if it be a mother liquor from which a crop of crystals has been removed, for the salt is almost certain to come down in a few hours as a crust over the bottom instead of isolated crystals. At least three-fourths of the crystals were cloudy, but these were found to give the same results in the cells as the perfectly clear ones. By varying the rate of evaporation and by using acid, neutral and basic solutions, it was attempted to obtain a larger proportion of clear crystals, but without success. The latter, especially, adhered so firmly to the bottom of the dish that they were apt to be broken in removal. This was obviated by decanting the mother liquor and pouring a little pure water over the crystals. In a few moments they became loose, without going into solution to any great extent, and were easily removed with a spatula. They were washed two or three times with water and preserved air-dried in bottles.

Zinc and cadmium sulphate solutions.—The saturated zinc or cadmium sulphate solutions required for making up the paste and for filling the cells were prepared by agitating an excess of purified salt with distilled water. In the case of zinc sulphate shaking with water heated to not more than 35° for at least a half hour was sufficient, while for cadmium sulphate mechanical stirring for three or four hours was required on account of the slowness with which it dissolved.

Mercurous sulphate.—The chemically pure mercurous sulphate at present obtainable on the market may contain as impurities basic mercurous sulphate, basic mercuric sulphate, traces of nitrate, etc., according to the method of preparation. In addition the size of grain of the commercial samples, usually prepared by rapid precipitation, may also have an influence on its electromotive properties. Such materials can not, therefore, be directly employed if the highest accuracy of reproduction is sought. A considerable number of samples of mercurous sulphate was prepared by the following methods:

- (a) By electrolysis, old apparatus.
- (b) By electrolysis, new apparatus.
- (c) By the action of fuming sulphuric acid on mercury.
- (d) By the reaction between sulphuric acid and mercurous nitrate.

(e) By the action of a dilute solution of nitric acid in sulphuric acid on mercury.

(f) By the reduction of mercuric sulphate by mercury.

(g) By the reduction of mercuric sulphate by sulphurous acid.

(h) By the recrystallization of commercial mercurous sulphate from sulphuric acid.

(i, k, l, m, n) By digestion of commercial mercurous sulphate with sulphuric acid.

All the samples were prepared in subdued light and preserved in the dark, to protect them from the action of light. Even diffused daylight soon darkens the salt.

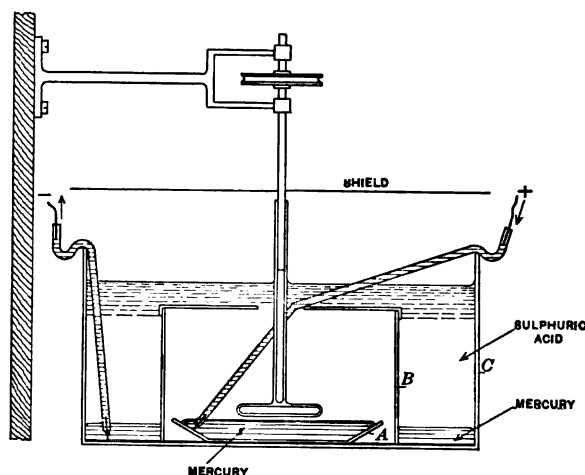


Fig. 1.—Apparatus for Electrolytic Preparation of Mercurous Sulphate (1904).

(a) *By electrolysis, old apparatus.*—In 1904 an electrolytic method of preparation was developed by one of the authors,³⁵ and also independently by Carhart and Hulett,³⁶ the mercurous sulphate being formed by the passage of a current at the surface of a mercury anode in sulphuric acid. Preliminary experiments, based on the reversibility of the Clark and Weston standard cells, as shown by the slight polarization with weak inverse currents, were made with cadmium sulphate solutions. Without stirring a crust of yellow basic mercurous sulphate formed on the surface of the mercury anode.

³⁵ Wolff: Trans. Amer. Electrochem. Soc., 5, p. 49; 1904.

³⁶ Ibid., 5, p. 59; 1904.

rous and mercuric sulphate was formed and the solution gave a very strong mercuric reaction, while with an acidified solution a beautiful crystalline product was obtained.

The anode mercury was contained in a shallow plate, *A* (fig. 1), resting on the bottom of a crystallizing dish, *B*, of slightly greater diameter, placed inside of a second larger crystallizing dish, *C*, which contained the cathode mercury. Both dishes were filled with dilute sulphuric acid, covering the inner dish to a depth of several centimeters. Electrical connections were made in the usual manner by means of glass tubes with platinum terminals fused into the lower ends and filled with mercury. To prevent the possible formation of basic compounds and mercuric sulphate, the solution near the surface of the anode was stirred continuously. In this way, and with a current density not greater than 0.25 ampere per square decimeter, the surface of the anode was kept in motion and the mercurous sulphate was swept over the edges of the plate as rapidly as it was

TABLE I.

Electrolytic Method, Old Apparatus.

Sample	Made	Conc. of H_2SO_4	Remarks
a_1	Jan., '04	5%	+5% $CdSO_4$
a_4^1	Mar., '04	2.5%	
a_5	Apr., '04	5%	} Slightly brownish on exposed surfaces Mar., '06
a_6	Apr., '04	5%	
a_7	May, '04	1:6	
a'_7	May, '04	1:6	a_7 digested with 25% H_2SO_4
a_8	May, '04	1:6	
a'_8	May, '04	1:6	From top of cover dish
a_9	June, '04	1:6	
a_{10}	Sept., '05	1:6	From inside dish
a'_{10}	Sept., '05	1:6	From outside dish
a_{11}	Sept., '05	1:6	From inside dish
a'_{11}	Sept., '05	1:6	From outside dish
a_{12}	Oct., '05	1:6	
a_{13}	Oct., '05	1:6	

¹Samples a_2 and a_3 not preserved.

formed. To prevent mechanical loss the inner dish was covered with a perforated crystallizing dish, through which the glass stirrer and anode connection passed. A metallic shield attached to the shaft of the stirrer prevented possible contamination by oil and particles of metal from the bearing.

Nine samples, of which seven were preserved, were made in 1904, and four others in 1905, using this form of apparatus, which was

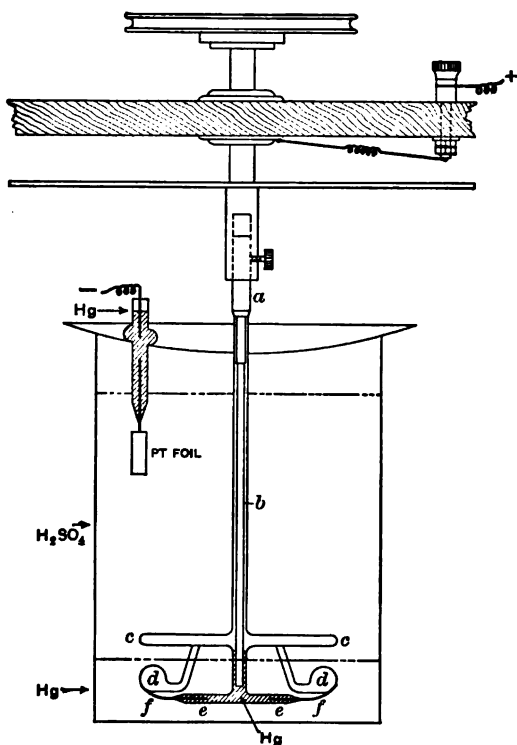


Fig. 2.—Apparatus for Electrolytic Preparation of Mercurous Sulphate (1905).

designed to give a white product free from finely divided mercury, as per Table I. The current density was kept below 0.25 ampere per square decimeter in the preparation of all the samples.

(b) *Electrolytic method, new apparatus.*—In the preparation of the remaining samples of electrolytic mercurous sulphate, the apparatus was so modified as to permit the use of greater current densities, which necessitated more efficient stirring. A layer of mercury 3 to 4 cm deep, purified as described above, was placed in a battery jar

about 12 cm in diameter and 20 cm in depth, and the jar nearly filled with dilute sulphuric acid. A piece of platinum foil, about 3 cm square, in the upper part of the solution served as cathode. The foil was welded to a short piece of platinum wire, the other end of which was sealed into the end of a short glass tube to serve as a mercury connection. A small bulb was blown about the middle of the tube, so that it rested in a hole through the glass cover of

the jar. When making samples (b_1) to (b_6) a simple glass T-stirrer revolving close to the surface of the mercury was used, connections to the mercury anode being made as shown in fig. 1. The surface of the mercury was soon covered with a dark coating, and in addition some of the mercurous sulphate accumulated around the anode tube. A second stirrer was then made by attaching near the ends of the cross arm short glass rods, flattened to form paddles, which dipped into the mercury, so that both the mercury and the electrolyte could be stirred vigorously (samples b_{10} , b_{11} , b_{12}). As this form of stirrer did not prevent the accumulation of mercurous sulphate around the anode tube, a combination stirrer and anode connection was constructed. The two cross arms (fig. 2), cc and ee were about 3 cm apart, and the lower arm had a short piece of platinum wire sealed into each end. The paddles, dd , made of glass rod 2 to 2.5 mm in diameter, flattened at the end, were sealed to the cross arm, cc , and the platinum terminals, ff , thus also making the construction more rigid and preventing the platinum wires from being broken off during cleaning. Electrical contact was made with the bearing of the rotating apparatus by means of a copper wire, b , soldered into the lower end of the brass cylinder, a , and extending into the cross tube, ee . The latter and the lower part of the stem were filled with mercury, and the soldered joint was protected from amalgamation by a coat of Khotinsky cement. The operation was watched as the speed of the stirrer, driven by an electric motor, decreased after the formation of sulphate began.

The best position of the stirrer is readily determined by trial, and is that position in which the mercury is vigorously rotated without being broken up into small globules. A speed of 100 to 200 revolutions per minute was employed.

The product formed was light to dark gray, from the presence of finely divided mercury, depending upon the current density, strength of acid, and rate of stirring employed, all of which were varied between wide limits without appreciably affecting the electromotive properties of the product. To minimize the possible influence of size of grain the stirring was always continued for some hours after interrupting the current.

The mercurous sulphate, separated from the excess of mercury, which interfered with its subsequent washing, by means of a sep-

arating funnel (using no grease on the stopcock) or by means of a pipette, was then transferred to a clean glass-stoppered bottle and preserved in the dark under part of the electrolyte employed. Further data on the samples prepared by the modified apparatus are given in the table below.

TABLE II.
Electrolytic Method, New Apparatus.

Sample	Made	Current Density	Conc. of H_2SO_4	Remarks
b ₁	Feb., '06	0.2 amp.	V=0.25	} Poor stirring
b ₂	"	0.2	0.50	
b ₃	"	0.2	0.75	
b ₄	"	0.2	0.25	
b ₅	"	0.2	0.50	
b ₆	"	0.2	0.75	
b ₇	"	0.5	0.25	
b ₈	"	0.5	0.50	
b ₉	"	0.5	0.75	
b ₁₀	"	1.0	0.25	
b ₁₁	"	1.0	0.50	
b ₁₂	"	1.0	0.75	
b ₁₃	"	5.0	0.50	
b ₁₄	Mar., '06	9.25	0.25	
b ₁₅	"	9.25	0.50	
b ₁₆	"	9.25	0.75	
b ₁₇	"	8	1	
b ₁₈	"	8	2	
b ₁₉	"	8	5	
b ₂₀	"	0.5	1	
b ₂₁	"	0.5	2	
b ₂₂	"	0.5	5	
b ₂₃	"	2	1	
b ₂₄	"	2	2	
b ₂₅	"	2	5	
b ₂₆	Dec., '06	5.25	1:6	
b ₂₇	"	5.25	1:6	
b ₂₈	"	5.25	1:6	
b ₂₉	Jan., '07	5.25	1:6	
b ₃₀	"	5.25	1:6	
b ₃₁	"	5.25	1:6	

(c) *By the action of fuming sulphuric acid on metallic mercury.*⁵⁷—Pure mercury was placed to a depth of about 3 mm in a porcelain dish and covered with four times its volume of fuming sulphuric acid, stirring from time to time and keeping the dish covered as far as possible on account of the acid fumes evolved. In spite of this the hood became filled with fumes. The action began in the cold but was greatly accelerated by the heat generated, so that the sulphate appeared in crystalline form after a few minutes. The operation was allowed to continue until action nearly stopped, when the reaction product was poured into a large beaker of 1:6 sulphuric acid, thoroughly stirred and allowed to settle. To avoid spattering, the beaker containing the dilute acid was covered with a perforated watch glass and the solution introduced through a narrow-stemmed funnel extending almost to the surface of the acid. The sulphuric acid was then decanted and the mercurous sulphate transferred with a little mercury and 1:6 sulphuric acid to a glass-stoppered bottle.

(d) *By the reaction between mercurous nitrate and sulphuric acid.*—Mercurous sulphate is ordinarily made by the reaction between mercurous nitrate and a soluble sulphate in order to avoid the presence of a large excess of free acid. For use in Clark and Weston cells precipitation by means of zinc or cadmium sulphate, respectively, has been recommended. It is, however, preferable to use comparatively strong sulphuric acid and to add the mercurous nitrate solution to it in order to avoid possible hydrolysis.

A concentrated solution of mercurous nitrate was prepared by the action of concentrated nitric acid on an *excess* of mercury.

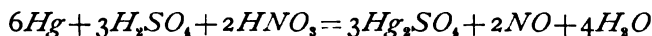
Sample d_1 was made by adding drop by drop to 1 liter of almost boiling sulphuric acid ($V=0.25$) 250 cc of the above mercurous nitrate solution, stirring continuously. The solution was kept hot by the aid of an electric stove. During the reaction, fumes of nitric acid were evolved. The product consisted of fairly large, pitted and irregular crystals, which were washed and preserved, as described above.

Sample d_2 was prepared by adding drop by drop 400 cc of the mercurous nitrate solution to a large excess of cold 1:4 sulphuric

⁵⁷ F. E. Smith: Electrician (Lond.), 55, 857; 1905.

acid continuously stirred with mercury in a large battery jar. The sample was then washed by decantation with acid of the same strength and preserved as described above. This is practically Smith's method *b*.³⁸ In both cases the stirring was continued over night to facilitate the reduction of any mercuric salt present.

(*e*) *By the action of dilute sulphuric acid, containing a small amount of nitric acid, on mercury.*—Six samples were made by this method, in which the preparation of mercurous nitrate is avoided. The reaction which takes place is that involved in the Lunge method for the estimation of nitrates, so that the nitric acid is almost completely eliminated from the solution.



The rate of the reaction depends on the concentration of the sulphuric acid, the amount of nitric acid present, and the temperature. The mercury is contained in a beaker and covered with sulphuric acid, the nitric acid added, and the whole stirred vigorously and continuously for some time after the disappearance of brown fumes of nitrogen peroxide. The product formed is quite gray from the presence of finely divided mercury, and after washing several times by decantation with 1:6 sulphuric acid it should be transferred to a glass-stoppered bottle with some of the acid. The conditions under which the various samples were formed are as follows:

Sample *e*₁ was obtained by vigorously stirring a solution consisting of 600 cc sulphuric acid (*V*=0.25) and 20 cc concentrated nitric acid with a considerable volume of pure mercury. After a few minutes, before starting the stirring, the mercurous sulphate began to separate out at the surface of the mercury. The beaker was then heated and the stirring begun. Shortly afterwards fumes of nitrogen peroxide began to appear, gradually increasing in volume, and finally disappearing. The heating and stirring were continued for an hour. The product, which settled rapidly, was in grayish aggregates of fairly regular crystals, as seen under the microscope.

³⁸ For the preparation of mercurous nitrate Smith recommends adding nitric acid until all the mercury has gone into solution. This is inadvisable owing to the formation of more or less mercuric nitrate. He also recommends that only enough mercurous nitrate solution to neutralize 30 per cent of the sulphuric acid should be added.

Sample e_2 was prepared at room temperature in a similar manner with a mixture of 800 cc 1:4 sulphuric acid and 10 cc nitric acid. The action was very much slower. No crystals appeared in two hours. The stirring was continued for thirty-six hours and a voluminous precipitate of fairly good crystals was obtained.

Sample e_3 was obtained at room temperature with 600 cc 1:2 sulphuric acid and 10 cc concentrated nitric acid. The action was much more rapid than in the previous case.

Sample e_4 was prepared by adding to 600 cc 1:4 sulphuric acid, previously heated almost to boiling, 2 cc concentrated nitric acid. There was no visible action after one hour, even after adding a small quantity of electrolytically prepared mercurous sulphate. On interrupting the stirring, however, the formation of the mercurous sulphate and nitrogen peroxide began almost immediately.

Sample e_5 was prepared in a similar manner, using 750 cc 1:4 sulphuric acid and 2 cc nitric acid. Crystals very irregular.

Sample e_6 was prepared in the same manner as the previous sample except that the solution was not heated and twice the quantity of nitric acid was used. The action was very slow in starting so that after waiting a considerable time the formation was started electrolytically by a current of 2 amperes for less than two minutes. The crystals were also very irregular.

Samples e_1 , e_2 , and e_3 were tested for nitric acid by brucine. The sulphuric acid in which the samples were prepared gave a mere trace for sample e_1 , a small quantity for sample e_2 , and a considerable quantity in sample e_3 , the latter two being prepared in the cold. The mercurous sulphate samples showed only mere traces of nitrate.

The same samples and the filtrates were also tested for ammonia by means of Nessler's solution, but the results were negative.

(f) *By reduction of mercuric sulphate by metallic mercury.*—Mercuric sulphate was prepared by adding 150 grams of pure mercury to 350 cc concentrated sulphuric acid, previously heated approximately to the boiling point. The heating was continued for some time after all the mercury had been dissolved, and the white product dissolved in 2.5 liters of 1:6 sulphuric acid, the solution remaining perfectly clear.

Sample f_1 was prepared by vigorously stirring 400 cc of the above solution, further diluted to about 2 liters with 1:6 sulphuric acid, with metallic mercury. The product formed was quite dark. The stirring was continued over night, and in the morning the acid showed no trace of mercuric sulphate.

Sample f_2 was prepared in a similar manner with 100 cc of the mercuric sulphate solution diluted with nine times its volume of a solution of sulphuric acid containing 200 grams per liter. After the action seemed completed an additional 100 cc of the mercuric sulphate solution was added and the stirring continued over night. No mercuric sulphate was found in the solution.

(g) *By reduction of mercuric sulphate with sulphurous acid.*—A considerable number of experiments were made under varying conditions on the action of sulphurous acid on mercuric sulphate. In most cases an approximately saturated solution of sulphurous acid, diluted with sulphuric acid and water so that the concentration of the sulphuric acid was at least 1:6, was introduced drop by drop into a similarly diluted mercuric sulphate solution and vigorously stirred. After some time the solution would become cloudy from the formation of mercurous sulphate and on interrupting the stirring the white product would collect at the bottom of the beaker. On the further addition of H_2SO_3 , the product would become gray. The crystals obtained were usually quite large and fairly good. Very different results were obtained by mixing the solutions quickly, the crystals being in the form of irregular plates; sometimes feathery or large prismatic crystals were obtained. They appeared cloudy when examined under the microscope. An excess of sulphurous acid should be avoided to prevent the further reduction to metallic mercury.

Sample g_1 was obtained by adding drop by drop a dilute solution of sulphurous acid in 1:6 sulphuric acid, to a similarly diluted solution of mercuric sulphate, previously heated and continuously stirred. The heating was continued for some hours and the stirring over night. The crystals formed were white and fairly regular.

Sample g_2 was prepared in a similar manner, but without heating the solution. The crystals obtained were white, but much channeled.

Sample *g*₃. Twenty-five grams of chemically pure mercuric sulphate were dissolved in one liter of 1:6 sulphuric acid and reduced by means of a slow current of sulphur dioxide, mixed with air, the solution being kept at 80°. It was then digested in a covered beaker for two days at the same temperature.

METHODS OF TREATMENT OF COMMERCIAL SAMPLES.

In the earlier stages of the work efforts were made to obtain the mercurous sulphate in the form of perfect crystals. It was first noticed that the character of commercial samples was very considerably modified by digesting them with 1:6 sulphuric acid on the steam bath. The commercial samples were very fine-grained, and when examined under the microscope showed hardly any regular crystalline structure with the magnification employed, while, after digesting, crystals of considerable size and some regularity were obtained. This suggested the possibility of obtaining perfect crystals by slow recrystallization from strong sulphuric acid. On slowly cooling long crystals of an acid sulphate were formed, which became whitish and opaque when treated with water, alcohol, anhydrous ether or even concentrated sulphuric acid, after they had been filtered by suction. When the solution was rapidly cooled fairly good crystals began to separate out when the solution was still quite warm.

Experiments were next made to precipitate the mercurous sulphate from its solution in strong sulphuric acid by gradual dilution with sulphuric acid of various concentrations not less than 1:6.

(*h*) *By crystallization from strong sulphuric acid.*—Sample *h*₁ was prepared as follows:

Concentrated sulphuric acid was heated to 150° and an excess of commercial mercurous sulphate added. Three hundred cc of the cloudy liquid was transferred to a beaker and stirred. It did not clear on adding fuming sulphuric acid. Three hundred cc of 1:2 sulphuric acid was added drop by drop. Then about 200 cc of 1:3 and finally 500 cc of 1:6 sulphuric acid were added in the same manner. The precipitated mercurous sulphate was in comparatively large crystals, which were, however, much channeled.

The warm mother liquor was decanted and divided into equal portions. To one was added an equal volume of hot water. Both

lots yielded crystals on cooling, those from the diluted portion being very perfect.

Sample h_1 was prepared by dissolving 50 grams of Kahlbaum mercurous sulphate in 250 cc of concentrated sulphuric acid previously heated to 150° . To the clear liquid was added an excess of mercury and 300 cc concentrated sulphuric acid. This was diluted by adding drop by drop 600 cc 1:2, 300 cc 1:3, and finally 1,500 cc 1:6 acid. After all the dilute acid had been introduced, it was rapidly cooled to room temperature, the mother-liquor decanted, and the mercurous sulphate washed twice with 1:6 sulphuric acid.

(h_2) Smith³⁹ has also described a method for preparing mercurous sulphate by recrystallization from sulphuric acid. This sample was made by closely following his specifications. Chemically pure mercurous sulphate as purchased was warmed in a covered evaporating dish with pure concentrated sulphuric acid and a small quantity of mercury to a temperature of about 150° for about ten minutes, the mixture being kept well stirred. After settling, the clear solution was poured into 1:6 sulphuric acid, when crystalline mercurous sulphate separated out. About ten times the bulk of dilute acid was employed and, to avoid spitting, the hot liquid was poured through a funnel having its stem immersed in the dilute acid. The mixture was well stirred, and after cooling the sulphuric acid was decanted and the mercurous sulphate was transferred to a glass-stoppered bottle.

(i, k, l, m, n) *By digestion with sulphuric acid.*—Sample i_1 was prepared by heating mercurous sulphate (Kahlbaum B) with mercury and concentrated sulphuric acid to 150° for some hours. Compact crystals were obtained when the solution was rapidly cooled.

Sample i_2 was one of a large number prepared by heating mercurous sulphate (Kahlbaum B) with strong sulphuric acid.

Further experiments were made to study the influence of digesting commercial samples of mercurous sulphate with dilute sulphuric acid under various conditions. Six samples purchased as chemically pure were available. Three of these were obtained from Kahlbaum at different times and the others from Merck, Schuchardt, and Gehe. Twenty-five grams of each of these six samples were digested with

³⁹F. E. Smith: *Electrician* (Lond.), 55, p. 857; 1905.

hot 1:4 sulphuric acid and mercury in March, 1906. The samples were ground in an agate mortar with the acid and transferred to a 1.5-liter beaker and vigorously stirred and heated to about 90° for several hours. The stirring was continued until the next morning. The Merck sample at first showed a decidedly yellowish color which disappeared during the night except above the level of the liquid.

In some cases the beakers were sheathed with asbestos and covered with watch glasses to reduce the formation of the yellow incrustation due to the condensation of water on the sides of the beaker and its action on the mercurous sulphate. The effect was further reduced in one case by blowing air into the beaker through a hole in the cover glass to prevent condensation, and by increasing the speed of the stirrer from time to time to sweep back any mercurous sulphate into the body of the liquid.

In the tables below these samples are designated as follows:

Source	Designation
Kahlbaum A	k ₁
Kahlbaum B	k ₂
Kahlbaum C	k ₃
Merck	k ₄
Schuchardt	k ₅
Gehe	k ₆

Five of the commercial samples were digested with 25 per cent sulphuric acid and four of them with 10 per cent acid since April, 1904. All six samples were digested with 1:6 acid and mercury since October, 1905, the bottles being shaken from time to time. In the table below these samples are designated as follows:

Treatment and Designation.			
Sample	25% acid	10% acid	1:6 acid
Kahlbaum A	1 ₁	m ₁	n ₁
Kahlbaum B	1 ₂	--	n ₂
Kahlbaum C	--	--	n ₃
Merck	1 ₃	m ₂	n ₄
Schuchardt	1 ₄	m ₃	n ₅
Gehe	1 ₅	m ₄	n ₆

Samples 1₁ to 1₅ were vigorously stirred (April, 1906) with mercury and acid for twenty-four hours and then preserved in bottles with mercury.

The paste.—This was always freshly prepared before setting up the cells. It is obvious from the irregular pitted character of the mercurous sulphate crystals, as shown under the microscope, that the greatest care must be taken in washing the sample to completely remove all traces of acid. This was done in a platinum Gooch crucible with a disk of hardened filter paper at the bottom. These disks were cut to size with a cork borer, warmed for some time with dilute nitric acid, washed until acid-free with hot distilled water and dried. A sufficient quantity of the mercurous sulphate, shaken up with the acid under which it was preserved, was poured into the crucible, care being taken not to transfer any of the mercury which interferes with the washing, the acid removed by suction, and the salt washed twice with small portions of 1:6 sulphuric acid to remove possible traces of mercuric sulphate formed by the action of air on particles of mercurous sulphate not covered by the acid. The acid was removed by the addition of 5 or 6 portions of redistilled absolute alcohol, care being taken to wash down the sides of the crucible. To completely remove the alcohol the sulphate was then washed three or four times with a saturated solution of zinc or cadmium sulphate, according to the type of cell to be set up, taking the same precautions as above. If the paste cracked or separated from the sides of the crucible more of the wash liquid was added and the contents of the crucible thoroughly stirred up before again applying suction. After scraping off the upper layer the mercurous sulphate was transferred to a small, clean, dry beaker, or crucible, mixed with one-third to one-half its volume of finely powdered zinc or cadmium sulphate crystals and sufficient saturated zinc or cadmium sulphate solution to make a moderately thin paste. A large excess of zinc or cadmium sulphate crystals in the paste should be avoided so that practically every part of the mercury surface will be in contact with the mercurous sulphate, thus securing the rapid attainment of saturation equilibrium. With white samples of mercurous sulphate, one-third the volume of mercury was added with the crystals of zinc or cadmium sulphate in making up the paste. To eliminate possible influence of size of grain the paste was stirred as little as possible in its preparation.

THE CELLS.

For facility in filling and sealing the H type was adopted. The size and dimensions, although not affecting the electromotive force determine the polarization produced by the passage of a current and the rapidity with which the cell assumes the temperature of its surroundings. Figure 3, drawn to half scale, gives the approximate dimensions of the cells used at the Bureau. Especial care was taken in sealing in the platinum wires and subsequent annealing. As recommended by Hulett, the platinum wire inside the cell was covered in most cases with a thin layer of glass to within 1 mm or

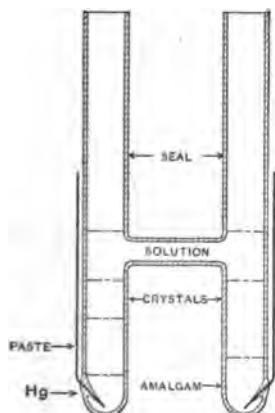


Fig. 3.

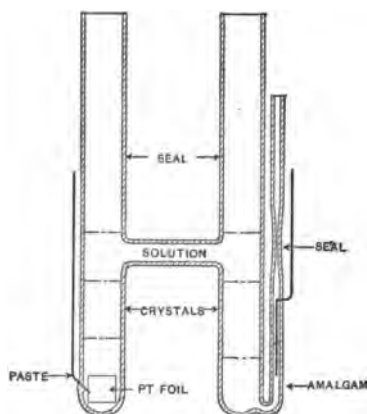


Fig. 4.

Sectional Views of Standard Cells (Half size).

less from the end. The wire should not be larger than B. & S. No. 28, to reduce the liability of cracking. No. 32 was used in most of our cells.

The Clark cells nevertheless frequently cracked at the point where the platinum terminal was sealed into the amalgam limb. This may be avoided by a construction recommended by the Reichsanstalt and shown in figure 4, in which the platinum terminal of the amalgam limb is sealed into a side tube while the amalgam is still liquid, contact being made by sucking the amalgam up into this tube. The platinum wire extends downward about 2 cm below the

point at which it is sealed into the tube and the amalgam should cover only its lower half, thus preventing the glass from cracking at the seal. This construction also reduces the chance of contact between the platinum and the electrolyte, and for this reason is preferable for portable cells. This form was adopted, for Clark cells, in our later work.

As the cells are to be sealed off above the cross arm after filling, they may be slightly constricted beforehand, but if the wall thickness is not greater than 0.75 mm, as was the case with our cells, this is not necessary. The length of the tube above the cross arm as shown in figures 3 and 4, although greater than absolutely necessary, facilitates the sealing but at the same time somewhat increases the difficulty of filling.

Cleaning the cell.—The cells and filling tubes when badly contaminated with grease, etc., were covered with chromic acid mixture and allowed to stand overnight. Longer contact with this cleaning mixture was avoided on account of the danger of forming lead chromate from the lead sealing-in glass employed.⁴⁰ The cells were then washed with distilled water, filled with aqua regia, which was allowed to remain in them for thirty minutes, and repeatedly washed with distilled water. The action of the aqua regia on the platinum wires facilitated subsequent amalgamation. Ordinarily the cleaning with chromic acid mixture could be omitted.

Amalgamation of the platinum terminals.—In order to minimize the possibility of contact between the electrolyte and the platinum terminals, especially from shaking in transport, they were amalgamated with a solution of pure mercurous nitrate in dilute nitric acid. The amalgamating solution was introduced into the cell and a weak current passed through it from a platinum wire anode to the platinum terminal externally connected to the negative pole of a battery. Sufficient mercury was deposited in a few minutes. In a type of portable cell to be described later, the limb intended to receive the paste was provided with a platinum foil electrode welded to the platinum wire. (Fig. 4.) In this case the whole surface was

⁴⁰ This was observed in one lot of cells that had been overlooked and had been left in the cleaning fluid for several weeks. The chromate was not removed by treatment for several days with nitric acid or aqua regia.

thoroughly amalgamated, for which a proportionately longer time was required. Especial care was taken as the amalgamated foil was employed to replace the mercury.

To remove every trace of the amalgamating solution the cell was next washed several times with dilute nitric acid, and finally repeatedly rinsed with distilled water. The amalgamated surface was usually "rinsed" with a small quantity of pure mercury, the cell then dried in an air bath at 110°.

Introduction of the materials.—The materials were easily and neatly introduced by means of filling tubes, care being taken not to allow the latter to come in contact with the walls of the cell.

The amalgam.—The zinc or cadmium amalgam, prepared as described above, was heated slightly above its melting point and a quantity of it sufficient to cover the platinum terminal to a depth of at least 10 mm transferred to the cell by means of a previously heated, clean, dry pipette. A pipette with a rounded end, at the center of which is a small hole, was found more satisfactory than one drawn out to a capillary, which the amalgam tended to clog up owing to cooling. After heating the pipette, which was provided at the upper end with a rubber tube, it was introduced below the surface of the amalgam, meanwhile blowing through it, to avoid the introduction of any oxide formed on the surface. A suitable amount of amalgam was then drawn into the pipette by gentle suction, which was released while the pipette was withdrawn, and then applied again sufficiently to prevent any more of the amalgam from running out. Particles of the amalgam adhering to the outside were then removed by wiping with filter paper and the pipette introduced into the cell to within 2 cm of the bottom. On releasing the suction the amalgam ran out freely, the amount introduced being regulated by again applying suction at the proper moment. The pipette was then withdrawn without touching the walls of the cell. In our earlier work a thin glass tube of slightly less diameter than the cell was used to protect the walls but after some practice this was found to be unnecessary.

Particles of amalgam which in spite of precautions occasionally adhered to the cell wall, and which could not exert any influence on

the electromotive force except when forming an integral part of the amalgam electrode and not fully covered with crystals, were removed by the aid of a glass rod. Great care was taken to prevent them from reaching the other limb and contaminating the mercury, which would of course seriously influence the electromotive force.

Introduction of the mercury.—At first the mercury was introduced by means of a pipette drawn out to a very fine capillary, but later a sufficient quantity to cover the platinum terminal to a depth of least 10 mm was simply poured into the cell, care being taken not to introduce any into the amalgam limb. By cautiously tilting the cell, air bubbles entrapped under the mercury could easily be removed.

Introduction of the paste.—The paste, prepared immediately before setting up the cell as described above, was introduced by means of a pipette with an opening 3 to 4 mm in diameter. Its consistency was such that it could readily be drawn up into and flow from the pipette; and therefore finely crushed zinc sulphate or cadmium sulphate crystals were employed. After filling the pipette with paste it was wiped with clean filter paper, and a small amount of the paste allowed to flow out. The end of the pipette was then brought close to the mercury and paste allowed to flow into the limb, to a depth of 1.5 to 2 cm, except in the cells first set up, care being taken to keep the walls of the cell clean and to avoid trapping air bubbles.

Introduction of the crystals and solution.—The amalgam and paste were next covered to a depth of about 1 cm with *saturated* zinc sulphate or cadmium sulphate solution, after which pulverized zinc sulphate or cadmium sulphate was introduced—a little at a time to avoid trapping air bubbles—to a depth of 1.5 to 2 cm by means of a wide-stemmed funnel. The cell was then filled slightly above the cross arm with *saturated* zinc sulphate or cadmium sulphate solution.

In order to maintain a concentration equilibrium at all temperatures to which the cell was likely to be exposed, a saturated solution of zinc or cadmium sulphate and a sufficient excess of the corresponding crystals over the amalgam and in the paste were employed. The layer of crystals in the mercury limb was intended as a further

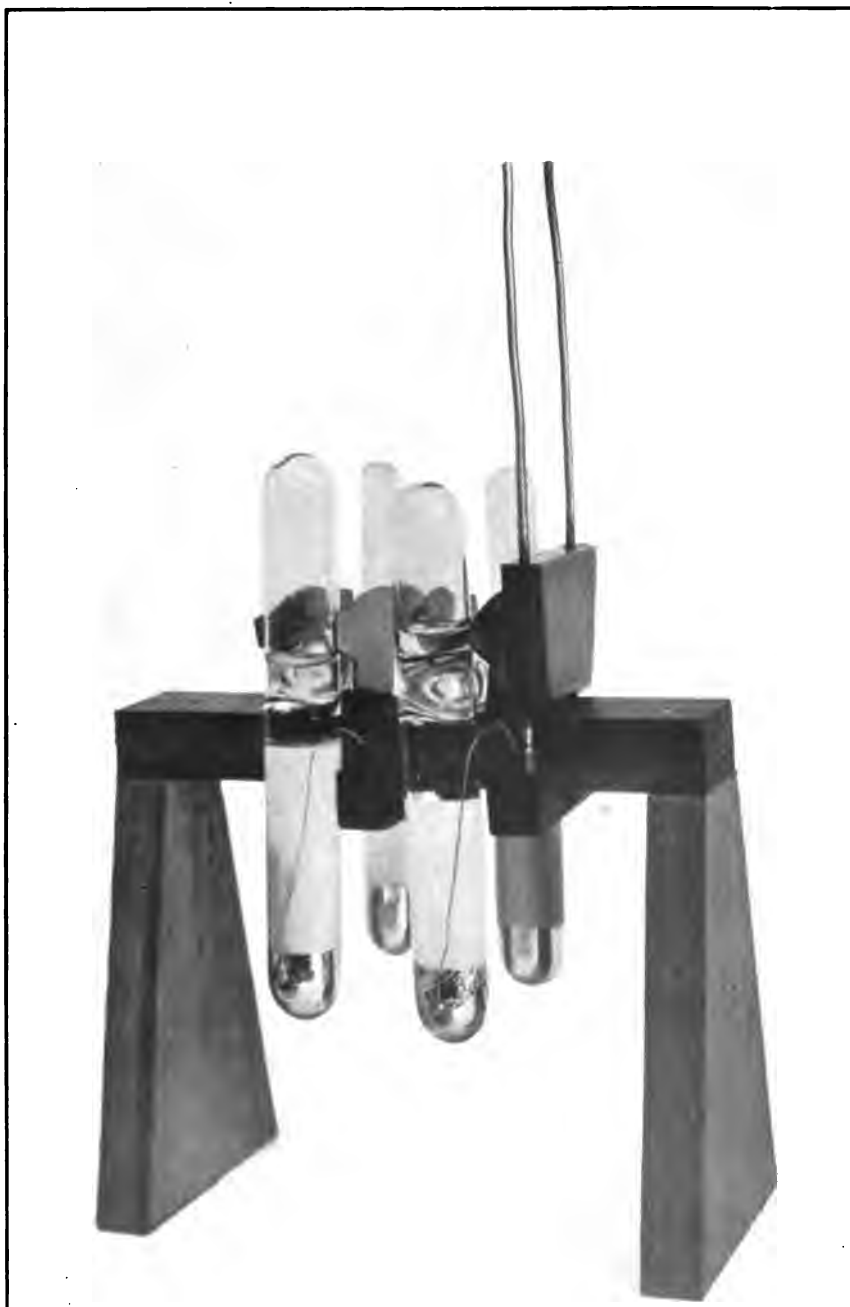


Fig. 5.—*Finished Cells showing Traveling Contacts and Method of Mounting.*

precaution against their leaching out of the paste, which may happen when the solution is not completely saturated.

*Sealing.*⁴¹—The cell was next hermetically sealed by the aid of two small horizontal blow-pipe flames applied to the cell wall from opposite directions. One limb of the cell was closed by a cork and the other by a cork nicked at one side, through which passed a glass rod to serve as a handle in drawing it out. The cell was gradually heated 2 to 3 cm above the level of the liquid, until the danger of cracking had passed, and then held in the flame, rotating meanwhile, until the tube almost collapsed, after which it was closed by drawing it out slowly while still in the flame. By judicious heating the seal could be nicely rounded by the expansion of the inclosed air. The second limb was sealed in a similar manner. The finished cell is shown in figure 5. Whenever, in filling the cell, the materials accidentally came in contact with the cell wall above the cross arm, where it was to be heated in sealing, it was cleaned by first wiping with filter paper slightly moistened with distilled water and then with dry paper.

THE COMPARING BATHS.

The relatively large temperature coefficient of the Clark cells made it necessary to provide for automatically regulating their temperature to within 0.01° . With old cells this regulation must be maintained for twenty-four hours or more previous to measurements, on account of the hysteresis frequently observed. All comparisons were therefore made in electrically heated and controlled kerosene baths.

The baths, 70 cm in diameter and 40 cm deep, were set up in a room in the basement of the laboratory, the temperature of which was automatically controlled to within 1° to 2° for most of the year

⁴¹Wright and Thompson: *Phil. Mag.* (5), **16**, p. 28; 1883.

Kahle: *Zs. f. Instrk.*, **13**, p. 298; 1893.

Callendar and Barnes: *Proc. Roy. Soc.*, **62**, p. 117; 1897.

Barnes: *Phys. Rev.*, **10**, p. 268; 1900.

Carhart and Hulett: *Trans. Am. Electrochem. Soc.*, **5**, p. 67; 1904.

by means of a thermostat pneumatically operating dampers in the hot and tempered air supply ducts. As there were no windows, the cells were also protected against any possible influence of direct or even diffused sun light.

The construction of the thermoregulator employed in the baths, although not embodying any essentially new features, is described

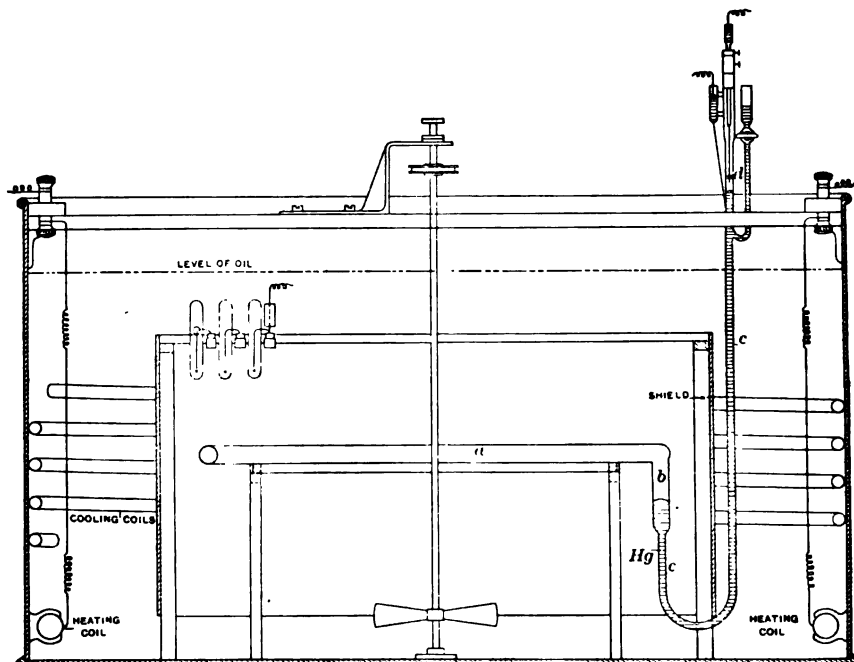


Fig. 6.—Comparing bath (sectional view).

rather fully below for the convenience of others who may wish to undertake standard cell work.

It consists of a reservoir in the form of a horizontal grid made of thin-walled glass tubing, 1.5 to 2 cm in diameter, containing the thermometric substance (toluene), the volume changes being transmitted by a U-shaped connecting tube, *c*, filled with mercury to a vertical capillary, *d*. (Figs. 6 and 7.) The reservoir, *a*, is provided with a downward extension, *b*, to allow for expansion and contraction of the toluene when the regulator is used at widely differing

temperatures, thus preventing the toluene from reaching the capillary. In filling, all air must be removed to eliminate the influence of variations in atmospheric pressure on the regulation. The capillary, *d*, 1 to 2 mm in diameter, is fused to a wider tube, *e* (fig. 7), to the upper end of which is cemented a bushing, *f*, fitting loosely enough to allow for the cement. The steel pin, *g*, with its upper end hollowed out to serve as a mercury cup, and passing through *f*, is fixed in position by the set screw, *h*, and has soldered to its long end a fine-pointed platinum wire, *i*. Five millimeters from the free end of the wire is fused a glass bead, *k*, to facilitate centering, but fitting the capillary loosely enough to permit the ready passage of mercury in either direction. It is advisable to have on the rod, *g*, a small steel collar, *p*, provided with a set screw to facilitate resetting after cleaning the capillary, which must be done occasionally. One end of a second platinum wire, *l*, is fused into the tube, *c*, below the capillary, and the other end into a small glass tube, *m*, attached to *e* by two short glass rods, and serving as a terminal cup.

The side tube, *n*, connected to *c*, as shown, to avoid trapping air bubbles when for any reason the bath may be allowed to cool considerably, is provided with a well-fitting stop-cock communicating with the small mercury reservoir, *o*.

The bath is brought approximately to the temperature at which it is to be regulated and sufficient mercury admitted or withdrawn (by gentle suction applied by means of a rubber tube) so as to bring the mercury level near the middle of the capillary. The pin, *g*, is then adjusted so as to make contact with the mercury when the desired temperature is reached, thus energizing a 50-ohm relay and increasing the external resistance in series with

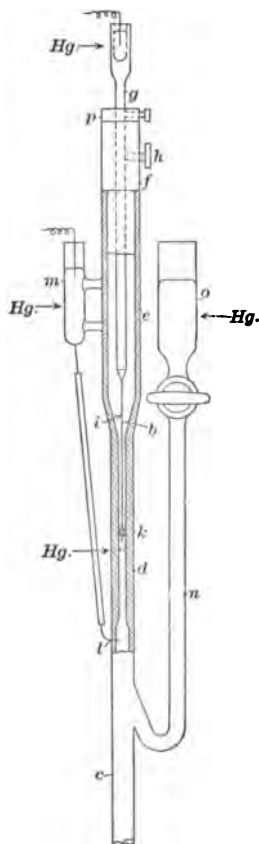


Fig. 7.—Thermoregulator.

the heating coils in the bath. Although the measurements here dealt with were all made at 25° , supplementary cooling had to be provided during the summer months, heat being abstracted by means of water circulation through a copper coil in the bath. The inlet and outlet of the coil were below the surface of the oil to prevent condensed moisture from getting into the bath. The flow of water was regulated so that somewhat more than enough cooling was effected, the difference being made up by electric heating. Owing to the variable temperature of the tap water, it was first passed through a coil in an ice chest.

In two of the baths the heating coils (Figs. 6 and 8) consist of four Ward-Leonard rheostat units, of advance wire wound in enamel on fire-clay tubes. These were mounted one in each quadrant by metal clips attached to the walls of the tank, external to the space occupied by the cells. In the third bath a grid built up of glass rods wound with bare advance wire and then shellacked was employed. With a 110-volt supply, a resistance of 80 to 125 ohms provided ample heating.

The external resistance, and if necessary the supplementary ice-water circulation, were adjusted so as to maintain the bath about 2° below the desired temperature, so that the additional heat supply required to produce regulation was as small as consistent with variations called for by changes in the room temperature, the effect of which was considerably reduced by covering the baths externally with sheet asbestos.

A most convenient and inexpensive form of rheostat, where a supply of incandescent lamps of various candlepowers is available, is shown in figure 8. The lamps in the two lower sockets determine the external resistance during the cooling period. When the relay circuit is open all the lamps are in parallel.

Stirring.—Uniformity of temperature throughout the bath was secured by an efficient centrally located stirrer producing an upward and rotary circulation. In addition the rectangular stand on which the cells were mounted was provided with a sheet metal shield extending to within 4 cm of the bottom of the bath (Fig. 6), thus directing the circulation and in addition protecting the cells from the direct influence of the cooling coil, and also in two of the tanks from the heating coils. Measurements made by a platinum ther-

mometer showed that the temperature of the space occupied by the cells was everywhere uniform to well within 0.01° , so that it was

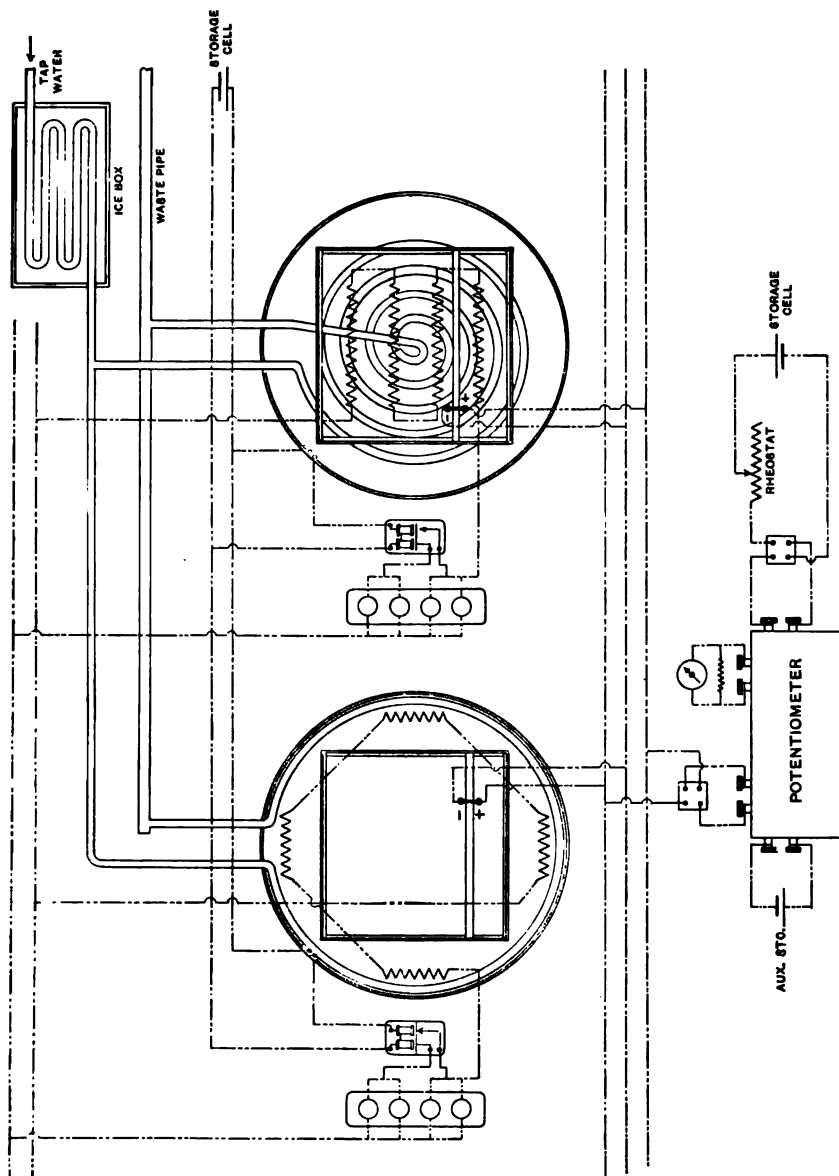


Fig. 8.—Plan View of Comparing Baths and Diagram of Connections.

hardly necessary to employ a grid thermoregulator to integrate the temperature of the bath.

The temperature variation during regulation is of course determined primarily by the volume and differential cubical expansion of the liquid employed in the thermostat and the diameter of the capillary. It is influenced, also, by temperature lag in the regulator and in the heating coils and by any contamination of the mercury surface at which contact is made. The lag is reduced by the form of regulator employed and by a proper adjustment of the external resistance, so that a minimum increase in the heating current is required for regulation.

The actual range can be calculated as follows:

Let W_1 = rate of heat loss in watts during cooling period

W_2 = added heating in watts for regulation

t_1 = cooling period in seconds

t_2 = heating " " "

Then

$$W_1 t_1 = (W_2 - W_1) t_2,$$

or

$$W_1 = W_2 \frac{t_2}{t_1 + t_2}.$$

Hence W_1 can be calculated.

The total heat loss in joules during the cooling period or gain during the heating period will therefore be

$$W_1 t_1 = W_2 \frac{t_1 t_2}{t_1 + t_2}.$$

The temperature range may then be found by dividing this quantity by the calculated heat capacity of the bath expressed in the same units.

The results thus arrived at are in good agreement with direct measurements with a platinum thermometer.

The cells were mounted (Figs. 6 and 8) on six parallel wooden strips, each accommodating 18, and held in place by sheet metal clips. The strips were supported by a metal framework, so that the electrodes of the cells were 25 cm above the bottom of the tank and 3 or 4 cm above the level of the grid of the thermoregulator. The cells were covered to a depth of at least 2 cm with high-grade kerosene.

Between the cells were mounted hard rubber blocks, each carrying two short copper rods, provided with a pair of holes for mercury

cups, one for a cell terminal and the other for external connections. The holes for the latter purpose were spaced at the same distance apart, so that any cell could be put in circuit by inserting a pair of stiff contact wires mounted on a hard rubber block. (Fig. 5.) This method was preferred on account of possible leakage troubles which might arise by connecting together all the positive or negative terminals and to facilitate the comparison of any number of cells in series.

Connections to the potentiometer were made by the aid of three insulated line wires extending over the baths and to which the leads from the contact wires and the potentiometer terminals could be connected in any desired manner by the aid of metal clips. (Fig. 8.) By this means the measurements were so greatly facilitated that the cells could easily be compared to parts per million at the rate of 100 per hour.

THE ELECTRICAL MEASUREMENTS.

The measurements given below were all made by means of a 5-dial Wolff potentiometer of the usual construction having a resistance of 10,000 ohms per volt, the units of the tenth ohm dial corresponding therefore to 10 microvolts. The potentiometer current was furnished by a storage cell of ample capacity, which was continuously left in circuit and covered with sheet asbestos to reduce temperature variations. Under these conditions the current, regulated by a manganin rheostat, was sufficiently constant for all purposes.

A high sensibility D'Arsonval galvanometer, made by the Weston Electrical Instrument Company, was employed throughout. A difference of 10 microvolts produced a deflection of at least 1 mm, so that the values could be readily measured to microvolts, even with the high resistance potentiometer employed.

The comparison of cells of the same type was made by the differential method, each cell being measured in opposition to one of the lot taken as a standard of reference. As the differences measured were small, no errors are introduced by possible variations of the potentiometer current likely to occur during a complete set of measurements. From the differences thus measured the results could be expressed in terms of any particular cell or the mean value of a

number by applying a suitable correction. This method has an additional advantage, particularly with Clark cells when in the same bath, as all vary simultaneously by an approximately equal amount with the minor temperature variations during regulation.

As a further check on the constancy of both the Clark and Weston cells their ratio was determined from time to time, both by the direct method in which a number of the cells of each type were separately measured and by the differential method in which the difference between series of five Clark and seven Weston cells was determined. The differential method had the advantage that the potentiometer corrections became almost negligible, as did also errors due to variations in the potentiometer current. In addition, it was practicable to determine the average difference during a complete cycle of regulation. The result thus obtained gives the mean ratio of the cells concerned.

Temperature measurements.—The influence of errors in measuring the temperature enters only in determining the value of the Weston in terms of the Clark cell, as in most of the work all the Clark cells were in the same bath.

The accuracy required in the temperature measurements is determined by the temperature coefficient of the Clark cell, almost one-tenth per cent per degree, corresponding to 0.01° where an accuracy of one part in 100,000 is sought. Mercury thermometers standardized at the Bureau were employed throughout; some checks have, however, been made by a platinum thermometer. In the earlier stages of the work less care was devoted to the determination of the Clark-Weston ratio, as new cells of both types were being constantly set up, and a single thermometer was used. In a redetermination of the temperature coefficients shortly to be made both mercury and platinum resistance thermometers will be employed.

Other possible sources of error.—The corrections of the potentiometer employed were determined from time to time to an accuracy of at least 1 part in 100,000. The close agreement of the values of Weston in terms of Clark cells obtained by the direct and differential methods (Table XVI) shows that no errors larger than the above amount can be thus introduced. The agreement also shows no appreciable errors are introduced by defective insulation which would affect the terminal potentials measured unequally. Measurements were, however, made of all insulation resistances, including

the resistance between cell terminals in the bath on a blank cell with the cross arm sealed off, but in every case the resistances were found to be so large as not to introduce an error as great as 1 part in 100,000 in the results. This was surprising, since the oil in two of the baths had been contaminated by the breakage of cells.

Thermoelectromotive forces, always very small, were eliminated in the usual manner by a simultaneous reversal of the potentiometer current and the connections of the cell circuit. The very small variations in the potentiometer current, not affecting the measurement of the differences between cells of the same type and in the differential method were practically eliminated in the direct comparison of Clark and Weston cells by a repetition of the observations in reverse order. The possible influence of hysteresis was practically eliminated by very close regulation of the baths.

TABULATION OF RESULTS.

The following tables give the results of a few of the numerous series of measurements on cells set up to test the electromotive properties of mercurous sulphate. A considerable number of other cells have been constructed for testing the other materials, different methods of washing the mercurous sulphate, influence of depth of paste, size of grain, and other possible causes of variation. A discussion of the results obtained, together with a study of the influence of added impurities, will be reserved for a later paper, as the mercurous sulphate has been found mainly, if not entirely, responsible for the variations observed.

In Tables III to XV, below, the first column gives the number of the cell, the second the date of setting up, the third the designation of the mercurous sulphate employed, the letter indicating the method of preparation and the subscript the number of the sample (see text); column four shows which lot of cadmium or zinc sulphate was employed, and column five gives similar information in regard to the amalgams. In the remaining columns are given the differences in microvolts between the individual cells and the mean of a number arbitrarily selected as a basis of reference.⁴² The three undated columns, marked *, **, and *** give the differences within one, two, and seven days, respectively, after the date of setting up the cells.

⁴² Weston cells Nos. 1, 2, 3, 4, 5, 6, 8, 9, 11, 12, 14, and 15, and Clark cells Nos. 27a, 28, 32, 34, and 39, data for which are given in heavy type in the tables.

TABLE III.

Weston Cells.—Electrolytic Mercurous Sulphate made with Old Apparatus.

[Differences in Microvolts from Mean of Reference Cells.]

Cell	Date	Mer- curous sul- phate	CdSO ₄	Cd Amal- gam	*	**	***	July 5, 1906	July 20	Aug. 4	Aug. 21
75	June 26, '06	a ₁	a ₁	b	+ 86	+ 77	+ 56	+ 57	+ 54	+ 47	+ 39
76	"	a ₄	a ₁	b	+ 61	+ 56	+ 41	+ 39	+ 35	+ 34	+ 33
64	June 25, '06	a ₅	a ₁	b	+109	+106	+ 87	+ 92	+ 84	+ 78	+ 81
65	"	a ₆	a ₁	b	+ 87	+ 83	+ 66	+ 67	+ 63	+ 58	+ 55
8	May 16, '06	a ₇	a ₁	b				+ 13	+ 12	+ 14	+ 13
40	May 9, '06	a ₇	a ₁	b				+102	+ 83	+ 74	+ 68
8a	June 28, '07	a ₇	a ₂	b	+147	+117	+107				
66	June 25, '06	a ₇	a ₁	b	+ 59	+ 55	+ 38	+ 39	+ 31	+ 32	+ 28
9	May 16, '06	a ₈	a ₁	b				- 9	- 8	- 6	- 7
41	May 9, '06	a ₈	a ₁	b				+ 68	+ 50	+ 45	+ 41
9a	June 28, '07	a ₈	a ₂	b	+ 47	+ 35	+ 31				
67	June 25, '06	a ₈	a ₁	b	+ 33	+ 34	+ 23	+ 26	+ 24	+ 25	+ 23
11	May 16, '06	a ₉	a ₁	b				+ 4	+ 4	+ 4	+ 4
42	May 9, '06	a ₉	a ₁	b				+ 48	+ 34	+ 30	+ 29
11a	June 28, '07	a ₉	a ₂	b	+ 68	+ 50	+ 46				
68	June 25, '06	a ₁₀	a ₁	b	+ 26	+ 25	+ 11	+ 9	+ 9	+ 9	+ 7
70	June 26, '06	a ₁₀	a ₁	b	+ 32	+ 29	+ 11	+ 14	+ 13	+ 8	+ 8
71	"	a ₁₁	a ₁	b	- 3	- 8	- 14	11	- 10	- 10	- 9
72	"	a ₁₁	a ₁	b	- 1	- 5	- 9	- 6	0	- 3	
73	"	a ₁₂	a ₁	b	+ 19	+ 13	+ 5	+ 9	+ 9	+ 4	+ 6
74	"	a ₁₂	a ₁	b	- 17	- 17	17	- 17	16	- 15	- 21

TABLE III.

Weston Cells.—Electrolytic Mercurous Sulphate made with Old Apparatus.

[Differences in Microvolts from Mean of Reference Cells.]

Cell	Sept. 13	Oct. 13	Nov. 2	Nov. 19	Dec. 1	Jan. 10, 1907	Mar. 1	Mar. 25	Apr. 24	May 24	June 26	July 23
75	+ 40	+ 29	+ 25	+ 21	+ 20	+ 11	+ 2	0	- 7	- 5	- 16	- 18
76	+ 27	+ 24	+ 19	+ 18	+ 16	+ 7	- 1	- 2	- 10	- 11	- 17	- 18
64	+ 73	+ 71	+ 68	+ 66	+ 62	+ 59	+ 50	+ 48	+ 41	+ 38	+ 32	+ 28
65	+ 40	+ 47	+ 46	+ 44	+ 43	+ 36	+ 38	+ 37	+ 34	+ 35	+ 31	+ 30
8	+ 12	+ 12	+ 11	+ 10	+ 12	+ 7	+ 11	+ 11	+ 9	+ 9	+ 8	+ 8
40	+ 65	+ 63	+ 56	+ 49	+ 46	+ 35	+ 24	+ 18	+ 10	+ 3	- 9	- 14
8a											+147	+ 65
66	+ 25	+ 24	+ 21	+ 22	+ 19	+ 18	+ 18	+ 19	+ 14	+ 14	+ 10	+ 10
9	+ 2	- 5	- 6	- 4	- 7	(-29) ¹						
41	+ 44	+ 48	+ 47	+ 38	+ 36	+ 27	+ 33	+ 33	+ 30	+ 29	+ 22	+ 23
9a											+ 47	+ 15
67	+ 20	+ 20	+ 20	+ 20	+ 18	+ 16	+ 18	+ 15	+ 15	+ 14	+ 12	+ 15
11	+ 3	+ 3	+ 3	+ 2	+ 1	+ 2	+ 1	+ 3	+ 1	+ 2	0	0
42	+ 30	+ 33	+ 31	+ 25	+ 26	+ 24	+ 25	+ 25	(1)			
11a											+ 68	+ 23
68	+ 3	+ 2	- 1	- 3	- 6	- 9		- 13	- 16	- 17	- 19	- 19
70	+ 3	0	0	+ 1	- 3	- 4	- 7	- 7	- 9	- 7	- 9	- 11
71	- 11	- 11	- 10	- 10	- 10	- 13	- 14	- 12	- 16	- 15	- 17	- 15
72					- 5	- 6	- 9	- 10	- 14	- 13	- 13	- 15
73	+ 2	+ 3	+ 1	+ 2	+ 1	- 1	0	- 1	- 3	0	0	- 5
74	- 20	- 22	- 26	- 23	- 18	- 20	- 23	- 24	- 19	- 20	- 22	- 22

¹ Oil in cell; seal found defective.

TABLE IV.

Weston Cells.—Electrolytic Mercurous Sulphate made with New Apparatus.

Concentration of Sulphuric Acid exceeding Gram-Molecular.

[Differences in Microvolts from Mean of Reference Cells.]

Cell	Date	Mer- curous sul- phate	CdSO ₄	Cd Amal- gam	*	**	***	July 5, 1906	July 20	Aug. 4	Aug. 21
26	May 9, '06	b ₁	a ₁	b				+ 24	+ 14	+ 12	+ 10
51	June 23, '06	b ₁	a ₁	b		- 6	- 8	- 12	- 10	- 10	- 9
27	May 9, '06	b ₂	a ₁	b				- 3	- 4	- 4	- 7
52	June 23, '06	b ₂	a ₁	b		+14	+28	+ 19	+ 14	+ 12	+ 9
28	May 9, '06	b ₃	a ₁	b				+ 35	+ 28	+ 22	+ 18
61	June 25, '06	b ₄	a ₁	b	+12	+12	+ 1	+ 4	+ 2	+ 1	+ 1
62	"	b ₅	a ₁	b	+ 7	+ 7	- 1	- 1	+ 1	0	0
63	"	b ₆	a ₁	b	- 1	0	-11	- 8	- 9	- 9	- 12
3	May 11, '06	b ₇	a ₁	b				+ 4	+ 4	+ 4	+ 3
3a	June 25, '07	b ₇	a ₂	b	+18	+ 4	+ 4				
29	May 9, '06	b ₇	a ₁	b				+ 21	+ 15	+ 12	+ 10
30	"	b ₈	a ₁	b				+ 12	+ 5	+ 3	+ 1
46	June 23, '06	b ₈	a ₁	b		+24	+18	+ 17	+ 18	+ 16	+ 17
46a	June 25, '06	b ₈	a ₁	b		- 6	-16	- 16	- 16	- 15	- 18
46b	June 25, '07	b ₈	a ₂	b	+29	+10	+ 9				
31	May 9, '06	b ₉	a ₁	b				+ 23	+ 16	+ 13	+ 10
47	June 23, '06	b ₉	a ₁	b				+ 2	+ 2	0	0
4	May 11, '06	b ₁₀	a ₁	b				- 7	- 6	- 4	- 6
4a	June 25, '07	b ₁₀	a ₂	b	+ 8	- 2	- 3				
32	May 9, '06	b ₁₀	a ₁	b				- 18	- 20	- 23	- 22
33	"	b ₁₁	a ₁	b				+ 7	+ 5	+ 3	+ 2
48	June 23, '06	b ₁₁	a ₁	b		- 2	- 6	- 7	- 9	- 10	- 8
34	May 9, '06	b ₁₂	a ₁	b				- 14	+ 10	+ 7	+ 7
49	June 23, '06	b ₁₂	a ₁	b		0	- 3	- 6	- 7	- 6	- 8
6	May 11, '06	b ₁₃	a ₁	b				-12	-12	- 10	-11
6a	Dec. 3, '06	b ₁₃	a ₂	a	- 7		-10				
6b	"	b ₁₃	a ₃	a	-10		- 9	Given to Prof. Hulett			
151	Dec. 15, '06	b ₁₄	a ₁	b	- 4	- 4	- 5				
5	May 11, '06	b ₁₄	a ₁	b				0	- 1	0	- 1
14	May 16, '06	b ₁₅	a ₁	b				- 5	- 2	- 1	- 1
37	May 9, '06	b ₁₅	a ₁	b				- 7	- 6	- 8	- 6
15	May 16, '06	b ₁₆	a ₁	b				-14	-14	-13	-15
38	May 9, '06	b ₁₆	a ₁	b				- 5	Given to Prof. Hulett		
15a	June 25, '07	b ₁₆	a ₂	b	+30	+12	+11				
178	Dec. 21, '06	b ₂₀	a ₂	a		+ 7	+ 8				
179	"	b ₂₁	a ₂	a		+ 8	+10				
180	"	b ₂₂	a ₂	a		- 8	- 9				

TABLE IV.

Weston Cells.—Electrolytic Mercurous Sulphate made with New Apparatus.

Concentration of Sulphuric Acid exceeding Gram-Molecular.

[Differences in Microvolts from Mean of Reference Cells.]

Cell	Sept. 13	Oct. 13	Nov. 2	Nov. 19	Dec. 1	Jan. 10, 1907	Mar. 1	Mar. 25	Apr. 24	May 24	June 26	July 23
26	+ 10	+ 12	+ 15	+ 11	+ 13	+ 6	+ 10	+ 10	+ 7	+ 6	+ 5	+ 3
51	- 12	- 11	- 12	- 10	- 11	- 11	- 12	- 12	- 12	- 13	- 13	- 15
27	- 6	- 5	- 3	- 4	- 4	- 3	- 3	- 4	- 6	- 7	- 8	- 9
52	+ 4	+ 4	+ 4	+ 2	+ 1	+ 3	- 1	0	- 11	- 2	- 3	- 3
28	+ 18	+ 18	+ 21	+ 18	+ 18	+ 18	+ 17	+ 17	+ 14	+ 13	+ 9	+ 9
61	+ 2	- 2	- 1	- 1	+ 1	- 2	- 1	+ 1	0	- 1	0	- 3
62	- 4	- 4	- 5	- 6	- 4	- 2	- 7	- 6	- 6	- 7	- 8	- 9
63	- 11	- 9	- 9	- 8	- 8	- 8	- 9	- 10	- 10	- 12	- 11	- 13
3	+ 5	+ 5	+ 4	+ 4	+ 1	+ 5	+ 7	+ 6	+ 7	+ 6	+ 9	+ 8
3a											+ 18	- 3
29	+ 11	+ 12	+ 15	+ 12	+ 14	+ 9	+ 11	+ 13	+ 12	+ 13	+ 10	+ 9
30	+ 1	+ 1	+ 6	+ 3	+ 3	+ 4	+ 1	+ 4	+ 1	+ 1	+ 1	- 1
46	+ 14	+ 14	+ 11	+ 13	+ 12	+ 7	+ 8	+ 10	+ 7	+ 7	+ 6	+ 6
46a	- 19	- 17	- 20	- 17	- 16	- 16	- 18	- 20	- 17	- 20	- 21	- 17
46b											+ 29	- 5
31	+ 10	+ 9	+ 14	+ 10	+ 11	+ 11	+ 9	+ 11	+ 7	+ 6	+ 4	+ 1
47	- 2	- 1	- 2	- 1	- 1	- 3	- 5	- 6	- 8	- 8	- 10	- 14
4	- 6	- 6	- 7	- 7	- 6	- 7	- 8	- 9	- 7	- 7	- 6	- 6
4a											+ 8	- 10
32	- 21	- 26	- 11	- 24	- 24	- 24	- 30	- 33	- 30	- 35	- 34	- 34
33	+ 1	0	+ 5	+ 1	+ 5	+ 1	- 1	0	- 1	+ 1	0	- 3
48	- 11	- 10	- 11	- 11	- 9	- 11	- 12	- 10	- 13	- 12	- 14	- 15
34	+ 7	+ 7	+ 10	+ 6	+ 9	+ 5	+ 7	+ 10	+ 6	+ 7	+ 4	+ 3
49	- 9	- 9	- 10	- 8	- 8	- 10	- 11	- 10	- 13	- 11	- 13	- 14
6	- 11	- 10	- 12	- 10	- 9	- 11	- 9	- 9	- 9	- 9	- 10	- 8
6a						- 14	- 14	- 12	- 14	- 13	- 16	- 15
6b												
151						- 2	- 10	- 8	- 7	- 4	- 7	
8	0	- 3	0	- 1	- 1	- 3	+ 3	+ 2	+ 2	+ 3	+ 2	+ 2
14	- 3	- 2	- 2	0	- 2	- 2	- 1	- 1	- 1	- 1	- 2	- 2
37	- 6		- 2	- 5	- 2	- 3	- 4	- 1	- 3	- 3	- 3	- 2
18	- 18	- 16	- 18	- 16	- 13	- 14	- 17	- 16	- 16	- 17	- 16	- 18
38												
15a											+ 30	- 5
178						+ 15	+ 13	+ 13			+ 14	+ 11
179						+ 21	+ 17	+ 18			+ 11	+ 5
180						- 7	- 12	- 10			- 8	- 11

TABLE IV—Continued.

Cell	Date	Mer- curous sul- phate	CdSO ₄	Cd Amal- gam	*	**	***	July 5, 1906	July 20	Aug. 4	Aug. 21
190	June 25, '07	b ₂₀	a ₂	b	+24	+20	+ 6				
191	"	b ₂₀	a ₂	b	- 1	0	- 8				
192	"	b ₂₁	a ₂	b	+ 1	0	- 4				
Exchange Samples											
12	May 16, '06	Hu- lett's I	a ₁	b				+26	+27	+29	+28
39	May 9, '06	"	a ₁	b				+ 42	+ 36	+ 38	+ 39
141	Dec. 3, '06	Hu- lett's II	a ₂	a	0		- 4	Given to Prof. Hulett			
141a	"	"	a ₂	a	+ 4		- 6				
141b	"	"	a ₂	a	+13		0	Given to George Washington University			
142	Dec. 15, '06	"	a ₂	a	+12	- 3	+ 5				
149	June 25, '07	Guthe	a ₂	b	+48	+42	+30				

TABLE V.

Weston Cells.—Electrolytic Mercurous Sulphate made with New Apparatus.

Concentration of Sulphuric Acid, Gram-Molecular or Less.

[Differences in Microvolts from Mean of Reference Cells.]

Cell	Date	Mer- curous sul- phate	CdSO ₄	Cd Amal- gam	*	**	***	July 5, 1906	July 20	Aug. 4	Aug. 21
50	June 23, '06	b ₂₀	a ₁	b		+19	+12	+ 9	+ 6	+ 2	- 4
53	"	b ₂₁	a ₁	b		+24	+25	+23	+29	+22	+19
54	"	b ₂₂	a ₁	b		+14	+11	+11	+13	+11	+ 7
55	"	b ₂₃	a ₁	b		- 3	- 5	- 4	- 1	- 3	- 4
56	"	b ₂₄	a ₁	b		+ 4	+ 3	+ 5	+ 9	+ 7	+ 5
57	"	b ₂₅	a ₁	b		+17	+15	+17	+17	+18	+16
58	"	b ₁₇	a ₁	b		+45	+27	+24	+19	+17	+14
69	June 25, '06	b ₁₇	a ₁	b	+ 7	+ 5	- 6	- 5	- 8	-17	- 9
59	"	b ₁₈	a ₁	b	+58	+54	+34	+34	+28	+19	+17
60	"	b ₁₉	a ₁	b	+20	+20	+ 6	+ 6	+ 6	- 3	- 2

TABLE IV—Continued.

Cell	Sept. 13	Oct. 13	Nov. 2	Nov. 19	Dec. 1	Jan. 10, 1907	Mar. 1	Mar. 25	Apr. 24	May 24	June 26	July 23
190											+ 24	— 5
191											— 1	— 17
192											+ 1	— 15
12	+ 26	+ 28	+ 29	+ 30	+ 28	+ 32	+ 30	+ 31	+ 31	+ 31	+ 31	+ 32
39	+ 37	+ 38	+ 38	+ 42	+ 35	+ 39	+ 39	+ 40	+ 37	+ 35	+ 36	+ 34
141												
141a						— 3	— 3	— 3	— 3	— 2	— 1	— 3
141b	Given to George Washington University											
142						+ 4	— 2	— 2	— 3	0	— 4	
189											+ 48	+ 25

TABLE V.

Weston Cells.—Electrolytic Mercurous Sulphate made with New Apparatus.

Concentration of Sulphuric Acid, Gram-Molecular or Less.

[Differences in Microvolts from Mean of Reference Cells.]

Cell	Sept. 13	Oct. 13	Nov. 2	Nov. 19	Dec. 1	Jan. 10, 1907	Mar. 1	Mar. 25	Apr. 24	May 24	June 26	July 23
50	— 1	— 4	— 1	— 3	— 3	— 5	—10	— 8	—11	—10	—13	—15
53	+18	+17	+19	+18	+18	+ 6	+15	+18	+16	+16	+15	+15
54	+ 9	+10	+11	+ 9	+12	+ 3	+ 8	+11	+10	+10	+10	+ 8
55	— 1	+ 1	+ 2	0	+ 1	— 1	0	+ 2	+ 2	+ 2	+ 1	— 2
56	+ 5	+ 7	+ 8	+ 4	+ 6	+ 2	+ 8	+11	+ 9	+10	+10	+ 8
57	+15	+18	+17		+16	+15	+18	+20	+18	+19	+20	+18
58	+10	+13	+10	+ 7	+ 9	+ 9	+10	+12	+11	+12	+10	+ 9
69	— 9	— 8	—10	— 9	— 9	—11	—12	—11	—11	—11	—11	—14
59	+13	+14	+11	+ 8	+ 8	+ 4	— 4	— 6	—12	—16	—23	—22
60	— 6	— 6	— 8	—10	— 8	—12	—12	—12	—13	—14	—16	—15

TABLE VI.

Weston Cells—Chemically Prepared Mercurous Sulphate.

[Differences in Microvolts from Mean of Reference Cells.]

Cell	Date	Mer- curous sul- phate	CdSO ₄	Cd	*	**	***	July 5, 1906	July 20	Aug. 4	Aug. 21
86	June 26, '06	c	a ₁	b	+ 4	+ 4	- 5	+ 1	+ 3	+ 4	+ 5
99	June 27, '06	d ₁	a ₁	b	+ 51	+38	+20	+21	+13	+ 7	+ 5
99a	June 28, '06	d ₁	a ₁	b	+ 13	+ 2	- 9	- 7	-11	-16	-15
84	June 26, '06	d ₂	a ₁	b	- 11	-14	-16	-17	-18	-15	-18
84a	Dec. 3, '06	d ₂	a ₂	a	- 19	-17	-16				
84b	"	d ₂	a ₃	a	- 15	-16	-16	Given to George Washington University			
1	May 11, '06	e ₁	a ₁	b	- 9			+ 7	- 1	- 1	- 8
43	May 9, '06	e ₁	a ₁	b				- 6	- 9	-11	-10
157	Dec. 15, '06	e ₁	a ₁	b	+ 23	- 3	- 5				
*1a	June 25, '07	e ₁	a ₂	b	+ 47	+32	+27				
103	June 27, '06	e ₂	a ₁	b	- 14	-16	-22	-21	-22	-23	-24
2	May 11, '06	e ₃	a ₁	b				- 4	- 3	- 2	- 8
44	May 9, '06	e ₃	a ₁	b				- 4	- 6	- 5	- 3
104	June 27, '06	e ₃	a ₁	b	- 17	-18	-20	-16	-14	-13	-13
2a	"	e ₃	a ₂	b	+ 32	+17	+11				
105	"	e ₄	a ₁	b	+ 3	- 5	-13	-11	-11	-12	-15
106a	June 28, '06	e ₅	a ₁	b	- 9	-11	-15	-15	-17	-22	-18
107	"	e ₆	a ₁	b	- 11	-15	-16	-16	-18	-25	-22
98	June 27, '06	f ₁	a ₁	b	- 4	- 7	-10	-12	-10	-11	- 8
97	"	f ₂	a ₁	b	- 4	- 7	- 7	- 9	- 8	-10	- 9
100	June 27, '06	g ₁	a ₁	b	+ 26	+15	- 4	- 4	-17	-17	-17
100a	June 28, '06	g ₁	a ₁	b	+ 66	+43	+22	+22	+12	+ 3	+ 2
101	June 27, '06	g ₂	a ₁	b	+121						
101a	June 28, '06	g ₂	a ₁	b	+112	+92	+78	+78	+59	+51	+48
163	Dec. 15, '06	g ₃	a ₂	a	+102	+81	+71				
95	"	h ₁	a ₁	b	+ 1	-10	-23	-23	-26	-20	-27
96	"	h ₂	a ₁	b	+ 32	+19	+ 2	+ 2	+ 2	- 1	- 2
85	June 26, '06	h ₃	a ₁	b	+ 4	+ 1	- 6	- 7	- 7	-10	-12

TABLE VI.

Weston Cells—Chemically Prepared Mercurous Sulphate.

[Differences in Microvolts from Mean of Reference Cells.]

Cell	Sept. 13	Oct. 13	Nov. 2	Nov. 19	Dec. 1	Jan. 10, 1907	Mar. 1	Mar. 25	Apr. 24	May 24	June 26	July 23
86	+ 3	+ 4	+ 1	+ 1	0	- 2	- 3	- 4	- 7	- 5	- 7	- 9
99	0	- 2	- 3	- 4	- 5	- 4	-10	-11	-13	-13	-14	-16
99a	-18	-16		-16	-14	-15	-18	-18	-19	-20	-20	-18
84	-18	-19	-21	-19	-18	-19	-27	-30	-27	-33	-33	-30
84a						-16	-20	-18	-18	-21	-19	-19
84b	Given to George Washington University											
1	- 3	- 3	- 4	- 4	- 5	- 4	- 4	- 9	- 8	- 8	- 8	- 5
43	-11	-11		-11	- 8	- 5	-11	- 9	-11	- 7	-12	-15
157						-10	-19	-20	-15	-15	-19	-19
*1a											+47	+ 9
103	-24	-26	-25	-23	-20	-23	-23	-26	-23	-25	-29	-25
2	- 2	- 2	- 3	- 3	- 4	- 2	0	0	- 1	- 2	0	- 2
44	- 3	- 3	- 3	- 1	- 2	+ 1	- 3	- 3	- 3		- 3	- 7
104	-15	-12	-14	-11	- 9	- 8	-12	-10	-10	-10	-10	-12
2a											+32	- 2
105	-16	-16	-20	-17	-14	-15	-19	-18	-18	In Europe		
106a	-18	-18		-20	-16	-18	-19	-20	-19	-19	-23	-19
107	-21	-22		-23	-19	-27	-33	-39	-46	-51	-54	-52
98	- 8	- 7	- 9	- 8	- 6	- 5	- 9	- 6	- 7		- 9	-14
97	- 9	- 8	-10	-10	- 9	- 7	- 9	-10	-13	-10	-11	-14
100	-20	-22	-21	-19	-16	-20	-20	-20	-19	-14	-22	-19
100a	0	0		0	0	+ 3	- 3	- 1	- 1	+ 3	- 2	- 3
101												
101a	+43	+43	+40	+40	+41	+42	+36	+36	+38	+39	+31	+25
163						+39	+14	+14	+10	+ 9	+ 4	+ 3
95	-28	-28	-28	-24	-24	-28	-29	-27	-28		-29	-28
96	- 5	- 4	- 4	- 7	- 5	- 4	-10	- 7	- 9	- 6	- 8	-12
85	-13	-12	-13	-14	-11	-15	-20	-23	-20	-27	-28	-27

TABLE VII.

Weston Cells—Specially Treated Commercial Samples of Mercurous Sulphate.

[Differences in Microvolts from Mean of Reference Cells.]

Cell	Date	Mer- curous sul- phate	CdSO ₄	Cd amalg.	*	**	***	July 5, 1906	July 20	Aug. 4	Aug. 21
93	June 27, '06	l ₁	a ₁	b	+ 44	+ 29	+ 14	+ 14	+ 12	+ 8	+ 9
94	"	l ₂	a ₁	b	+ 24	+ 13	+ 5	+ 5	- 2	0	0
78	June 28, '06	k ₁	a ₁	b	- 22	- 24	- 19	- 19	- 18	- 22	- 18
79	June 26, '06	k ₂	a ₁	b	+ 10	+ 8	+ 5	+ 4	+ 5	+ 4	+ 6
80	"	k ₃	a ₁	b	+ 15	+ 16	+ 18	+ 19	+ 23	+ 21	+ 22
81	"	k ₄	a ₁	b	+ 6	+ 4	- 1	- 1	+ 2	0	0
82	"	k ₅	a ₁	b	+ 13	+ 13	+ 9	+ 10	+ 14	+ 15	+ 14
83	"	k ₆	a ₁	b	+ 17	+ 16	-----	-----	-----	+ 2	+ 6
108	June 27, '06	l ₁	a ₁	b	+ 6	+ 4	+ 1	+ 4	+ 7	+ 8	+ 8
116	June 28, '06	l ₂	a ₁	b	+ 51	+ 51	+ 49	+ 54	+ 55	+ 50	+ 51
109	June 27, '06	l ₃	a ₁	b	+ 11	+ 4	- 3	0	+ 4	+ 5	+ 5
114	June 28, '06	l ₄	a ₁	b	+ 54	+ 64	+ 55	+ 61	+ 59	+ 52	+ 54
115	"	l ₅	a ₁	b	+ 39	+ 40	+ 36	+ 41	+ 34	+ 34	+ 35
117	June 28, '06	m ₁	a ₁	b	+ 34	+ 25	+ 19	+ 28	+ 28	+ 24	+ 28
119	"	m ₂	a ₁	b	+ 24	+ 15	+ 17	+ 17	+ 23	+ 19	+ 24
118	"	m ₃	a ₁	b	+ 79	+ 75	+ 71	+ 79	+ 79	+ 75	+ 76
120	"	m ₄	a ₁	b	+ 76	+ 75	+ 69	+ 78	+ 78	+ 74	+ 76
87	June 27, '06	n ₁	a ₁	b	+ 36	+ 33	+ 29	+ 33	+ 39	+ 40	+ 42
88	"	n ₂	a ₁	b	+ 78	+ 74	+ 73	+ 72	+ 76	+ 80	+ 80
89	"	n ₃	a ₁	b	+114	+108	+103	+107	+112	+113	+112
90	"	n ₄	a ₁	b	+ 24	+ 19	+ 15	+ 18	+ 22	+ 20	+ 22
91	"	n ₅	a ₁	b	+123	+123	+117	+118	+124	+123	+122
92	"	n ₆	a ₁	b	+ 74	+ 73	+ 74	+ 73	+ 71	+ 68	+ 67

TABLE VII.

Weston Cells—Specially Treated Commercial Samples of Mercurous Sulphate.

[Differences in Microvolts from Mean of Reference Cells.]

Cell	Sept. 13	Oct. 13	Nov. 2	Nov. 19	Dec. 1	Jan. 10, 1907	Mar. 1	Mar. 25	Apr. 24	May 24	June 26	July 23
93	+ 5	+ 6	+ 4	+ 5	+ 4	+ 6	- 2	0	- 2	- 2	- 3	- 4
94	- 5	- 3	- 5	- 6	- 6	- 5	- 12	- 10	- 13	- 15	- 18	- 18
78	- 20			- 19	- 15	- 17	- 20	- 22	- 23	- 30	- 33	- 30
79	+ 3	+ 5	+ 5	+ 5	+ 3	+ 7	+ 3	+ 3	0	0	- 5	- 10
80	+ 21	+ 24	+ 25	+ 28	+ 27	+ 28	+ 26	+ 25	+ 24	+ 25	+ 24	+ 22
81	- 1	+ 1	0	+ 1	0	+ 2	- 1	- 1	- 1	- 2	- 4	- 7
82	+ 12	+ 14	+ 14	+ 15	+ 14	+ 16	+ 12	+ 13	+ 12	+ 12	+ 11	+ 8
83	0	+ 8	+ 5	+ 8	+ 8	+ 16	+ 15	+ 15	+ 16	+ 15	+ 10	+ 7
108	+ 5	+ 7	+ 5	+ 7	+ 4	+ 11	+ 3	+ 4	+ 4	+ 2	- 9	- 12
116	+ 51	+ 60	+ 54	+ 50	+ 52	+ 55	+ 50	+ 51	+ 51	+ 52	+ 50	+ 48
109	+ 3	+ 7	+ 5	+ 3	+ 5	+ 9	+ 3	+ 3	+ 3	+ 1	- 13	- 25
114	+ 54	†+ 64	+ 59	+ 57	+ 59	+ 64	+ 59	+ 55	+ 61	+ 60	+ 59	+ 55
115	+ 32	†+ 42	+ 35	+ 31	+ 32	+ 35	+ 30	+ 30	+ 31	+ 30	+ 23	+ 4
117	+ 28	†+ 37	+ 31	+ 28	+ 26	+ 31	+ 23	+ 21	+ 21	+ 13	+ 11	+ 9
119	+ 22	†+ 33	+ 26	+ 27	+ 24	+ 31	+ 26	+ 28	+ 26	+ 28	+ 27	+ 21
118	+ 78	†+ 87	+ 84	+ 81	+ 83	+ 86	+ 83	+ 85	+ 86	+ 86	+ 86	+ 82
120	+ 74	†+ 84	+ 79	+ 76	+ 78	+ 79	+ 75	+ 76	+ 76	+ 77	+ 74	+ 72
87	+ 39	+ 41	+ 40	+ 39	+ 41	+ 42	+ 36	+ 35	+ 35	+ 29	+ 22	+ 21
88	+ 75	+ 77	+ 77	+ 79	+ 78	+ 81	+ 76	+ 77	+ 78	+ 80	+ 77	+ 74
89	+108	+111	+109	+110	+110	+113	+106	+108	+107	+110	+107	+103
90	+ 20	+ 23	+ 21	+ 22	+ 24	+ 28	+ 23	+ 24	+ 25	+ 26	+ 22	+ 19
91	+118	+120	+119	+121	+120	+124	+120	+121	+122	+124	+121	+119
92	+ 65	+ 65	+ 65	+ 65	+ 64	+ 73	+ 69	+ 69	+ 70	+ 71	+ 69	+ 67

† Cells transferred to other bath.

TABLE VIII.

Clark Cells—Electrolytic Mercurous Sulphate made with old Apparatus:

[Differences in Microvolts from Mean of Reference Cells.]

Cell	Date	Mercurous sulphate	ZnSO ₄	Zn Amalg.	***	May 1, 1906	May 28	June 20	July 20	Aug. 4
57	Nov. 20, '06	a ₁	a	b	+46					
58	"	a ₄	a	b	+34					
59	"	a ₅	a	b	+38					
60	"	a ₆	a	b	+34					
42	Apr. 12, '06	a ₇	b	a		+1	-6	+12	+5	-17
42a	Nov. 23, '06	a ₇	a	b	+27					
61	Nov. 20, '06	a' ₇	a	b	+20					
43	Apr. 12, '06	a ₈	b	a		+41	+39	+46	+38	+37
62	Nov. 20, '06	a ₈	a	b	+15					
63	"	a' ₈	a	b	+31					
44	Apr. 12, '06	a ₉	b	a		-2	+1	-6	-18	-14
44a	Nov. 23, '06	a ₉	a	b	+44					
64	Nov. 20, '06	a ₁₀	a	b	+11					
65	"	a' ₁₀	a	b	+25					
66	"	a ₁₁	a	b	+5					
67	"	a' ₁₁	a	b	+4					
68	"	a ₁₂	a	b	+10					
69	"	a ₁₂	a	b	+2					

Cells 57, 58, 60, 42, 62, 63, 67, 68, 69 found cracked and layer of zinc sulphate lifted. Replaced in bath after crystals were shaken back.

TABLE IX.

Clark Cells—Electrolytic Mercurous Sulphate made with New Apparatus.

Concentration of Sulphuric Acid exceeding Gram-Molecular.

[Differences in Microvolts from Mean of Reference Cells.]

Cell	Date	Mercurous sulphate	ZnSO ₄	Zn amalg.	*	**	***	May 1, 1906	May 28	June 20	July 20
27a	Apr. 12, '06	b ₁	b	a				+5	-1	+5	+5
27b	"	b ₁	b	a				0	-1	-2	0
27c	Nov. 23, '06	b ₁	a	b		+20	+8				
28	Apr. 12, '06	b ₂	b	a				+4	+2	-3	-2
28a	Nov. 23, '06	b ₂	a	b		+13	+1				
28b	June 28, '07	b ₂	a	b	+22	+21	+9				
29	Apr. 12, '06	b ₂	b	a				-1			
29a	Nov. 23, '06	b ₂	a	b		+13	+7				

TABLE VIII.

Clark Cells—Electrolytic Mercurous Sulphate made with old Apparatus.

[Differences in Microvolts from Mean of Reference Cells.]

Cell	Sept. 13	Oct. 13	Nov. 2	Nov. 19	Dec. 1	Jan. 10, 1907	Mar. 1	Mar. 26	May 24	July 23.
57					+41	+20	+ 2	+ 8	-21	+ 6
58					+31	+14	-21	+12	broken	
59					+33	+28	+32	+33	+30	+30
60					+29	+15	+19	+20	+20	+20
42						+10	-10	- 3	-10	-13
42a					+21	+14	+21	+20	+19	+17
61					+18	+13	+19	+19	+20	+18
43	+37	+35	+35	+34	+28	+30	+34	+32	+31	+26
62					+14	- 7	cracked			
63					+28	+ 5	+19	+12	+ 5	+21
44	-10	-8	-11	-5			broken			
44a					+42	+29	+33	+33	+32	+29
64					+10	+ 2	+ 6	+ 7	+ 6	+ 6
65					+24	+15	+17	+18	+18	+ 4
66					+ 3	+ 1	+ 5	+ 5	+ 7	+ 7
67					+ 1	- 5	+ 6	+ 7	+ 7	+12
68					+ 5	- 7	- 3	- 1	broken	
69					0	-22	-18	-20	broken	

Cells 57, 58, 60, 42, 62, 63, 67, 68, 69 found cracked and layer of zinc sulphate lifted. Replaced in bath after crystals were shaken back.

TABLE IX.

Clark Cells—Electrolytic Mercurous Sulphate made with New Apparatus.

Concentration of Sulphuric Acid exceeding Gram-Molecular.

[Differences in Microvolts from Mean of Reference Cells.]

Cell	Aug. 4	Sept. 13	Oct. 13	Nov. 2	Nov. 19	Dec. 1	Jan. 10, 1907	Mar. 1	Mar. 26	May 24	July 24
27a	+ 4	+ 4	+ 4	+ 4	+ 5	+ 5	+ 4	+ 4	+ 5	+ 7	+ 4
27b		Broken									
27c						+ 6	+ 6	+ 6	+ 6	+ 7	+90 ¹
28	- 4	- 5	- 6	- 5	- 4	- 4	- 6	- 7	- 7	- 4	- 5
28a						- 1	+ 1	+ 1	+ 1		
28b											+10
29											
29a						+ 5	+ 2	+ 3	+ 3	+ 8	+ 2

¹ High resistance. Seal defective?

TABLE IX—Continued.

Cell	Date	Mer- curous sul- phate	ZnSO ₄	Zn amalg.	*	**	***	May 1, 1906	May 28	June 20	July 20
73	Nov. 23, '06	b ₁	a	b		+19	+ 7				
74	"	b ₂	a	b		+28	+17				
75	"	b ₃	a	b		+24	+ 8				
30	Apr. 12, '06	b ₇	b	a				+15	+19	+16	+15
30a	Nov. 23, '06	b ₇	a	b		+16	+ 9				
30b	"	b ₇	a	b		+18	+19				
31	Apr. 12, '06	b ₈	b	a				- 6	+ 3	- 2	Broken
31a	Nov. 27, '06	b ₈	a	b	+ 9	+ 6	+ 6				
31b	Nov. 23, '06	b ₈	a	b		+ 9	- 3				
31c	Nov. 27, '06	b ₈	a	b	0	0	+ 2				
31d	"	b ₈	a	b	+14	+12	+ 4				
32	Apr. 12, '06	b ₉	b	a				+ 2	+ 6	+ 8	+ 5
32a	Nov. 23, '06	b ₉	a	b		+27	+17				
32b	June 28, '07	b ₉	a	b	+14	+13	+ 8				
33	Apr. 12, '06	b ₁₀	b	a				- 5	- 5		Broken
33a	Nov. 23, '06	b ₁₀	a	b			+ 6				
34	Apr. 12, '06	b ₁₁	b	a				+ 2	+ 1	+ 2	- 1
34a	Nov. 23, '06	b ₁₁	a	b	+ 7	+ 7	+ 4				
34b	June 25, '07	b ₁₁	a	b	+22	+20	+14				
35	Apr. 12, '06	b ₁₂	b	a				+ 4	+ 5	+ 6	+ 5
36a	Nov. 23, '06	b ₁₂	a	b		+ 8	+ 6				
36b	Dec. 3, '06	b ₁₂	a	b		+ 8					
36c	"	b ₁₂	a	b		+ 8					
37	Apr. 12, '06	b ₁₄	b	a				+10	+10	0	+ 3
38	"	b ₁₅	b	a				+ 9	+10	+ 4	+10
39	"	b ₁₆	b	a				-13	-10	-10	- 5
39a	Nov. 23, '06	b ₁₆	a	b		+ 7	+ 4				
39b	June 25, '07	b ₁₆	a	b	+ 6	+ 9	+ 7				
40	Apr. 11, '06	Hu- lett's I	b	a				+35	+39		
41	"	Hu- lett's I	b	a				+24	+30	+30	+32
70	Nov. 20, '06	Hu- lett's I	a	b			+53				
71	"	Hu- lett's II	a	b			+17				
71a	Dec. 3, '06	Hu- lett's II	a	b		+16					
71b	"	Hu- lett's II	a	b		+17					
111	June 28, '07	Guthe	a	b	+51	+49					

[illegible]

TABLE X.

Clark Cells—Electrolytic Mercurous Sulphate made with New Apparatus.

Concentration of Sulphuric Acid, Gram-Molecular or Less.

[Differences in Microvolts from Mean of Reference Cells.]

Cell	Date	Mercurous sulphate	ZnSO ₄	Zn amalg.	Nov. 26, 1906	Nov. 27	Nov. 28
48	Nov., '06	b ₃₀	a	b	+22	+17	+17
49	"	b ₂₁	a	b	+23	+15	+15
50	"	b ₂₂	a	b	+25	+21	+18
51	"	b ₂₃	a	b	+19	+15	+11
52	"	b ₂₄	a	b	+23	+22	+17
53	"	b ₂₅	a	b	+36	+34	+32
54	"	b ₁₇	a	b	+13	+17	+ 3
55	"	b ₁₈	a	b	+17	+13	+ 8
56	"	b ₁₉	a	b	+26	+23	+20

TABLE XI.

Clark Cells—Chemically Prepared Mercurous Sulphate.

[Differences in Microvolts from Mean of Reference Cells.]

Cell	Date	Mercurous sulphate	ZnSO ₄	Zn amalg.	*	**	***	May 1, 1906	May 26	June 20	July 20
78	Nov. 23, '06	c	a	b			+ 7				
79	"	d ₁	a	b			+ 9				
76	"	d ₂	a	b			+ 5				
46	Apr. 12, '06	e ₁	b	a				-12	-14	-20	-17
46a	Dec. 3, '06	e ₁	a	b			+ 1				
46b	"	e ₁	a	b			- 3				
86	Nov. 27, '06	e ₁	a	b	+ 1	- 2	- 1				
86a	June 28, '07	e ₁	a	b		+ 7	+ 5				
87	"	e ₂	a	b	0	- 2	- 2				
88	"	e ₂	a	b	- 3	- 2	+ 1				
89	"	e ₄	a	b	+12	+10	+ 6				
90	"	e ₅	a	b	+ 6	+ 2	+ 1				
91	"	e ₆	a	b	- 2	- 4	+ 1				
91a	"	e ₆	a	b	- 1	- 4	- 4				
45	Apr. 12, '06	f ₁	b	a				+ 4	+ 3	+ 7	+ 7
47	"	f ₂	b	a				- 5	- 3	- 7	0
84	Nov. 27, '06	g ₁	a	b	+36	+19	+11				
85	"	g ₂	a	b	+26	+19	+11				
82	Nov. 27, '06	h ₁	a	b	0	-11	-12				
83	"	h ₂	a	b	+21	+ 8	+ 4				
77	Nov. 23, '06	h ₃	a	b			+ 8				

TABLE X.

Clark Cells—Electrolytic Mercurous Sulphate made with New Apparatus.

Concentration of Sulphuric Acid, Gram-Molecular or Less.

[Differences in Microvolts from Mean of Reference Cells.]

Cell	Dec. 1	Dec. 6	Jan. 4, 1907	Jan. 10	Jan. 31	Mar. 1	Mar. 26	May 24	July 23
48	+15	+15	+11	+14	+15	+15	+14	+15	+11
49	+13		+13	+ 4	+ 5	- 2	+ 5	+ 7	+ 8
50	+15	+16	+ 7	+15	Cracked				- 5
51	+14	+14	+10	+22 Lifted					
52	+20	+21	+20	+23	+24	+23	+24	+25	+23
53	+34	+30	+28	+31	+33	+35	+37		
54	+ 4	+ 5	+ 2	+ 5	Lifted				
55	+ 8	+ 8	+ 6	+ 8	+ 9	+12	+ 8	+10	+ 9
56	+17	Broken							

TABLE XI.

Clark Cells—Chemically Prepared Mercurous Sulphate.

[Differences in Microvolts from Mean of Reference Cells.]

Cell	Aug. 4	Sept. 13	Oct. 13	Nov. 2	Nov. 19	Dec. 1	Jan. 10, 1907	Mar. 1	Mar. 26	May 24	July 23
78						+ 8	+ 7	+16	+18	+17	+13
79						+11	+ 5	Cracked			+ 9
76						+ 6	- 4	+ 1	- 2	+ 6	+29
46	-14	-11	-12	-13	- 8	- 7	- 8	- 4	- 4	- 4	- 4
46a							- 8	- 4	- 3	- 4	- 4
46b							- 8	- 2	- 4	- 4	
86							-17	-17	-13	-12	- 8
86a											+ 3
87							- 7	- 3	- 3	- 2	- 1
88							- 2	+ 3	+ 4	+ 7	+ 4
89							- 2	0	+ 1	+ 2	+ 1
90							- 7	- 1	- 2	- 3	- 3
91							- 8	- 2	- 2	- 2	- 3
91a							- 8	- 3	- 2	- 2	+ 1
45	+ 8	+11	+ 9	+ 8	+11	+ 8	+ 9	+13	+14	+15	+13
47	+ 1	+ 1	- 1	+ 1	+ 1	- 1	- 1	+ 4	+ 4	+ 3	- 1
84						+17	+ 5	Cracked			
85						+17	-13	- 3	- 2		+ 5
82						-13	-17	-13	-13	-13	-11
83						+ 6	- 3	+ 3	+ 2	+ 2	+ 2
77						+10	+ 9	+11	+11	+12	+11

TABLE XII.

Clark Cells.—Specially Treated Commercial Samples of Mercurous Sulphate.

[Differences in Microvolts from Mean of Reference Cells.]

Cell	Date	Mer- curous sul- phate	ZnSO ₄	Zn Amaig.	*	**	***	May 1, 1906	May 28	June 20	July 20
80	Nov. 27, '06	l ₁	a	b	+10	+ 3	+ 1	-----	-----	-----	-----
81	"	l ₂	a	b	+ 3	- 2	- 3	-----	-----	-----	-----
21	April 11, '06	k ₁	b	a	-----	-----	-----	+ 2	+ 2	- 6	- 3
22	"	k ₂	b	a	-----	-----	-----	+ 23	+ 25	+ 15	+ 17
23	"	k ₃	b	a	-----	-----	-----	+ 55	+ 50	+ 56	+ 51
23a	Nov. 23, '06	k ₃	a	b	-----	-----	+29	-----	-----	-----	-----
24	April 11, '06	k ₄	b	a	-----	-----	-----	+ 35	+ 27	+ 19	+ 20
24a	Nov. 27, '06	k ₄	a	b	+11	+ 9	+ 9	-----	-----	-----	-----
25	April 11, '06	k ₅	b	a	-----	-----	-----	+ 50	+ 47	+ 47	+ 48
25a	Nov. 23, '06	k ₅	a	b	-----	-----	+26	-----	-----	-----	-----
26	April 11, '06	k ₅	b	a	-----	-----	-----	+ 16	+ 17	+ 29	+ 20
92	Nov. 27, '06	l ₁	a	b	+28	+26	+29	-----	-----	-----	-----
93	"	l ₂	a	b	+57	+57	+61	-----	-----	-----	-----
94	"	l ₃	a	b	+25	+25	+20	-----	-----	-----	-----
95	"	l ₄	a	b	+71	+77	+77	-----	-----	-----	-----
96	"	l ₅	a	b	+47	+52	+52	-----	-----	-----	-----
15	April 11, '06	n ₁	b	a	-----	-----	-----	+ 59	+ 65	+ 55	+ 61
16	"	n ₂	b	a	-----	-----	-----	+112	+121	+111	+115
16a	Nov. 27, '06	n ₂	a	b	+76	+77	+78	-----	-----	-----	-----
17	April 11, '06	n ₃	b	a	-----	-----	-----	+143	+147	Broken	
17a	Nov. 23, '06	n ₃	a	b	-----	-----	+76	-----	-----	-----	-----
18	April 11, '06	n ₄	b	a	-----	-----	-----	+ 74	+ 71	+ 59	+ 62
19	"	n ₅	b	a	-----	-----	-----	+162	+157	+145	+146
20	"	n ₅	b	a	-----	-----	-----	+ 96	+ 97	+ 93	+ 93

TABLE XII.

Clark Cells—Specially Treated Commercial Samples of Mercurous Sulphate.

[Differences in Microvolts from Mean of Reference Cells.]

Cell	Aug. 4	Sept. 13	Oct. 13	Nov. 2	Nov. 19	Dec. 1	Jan. 10, 1907	Mar. 1	Mar. 26	May 24	July 23
80						+ 1	- 7	- 3	- 2	- 1	- 6
81						- 4	- 8	- 3	- 3	- 1	- 3
21	0	0	- 2	- 4	- 4	- 4	+ 2	+ 2	+ 1	- 1	- 5
22	+ 17	+ 21	+ 24	+ 25	+ 23	+ 25	+ 28	+ 33	+35	+ 28	+12
23	+ 48	+ 49	+ 53	+ 48	+ 49	+ 46	+ 49	+ 52	+53	+ 55	+52
23a						+ 30	+ 31	+ 35		+ 36	+37
24	+ 17	+ 19	+ 18	+ 17	+ 14	Broken					
24a						+ 7	+ 6	+ 2	- 2	0	+14
25	+ 46	+ 49	+ 50	+ 48	+ 53	+ 52	+ 56	+ 58	+59	+ 58	+45
25a						+ 25	+ 26	+ 30	+33	+ 32	+29
26	+ 20	+ 17	+ 20	+ 18	+ 19	+ 16	+ 20	+ 24	+24	+ 22	+16
92						+ 24	+ 20	+ 24	+28	+ 22	+ 9
93						+ 55	+ 57	+ 63	+60	+ 60	+59
94						+ 23	- 1	Broken			
95						+ 75	+ 67	+ 73	+71	+ 72	+73
96						+ 50	+ 42	+ 41	+43	Broken	
15	+ 60	+ 60	+ 57	+ 56	+ 60	+ 51	+ 42	+ 35	+39	+ 33	+29
16	+121	+114	+114	+118	+118	Broken					
16a						+ 75	+ 75	+ 81	+81	+ 79	+79
17											
17a						+ 74	+ 67	Cracked			
18	+ 66	+ 65	+ 64	+ 63	+ 67	+ 67	+ 68	+ 73	+72	+ 68	+65
19	+149	+146	+146	+152	+150	+150	+141	+142	+89	Cracked	
20	+ 94	+ 93	+ 93	+ 94	+ 95	+ 92	+ 93	+ 99	+98	+100	+99

TABLE XIII.

Clark Cells.—Set up with Commercial Samples of Mercurous Sulphate according to Old Specifications.

[Differences in Microvolts from Mean of Reference Cells.]

Cell	Date	Mercurous Sulphate	ZnSO ₄	Zn Amalg.	May 1, 1906	May 28	June 20	July 20	Aug. 4
1	April 9, '06	Kahlbaum A	b	a	+ 387	+ 353	+ 327	+ 320	+ 319
2	"	"	b	a	+ 442	+ 379	+ 378	+ 366	+ 366
3	"	"	b	a	+ 442	+ 375	+ 372	+ 362	+ 360
10	"	Kahlbaum B	b	a	+ 526	+ 489	+ 467	+ 489	+ 482
14	April 11, '06	Kahlbaum C	b	a	+ 305	+ 245	+ 241	+ 221	+ 212
6	April 9, '06	"	b	a	+ 404	+ 302	+ 295	+ 272	+ 265
5	"	Merck	b	a	+ 282	+ 248	+ 248	+ 244	+ 249
12	"	Schuchardt	b	a	+2202	+1652	+1587	+1366	+1275
13	"	"	b	a	+2172	+1646	+1585	+1350	+1241
8	"	Gehe	b	a	+ 412	+ 377	+ 376	+ 373	+ 376
9	"	"	b	a	+ 422	+ 381	+ 383	+ 377	+ 379

TABLE XIV.

Exchange Cells.

[Differences in Microvolts from Mean of Reference Cells.]

WESTON CELLS.

Cell		Date	Mercurous Sulphate	CdSO ₄	Cd Amalg.	July 5, 1906	July 20
F7	Hulett	Feb. 15, '04	1:6H ₂ SO ₄ D=.25 ...	Kahl. recrystallized	12½ % C.P. Merck		- 62
O4	Hulett						
O5	Hulett						
A3	Hulett	Oct. 21, '05	1:6H ₂ SO ₄ D=.82 ...	Kahl. recrystallized	12½ % C.P. Merck		+ 48
A52	Hulett	July, '06	1:6H ₂ SO ₄ D=.35 ...	Kahl. recrystallized	12½ % C.P. Merck		+ 34
1	Nat. Phy. Lab.	Feb., '06	Fuming Sulphuric.		9 % Cd.....	-80	- 78
2	Nat. Phy. Lab.	Feb., '06	Fuming Sulphuric.		9 % Cd.....	-85	-104
C1	Guthe			Merck and Baker, recrystallized	12½ % Kahl.		
5	Carhart	July 21, '06	El. 1:9H ₂ SO ₄				
8	Carhart	July 25, '06	El. 1:9H ₂ SO ₄				
1	W. W. Strong.	June 15, '07	b29		12.1 % Kahl. El.		
2	W. W. Strong.	June 15, '07	b29		"		
3	W. W. Strong.	June 15, '07	b29		"		

TABLE XIII.

Clark Cells.—Set up with Commercial Samples of Mercurous Sulphate according to Old Specifications.

[Differences in Microvolts from Mean of Reference Cells.]

Cell	Sept. 13	Oct. 13	Nov. 2	Nov. 19	Dec. 1	Jan. 10, 1907	Mar. 1	Mar. 26	May 24	July 23
1	+ 307	+307	+303	+300	+297	+297	+294	+290	+287	+267
2	+ 359	+357	+352	+355	+351	+256	+334	+328	+344	+311
3	+ 351	+352	+347	+349	+348	+348	+350	+344	+326	+328
10	+ 473	+474	+470	+472	+470	+473	+474	+468	+473	+461
14	+ 190	+181	+178	+177	+172	+172	+172	+163	+158	+142
6	+ 246	+237	+240	+226	+219	+214	+203	+194	+187	+170
5	+ 241	+242	+230	+240	+239	+240	+239	+233	+230	+217
12	+1066	+937	+854	+790	+745	+598	+414	+322	+180	Cracked
13	+ 977	+790	+669	Cracked						+ 27
8	+ 369	+371	+370	+369	+367	+371	+373	+368	+372	+364
9	+ 373	+378	+377	+379	Broken					

TABLE XIV.

Exchange Cells.

[Differences in Microvolts from Mean of Reference Cells.]

WESTON CELLS.

Cell	Aug. 4	Aug. 21	Sept. 13	Oct. 13	Nov. 2	Nov. 19	Dec. 1	Jan. 10, 1907	Mar. 1	Mar. 25	Apr. 24	May 24	June 26	July 23
F7	-67	-80	- 88	- 96	-118	-121	-126	-150	-227	-306	-375	-417	-466	-500
O4	+47	+47	+ 47	+ 47	+ 49	+ 50	+ 45	+ 50	+ 46	+ 47	+ 46	+ 49	+ 51	+ 51
O5	+17	+25	+ 26	+ 35	- 23	- 21	+ 8	- 43	- 25	0	+ 6	- 17	-119	-956
A3	+46	+48	+ 47	+ 49	+ 49	+ 50	+ 47	+ 52	+ 48	+ 48	+ 47	+ 44	+ 41	+ 42
A52	+30	+30	+ 27	+ 28	+ 30	+ 32	+ 32	+ 29	+ 29	+ 30	+ 29	+ 29	+ 28	+ 29
1	-78	-91	- 95	-107	-113			-130	-165	-173	-186	-208	-226	-242
2	-98	-99	-107	-111	-126		-112	-128	-112	-129	- 96	-125	-122	-107
C1								+ 26	+ 25	+ 23	+ 22	+ 17	+ 18	+ 16
5								+ 4	0	0	- 1	0	- 3	- 3
8								+ 12	+ 11	+ 13	+ 9	+ 10	+ 9	+ 7
									June 23	June 24	June 26	June 29	July 10	July 23
1									+38	+29	+27	+20	+ 8	+ 1
2									+53	+41	+36	+30	+14	+ 6
3									+10	-15	-16	-16	-17	-23

TABLE XIV—Continued.

CLARK CELLS.

Cell		Date	Mercurous Sulphate	ZnSO ₄	Zn Amalg.	July 20, 1906	Aug. 4
H4	Hulett	Mar. 15, '04	1:6 H ₂ SO ₄ D=.25...	Sch. twice re-crystallized	10 % C. P. — Baker	0	-9
H18	Hulett	Feb. 25, '06	1:6 H ₂ SO ₄ D=.82...	El. A twice re-crystallized	10 % C. P. — Baker	-2	+6
b1	Guthe		Hulett	C.P. Sch. re-crystallized	10 %		
c2	Carhart	Nov. 14, '06	El. 1:6 H ₂ SO ₄				

TABLE XV.

Weston Cells.

(Set up by Mr. P. I. Wold.)

[Differences in Microvolts from Mean of Reference Cells.]

1906

Cell	Date	Hg ₂ SO ₄	CdSO ₄	Cd Amalg.	Hg.	June 30, a. m.	June 30, p. m.	July 3
20	June 8, '06	1:6 H ₂ SO ₄	Recrystallized	12½ % by Electrolysis	Vac. distilled	-19	-11	-23
27	"	D=.3 Amp.	"	"		+10	+16	-10
21	"	"	"	"		-5	-2	-18
24	"	"	"	"		0	+6	-11

1907

Cell	Date	Hg ₂ SO ₄	CdSO ₄	Cd Amalg.	Hg.	June 14	June 14	June 15
42	April, '07	1:6 H ₂ SO ₄ D=3	Recrystallized	By Electr.	Vac. distilled	-7	-9	-9
46	"	" D=1	"	" "		+2	0	+1
49	"	" D=1	"	Melted		-25	-21	-22
50	"	" D=1	"	"		+14	+13	+13
55	"	Hulett's	"	By Electr.		-17	-17	-15
57	"	1:6 H ₂ SO ₄ D=.4	"	" "		-52	-53	-58

TABLE XIV—Continued.

CLARK CELLS.

Cell	Sept. 13	Oct. 13	Nov. 2	Nov. 19	Dec. 1	Jan. 10, 1907	Mar. 1	Mar. 26	May 24	July 23
H4	- 8	- 5	- 7			+ 1	0	- 3	- 1	
H18	+14	+16	+13	+18	+15	+22	+25	+30	+22	+21
b1						+71	+58	+65	+63	
c2						+14	+44	- 8	-18	

TABLE XV.

Weston Cells.

(Set up by Mr. P. I. Wold.)

[Differences in Microvolts from Mean of Reference Cells.]

1906

Cell	July 5, a. m.	July 5, p. m.	July 6	July 9	July 11	July 20	July 24	Aug. 4	Aug. 17	Sept. 11	Sept. 13
20	-16	-11	-12	-13	-12	-17	-11	-23	-12	- 6	-15
27	- 4	+ 4	- 1	- 3	- 2	-13	- 8	-14	-17	- 9	-11
21	-17										
24	- 9										

1907

Cell	June 17	June 18	June 19	June 20	June 21	June 22	June 24	June 26	June 29	July 10	July 23
42	- 7	-10	- 9	- 9	- 9	-12	- 9	- 9	-11		
46	+ 5	+ 1	0	+ 2	+ 3	+ 1	+ 4	+ 2	+ 1		
49	-18	-20	-19	-19	-20	-22	-19	-19	-14		
50	+13	+16	+15	+16	+16	+15	+18	+17	+18		
55	-15	-16	-16	-16	-16	-16	-14	-15	-12		
57	-58	-61	-60	-61	-64	-65	-62	-59	-53	-68	-69

DISCUSSION OF RESULTS.

Weston cells.—An inspection of the figures given in Tables III to VII shows that there is a highly satisfactory agreement between cells made with all the samples of mercurous sulphate prepared by methods *a* to *k*, inclusive, with the exception of a few of the white samples prepared in 1904, of which only scant quantities were preserved. In addition, two of these, *a*₆ and *a*₈, were somewhat discolored, probably by the action of light.

In cells Nos. 26 to 44, inclusive, the first Weston cells set up in 1906, the platinum terminals inside the cell were not amalgamated. Perhaps for this reason the agreement was not as close as could have been wished, as duplicates with amalgamated terminals showed a much smaller deviation from the mean. In general the cells with abnormally high initial values, with the exception of those in Table VII, have shown a decided decrease, in a few cases dropping below the normal value. Cells 9 and 42 showed a sudden decrease in value, but in each case they were found to have cracked.

The most satisfactory results, on the whole, were obtained with cells in which gray samples of electrolytic sulphate were employed, as shown in Table IV. A comparison of the values on July 20, 1906, and July 23, 1907, shows an average decrease of 6 microvolts for the interval. (Table XVII.)

Even with the samples of electrolytic mercurous sulphate prepared with more dilute sulphuric acid (*V*=1, 2, and 5), Table V, the agreement is very good and with the exception of No. 59, which had the highest initial value, the average relative change within the above-mentioned interval being 13 microvolts.

The agreement of the cells in Table VI, in which chemically prepared sulphate was used, compared with those made with electrolytic sulphate, is excellent; relatively large changes are shown, however, by the three samples made by reduction of mercuric sulphate by sulphurous acid, which gave abnormally high initial values, and by cell 107.

From Table VII it may be seen that commercial samples of mercurous sulphate, when digested with sulphuric acid, no longer show the large initial changes ordinarily observed when this treatment is omitted, and that digestion with hot 1:4 sulphuric acid, series *i* and *k*,

brings the commercial samples into excellent agreement with those prepared electrolytically. With few exceptions, the relative changes are very small. Most of the samples of series *l*, *m*, and *n*, which were prepared by allowing commercial mercurous sulphate to stand for a long time in contact with sulphuric acid, gave slightly high initial values which have persisted, in marked contrast with the usual behavior in such cases.

Clark cells.—The agreement of the Clark cells is on the whole somewhat better than that of the Weston cells, though, on account of their tendency to crack and the fact that many of them were not set up until November, 1906, the tables are less complete than in the case of the Weston cells. Although some of the Clark cells are more than a year old, the formation of gas at the amalgam limb, so frequently mentioned, has not been observed in cells which cracked in the bath, and which, therefore, were completely immersed in the kerosene. In such cases the development of a crack was generally followed by infinite resistance owing to the penetration of kerosene into the cell, in consequence of the slightly diminished pressure produced in sealing. As shown in the tables the cells resumed their normal values on cautiously shaking down the crystals which had been lifted by the layer of kerosene. In cases where the crack was so large as to expose the amalgam the electromotive force rapidly decreased.

Tables VIII to XII give the results obtained in the same order as in Tables III to VII. The deviations from the mean for the same sample of mercurous sulphate will, in general, be found to agree closely for both types of cell.

Table XIII gives the measurements on Clark cells set up with six untreated commercial samples of mercurous sulphate in accordance with the old specifications, in which the zinc sulphate solution was treated with an excess of zinc oxide, then with mercurous sulphate and filtered, while the latter was washed with water. These cells were set up in April, 1906, and had very high initial values in May, when first measured, and after the lapse of a year are, on the average, 280 microvolts above the mean.

Exchange cells.—During the progress of the work Clark and Weston cells were exchanged with Professors Hulett, Carhart, and Guthe, and in June, 1906, two Weston cells were kindly sent us by

the National Physical Laboratory of England. These have been kept under continuous observation, and the results obtained are given in Table XIV. This table also gives the measurements on three Weston cells set up by Mr. W. W. Strong at the Johns Hopkins University with mercurous sulphate, mercury, and cadmium furnished by the Bureau, the cadmium sulphate being purified by him.⁴³

In July, 1906, a number of Weston cells set up by Mr. P. I. Wold, of Cornell University, were measured at the Bureau; the results obtained on these and also on others received in June, 1907, are given in Table XV. Further data on these and the exchange cells, kindly placed at our disposal by Professors Carhart, Hulett, and Guthe, and by Mr. Wold, are given in the tables.

Professor Hulett's cells, with the exception of F_7 and O_8 , have not appreciably changed with respect to our cells during the period in which they have been under observation. Both of these cells, as well as O_6 , have defective seals which have allowed the penetration of oil. The paste of F_7 is quite brownish and the amalgams of F_7 and O_6 are slightly coated with a black deposit, possibly finely divided mercury. In addition, the values of O_6 show irregular changes.

The agreement of the cells of Guthe and von Ende⁴⁴ with the Bureau's cells indicates that the Clark cells obtained by them from Professor Hulett, constructed in 1904 and used as their basis of reference, must have suffered some change, possibly due to "exposure for some weeks to diffused light." Although Guthe and von Ende found relatively large initial changes in the electromotive force of their Weston cells, the measurements made at the Bureau on one of them showed that it had reached a practically constant value in close agreement with the Bureau cells when received.

The results obtained with the exchange cells, with the exception of cells F_7 and O_6 and the two cells from the National Physical Laboratory of England, which also contain oil, show that cells set up by different observers are in satisfactorily close agreement. This is also shown by the cells set up at the Bureau with samples of

⁴³ Three abnormally low Clark cells, two probably inverted, were also received from Mr. Strong.

⁴⁴ Phys. Rev., 24, p. 214 (1907).

mercurous sulphate obtained from Professors Hulett and Guthe. (Tables IV and IX.)

TABLE XVI.

Values of Weston Cells in Terms of Clark Cells.¹

Date	Diff. method	Direct method	Date	Diff. method	Direct method
June 1, '06	1.018859		Sept. 13, '06	1.018892	1.018890
June 2	65		Sept. 21	72	81
June 4	58	1.018854	Oct. 13	77	70
June 7	65	66	Nov. 3	70	74
June 9	51	59	Nov. 6	74	77
June 11	66	67	Nov. 17	68	76
June 13	62	65	Nov. 19, A. M.	66	
June 18		63	Nov. 19, P. M.	69	
June 19	61	64	Nov. 24	79	
June 20	63	64	Dec. 8	79	83
June 22	61	63	Mar. 2, '07	70	73
July 14	78	82	Mar. 27	64	
July 20		88	May 29	69	68
July 24	73	80	July 26	68	72
July 26	76	81	July 29	59	
Aug. 21	84	99	July 30	69	
			Mean	1.018869	1.018873

¹ Weston cells 1, 2, 3, 4, 5, 6, 8, 9, 11, 12, 14, and 15. Clark cells 27a, 28, 32, 34, and 39.

Relative values of Weston and Clark cells.—Table XVI shows the values of the mean of the twelve Weston cells, taken as the basis of reference, in terms of five Clark cells similarly used. The measurements were made by the direct and differential methods. In the former, the five Clark cells and seven of the twelve Weston cells were separately measured, and in the latter the cells of each set were arranged in series and the difference obtained by placing the sets in opposition. In the computations the value of the mean of the Clark cells was taken as 1.42110 volts at 25°, derived from the value, 1.434 volts at 15°,⁴⁵ adopted by the Chicago Congress for the Clark cell set up in accordance with the old specifications. A correction of -0.00030 volt was made for the average difference in electromotive force obtained with the specially prepared mercurous sulphate.

On account of breakage the same five Clark cells were not employed throughout the whole period, but the results were reduced

⁴⁵ $E_t = E_{15} - .00119(t - 15) - .000007(t - 15)^2$.

to the mean of the five which lasted longest. The new cells substituted for the broken ones agreed closely with the latter, so that the necessary corrections in no case exceeded five microvolts.

The close agreement between the results obtained by the direct and differential methods indicates that errors incident to electrical measurements are practically eliminated. The values obtained on the dates given can therefore be affected only by errors in temperature measurement ($0.01^{\circ} = 9$ microvolts), by possible hysteresis due to occasional interruptions in the regulation of the baths, and to possible ageing. The variations observed are, with two or three exceptions, so slight that they can be accounted for by an error of $\pm 0.01^{\circ}$ in reading the temperature. It may, therefore, be safely concluded that the Weston cells in terms of which the results are expressed have not changed with reference to the Clark cells, during the fourteen months, by as much as 20 microvolts. This conclusion is also confirmed, as shown in the tables, by the close agreement of cells set up at different times with the same sample of mercurous sulphate.

SUMMARY OF RESULTS.

For ready comparison of the results obtained with both Weston and Clark cells with mercurous sulphate made by the different methods, Tables XVII and XVIII are given. These show the mean differences and the average deviations in microvolts from the mean of the reference cells for each set of samples. Only those cells are included which were under continuous observation for the whole period, with the exception of exchange cells and those set up with mercurous sulphate obtained from Professors Hulett and Guthe. The last column in each table gives the relative changes from the beginning to the end of the period covered, which, in the case of the Clark cells is only eight months, as most of them were set up in November, 1906. The number of Clark cells was also smaller on account of the breakage.

The largest change for the Weston cells, as shown in Table XVII, is found with the white samples of mercurous sulphate prepared in the old apparatus, and is no doubt partly to be accounted for by the somewhat high initial values of some of them, as in practically all the other cases where a slight decrease is also shown. Part of the

decrease may be due to the gradual leaching out of traces of sulphuric acid obstinately retained by the irregularly pitted crystals of mercurous sulphate. Although these changes are relative, an in-

TABLE XVII.

Summary of Results.

Weston Cells—Mean Differences and Average Deviations, in Microvolts, for Each Series of Mercurous Sulphate Samples.

Table	Method	No. Samples No. Cells.		July 20, '06		October 13, '06		January 10, '07		April 24, '07		July 23, '07		Relative Change in One Year
				Mean	Ave. Devia- tion	Mean	Ave. Devia- tion	Mean	Ave. Devia- tion	Mean	Ave. Devia- tion	Mean	Ave. Devia- tion	
III	a	14	15	+28	31	+20	24	+10	17	+4	16	-1	16	-29
IV	b	17	28	+3	11	+1	10	0	10	-1	10	-3	10	-6
V	b	9	10	+12	14	+6	10	+1	7	+2	11	-1	12	-13
VI	c	1	1	+3	3	+4	4	-2	2	-7	7	-9	9	-12
	d	2	3	-5	14	-12	12	-12	12	-30	30	-31	31	-26
	e	5	8	-11	11	-12	12	-11	11	-15	15	-18	18	-7
	f	2	2	-9	9	-8	8	-6	6	-10	10	-14	14	-5
	g	2	3	+18	29	+7	22	+8	22	+6	19	+1	16	-17
VII	h	3	3	-10	12	-15	15	-16	16	-19	19	-22	22	-12
	i	2	2	+5	7	+2	4	0	6	-8	8	-11	11	-16
	k	6	6	+5	11	+5	12	+9	14	+5	13	-2	14	-7
	l	5	5	+32	32	+36	36	+35	35	+30	30	+14	29	-18
	m	4	4	+52	52	+60	60	+57	57	+52	52	+46	46	-6
	n	6	6	+74	74	+73	73	+77	77	+73	73	+67	67	-7

EXCHANGE SAMPLES.

IV	Hulett I.....	1	2	+32	32	+32	32	+36	36	+36	36	+33	33	+1
	Hulett II.....	1	2					0	4	-3	3			
	Guthe.....	1	1									+25	25	

EXCHANGE CELLS.

XIV & XV	Hulett.....	3	3			+40	40	+44	44	+41	41	+41	41	+1
	Guthe.....	1	1					4	4	-1	1	-3	3	-7
	Carhart.....	1	2					+6	6	+4	5	+2	5	-4
	Wald, '06.....	1	2	-15	15									
	Wald, '07.....	4	6							-14	20	-12	18	
	Strong.....											-5	10	

¹ June 14, '07.² June 29, '07.

spection of Table XVI will show that practically no change has taken place in the twelve Weston cells, chosen as standards, with reference to the corresponding Clark cells. This is also shown by the close agreement of duplicate Weston and Clark cells set up from time to time. (Tables III to XII.) One lot of the former, particularly those in Table III, set up June 28, 1907, show relatively large initial values,

TABLE XVIII.

Summary of Results.

Clark Cells—Mean Differences and Average Deviations, in Microvolts, for Each Series of Mercurous Sulphate Samples.

Table	Method	No. Samples	No. Cells	Dec. 1, '06		March 1, '07		May 24, '07		July 24, '07		Relative Change in Nine Months
				Mean	Ave. Deviation	Mean	Ave. Deviation	Mean	Ave. Deviation	Mean	Ave. Deviation	
VIII	a	10	12	+ 23	23	+ 18	18	+ 14	18	+ 16	16	- 7
IX	b	9	10	+ 4	7	+ 6	9	+ 9	9	+ 6	7	+ 2
X	b	4	4	+ 14	14	+ 12	13	+ 14	14	+ 13	13	- 1
	c	1	1	+ 8	8	+ 16	16	+ 17	17	+ 13	13	+ 5
	d	2	2	+ 8	8	+ 5	5	+ 7	7	+ 19	19	+11
XI	e	6	9			- 3	4	- 2	4	- 2	3	+ 1
	f	2	2	+ 4	4	+ 8	8	+ 9	9	+ 6	7	+ 2
	g	1	1	+ 17	17	- 3	3			+ 5	5	-12
	h	3	3	+ 1	10	0	9	0	9	+ 1	8	0
	i	2	2	- 2	2	- 3	3	- 1	1	- 4	4	- 2
XII	k	6	7	+ 24	25	+ 29	29	+ 28	28	+ 28	29	+ 4
	l	3	3	+ 51	51	+ 53	53	+ 51	51	+ 47	47	- 4
	n	4	4	+ 71	71	+ 72	72	+ 70	70	+ 68	68	- 3
XIII	Old Specifications	5	8	+308	308	+305	305	+298	298	+282	282	-26

EXCHANGE SAMPLES.

IX	Hulett I.....	1	2	+ 43	43	+ 44	44	+ 46	46	+ 47	47	+ 4
	Hulett II.....	1	1			+ 19	19	+ 19	19	+ 18	18	- 1
	Guthe.....	1	1							+ 49	49	

EXCHANGE CELLS.

XIV	Hulett.....	2	2			+ 12	12	+ 11	12			
	Guthe.....	1	1			+ 58	58	+ 63	63			
	Carhart.....	1	1			+ 44	44	- 18	18			

but at the present time are in practical agreement with those set up twelve months before.

The relative changes in the Clark cells in the eight months are smaller than in the case of the Weston cells. Excluding those set up in accordance with the old specifications, the changes, which are both positive and negative, are less than 1 part in 100,000.

Size of grain.—In view of the work of von Steinwehr on the influence exerted upon the electromotive force by varying size of grain, efforts were made in some cases to obtain samples as coarse as possible, but the close agreement in both Clark and Weston cells of practically all the samples prepared by methods *a* to *k*, inclusive, shows that special precautions in this direction are hardly necessary. Many of the samples were examined under the microscope, and it was found that there were very few particles measuring less than one micron, the average dimensions being considerably greater. On the other hand, it is unnecessary to attempt to obtain very coarse-grained samples on account of the marked hysteresis, due to the small surface exposed to the solvent action of the electrolyte, as shown also by von Steinwehr.

The irregular pitted character of the crystals of all the samples of mercurous sulphate, which was mentioned above (p. 28), may explain, in part at least, the gradual decrease in the electromotive force of some of the cells, on account of the difficulty of completely removing the acid in which the sample was made and preserved.

A study has also been made of the electromotive differences of cadmium sulphate obtained from various makers or made directly from the "C. P." metal. Twenty cells, set up with clear and cloudy crystals, obtained by the recrystallization of 8 samples, from acid, neutral and basic solutions, show an average deviation from the mean of all of ± 9 microvolts and with a range of $+28$ and -26 .

Further work has been done on the remaining materials and on other factors which might influence the electromotive force, i. e., the size of grain, depth of paste, influence of diffusion and influence of impurities. A determination of the temperature coefficients will also be made.

GENERAL CONCLUSIONS.

The principal conclusions to be drawn are—

1. That samples of mercurous sulphate of uniform electromotive properties can be prepared by a number of different methods.

2. The remaining materials used in the construction of standard cells can easily be obtained in an exceedingly pure state, and, besides, the ordinary impurities even in relatively large quantity, exert practically no influence on the electromotive force, so that Clark and Weston standard cells are reproducible to within 2 or 3 parts in 100,000.

3. When set up with proper materials the cells generally reach a practically constant value within a few days, and the electromotive force of both types can be depended on well within the above limits for at least one year.

4. The close agreement of the numerous samples prepared by different methods and under varying conditions shows that any effect due to size of grain may be disregarded.

5. The results obtained with exchange cells and samples of mercurous sulphate further establish the reproducibility of the cell.

6. Our experience with exchange cells, sent to and received from other investigators, shows that with ordinary precautions cells may be transported considerable distances without influencing the electromotive force.

7. The labor of preparing suitable materials for use in standard cells is much greater than the work of setting up the cells when all materials are ready. If these materials were to be furnished by the various national physical laboratories, standard cells could be set up by any investigator in a very short time, and the standard cell could well serve as one of the two *fundamental* electrical standards.

If the ampere as defined by the quantity of silver deposited per second were to be chosen as one of the two fundamental units (the ohm being the other), the standard cell would have to be depended upon between coulometer measurements to maintain the standard. Inasmuch as these measurements are somewhat laborious it is probable that they would be made quite infrequently. The standard cell has, however, shown itself to be sufficiently constant to justify trusting it for considerable periods, and if new cells set up occasionally agree with the old it would be necessary to resort to silver coulometer measurements only at considerable periods of time. This suggests, indeed, that the use of the silver coulometer as one of the two fundamental standards is of doubtful utility, as it seems improbable that it has sufficient advantage from the point of view of repro-

ducibility (if, indeed, it is as reproducible as the standard cell) to compensate for its greater inconvenience.

In the opinion of the authors the standard cell is to be preferred to the silver coulometer for another reason, namely, that practically all voltage and current measurements of precision are made by the potentiometer method, and thus directly in terms of the standard cell or a combination of a standard cell and a standard resistance.

The authors wish to acknowledge the valuable assistance rendered by Mr. M. P. Shoemaker, both in the preparation of the cells and in the electrical measurements.

APPENDIX.

REPORT ON STANDARD CELLS EXCHANGED WITH FOREIGN LABORATORIES.

On May 9, 1907, twelve Clark and twelve Weston cells, which had been set up at the Bureau of Standards and kept under observation for some time, were taken abroad by Dr. G. K. Burgess of this Bureau. They were carefully packed in tin cases after being separately wrapped in cotton and paraffined paper. The cans were carried by hand in a traveling bag, and kept in an upright position during transit. Rough weather was encountered for about three days, both on the outgoing and the return ocean passages. Six of the cells, three Clark and three Weston, were constructed with the aim of securing portability by the aid of an asbestos plug in each limb, held in place by a perforated glass tube fused to the cell walls.

Comparisons of all the cells were first made at the English National Physical Laboratory between May 25 and May 31, the Weston cells being compared with an equal number of cells of the same type, set up by Mr. F. E. Smith, four of which were kindly given the Bureau of Standards and four each to the Reichsanstalt and to Professor Mascart. The results show that the mean value of the Bureau Weston cells is less by three microvolts than the mean of the National Physical Laboratory cells with which they were compared. The results given in the tables were reduced to the Bureau basis of comparison by the aid of the measurements made

May 9 and August 22 on six of the Weston cells which were brought back to Washington.

According to a letter from Mr. Smith, the mean of all the Weston cells (more than 100) of the National Physical Laboratory is about 20 microvolts greater than the mean of the Bureau cells compared, so that the mean of all would be about 10 microvolts greater than the Bureau basis of reference, which, as stated above, is the mean of twelve Weston cells set up in May, 1906.

Eight of the Bureau Weston cells were next measured at the Laboratoire Central d'Électricité "under conditions which allowed an approximation to 1 part in 100,000." The maximum deviation of these cells from their mean was found to be 0.00002 volt, and the difference between this mean and the mean of the Weston standard cells of the Laboratoire Central was found to be of the order of 0.00001 volt. Eight of the English Weston cells compared under the same conditions showed a maximum deviation from their mean of about 0.00003 volt, and their mean differed from the mean of the Weston cells of the Laboratoire Central by about 0.00001 volt. A second set of comparisons, made at Paris in August and for which a formal report has not yet been received, confirms the first measurements on the Bureau cells, while four Weston cells constructed at the National Physical Laboratory had a mean electromotive force about 0.00003 volt greater than the mean of those of the Laboratoire Central d'Électricité. The above comparisons were made in kerosene baths which were stirred during the measurements. No temperature control was provided, so that the differences found may be partly accounted for by a slight hysteresis. Eight of the Weston cells of the Bureau and eight of the English cells were compared at Berlin, June 20 and 21, between the two comparisons at Paris. A preliminary report of the measurements made at Berlin has been received with the request to withhold publication for the present, so that the results of comparisons with the Reichsanstalt cells are omitted. We have, however, taken the liberty of calculating the relative values of the Bureau and the National Physical Laboratory cells to our basis of reference by the aid of measurements made May 9 and August 22 on Weston cells 105, P8, P9, P10, and 187, assuming the small rate of change in their mean to have been uniform in the interval. The comparisons at the National Physical Laboratory

were reduced in the same manner. Two Weston cells were kindly placed at the disposal of the Bureau by the Laboratoire Central d'Électricité and two by the Reichsanstalt.

Clark cells.—No measurements on the Clark cells were made at the Laboratoire Central d'Electricité. Those at the National Physical Laboratory showed a discrepancy in cells 35 and 28a, set up April 12, 1906, and November 23, 1906, respectively. As these measurements were not made in a thermostatically controlled bath, and as, in addition, the first measurements made on their return to Washington show similar differences, it is highly probable that the discrepancy of May 25–31 is due to hysteresis, which would be greater in the older cells, the others having been set up April 30, 1907. The comparisons made in Washington on May 9 on cells 35, 28a, 97, 98, 99, and 109, before their departure, and on August 22 on their return, show that the average change of the Clark cells has been even less than that of the Weston cells. The measurements made at London and Berlin were reduced to the Bureau basis in the same manner as described above for the Weston cells.

In the accompanying Tables XIX and XX, column 1 gives the numbers of the cells, column 2 the dates on which they were set up, columns 3, 4, 5, and 6 data in reference to the materials employed, and the remaining columns the differences in microvolts from the mean of twelve Weston cells and five Clark cells taken respectively as the basis of reference. The comparisons on May 9 and August 22 of the six Weston cells which had been measured at the three foreign laboratories show a mean decrease of six microvolts on their return to Washington, the change in only one cell exceeding ten microvolts. Practically all of the change of the cells set up April, 1907, is to be attributed to a small decrease frequently observed in cells newly set up, as is shown by check cells set up at the same time which remained in Washington. The Weston table includes the measurements on the exchange cells made immediately after their receipt at the Bureau. It will be noted that one of the cells of the National Physical Laboratory had a somewhat high initial value, but is rapidly approaching the mean of its companion cells.

The measurements on the two cells received from the Reichsanstalt are also given for the sake of completeness, although, on account

TABLE XIX.

Tabulation of Results.

Weston Cells—Differences in microvolts from mean of reference cells.

Cell	Date	Hg ₂ SO ₄	CdSO ₄	Cd-Amalg.	July 5, 1906	Sept. 13	Nov. 19	Jan. 10, 1907	Mar. 1	Mar. 25
19	June 22, '06	D=9.25V=.5	Kahl.2 Cloudy	12½ Kahl. El.	- 9	- 7	- 7	- 4	- 8	- 7
105	June 27, '06	Lunge IV	"	"	-11	-16	-17	-15	-19	-18
					Apr. 23, 1907	Apr. 30	May 1	May 2	May 3	May 8
P8	April 13, '07	#1 Dec. 21, '06	Kahl.3 Cloudy	12½ Kahl. El.	- 8		- 7	-10	- 8	- 3
P9	"	1:6 H ₂ SO ₄	"	"	-13		-12	-13	-11	- 7
P10	"	D=5.25	"	"	-13		-12	-12	-11	- 7
¹ P11	"	"	"	"	- 9		- 8	- 9	- 7	- 4
¹ P12	"	"	"	"	-13		-12	-13	-11	- 6
¹ P13	"	"	"	"	-10		-11	-11	- 9	- 6
181	April 30, '07	#2 Dec. 21, '06	"	"		+12	+ 8	+ 3	+ 4	+ 1
182	"	1:6 H ₂ SO ₄	"	"		+ 6	+ 3	0	+ 2	- 3
183	"	D=5.25	"	"		+ 5	+ 4	- 2	0	+ 4
184	"	"	"	"		+ 8	+ 7	+ 1	+ 3	+ 4
185	"	"	"	"		+ 6	+10	+ 5	+ 6	+ 5
186	"	"	"	"		- 1	- 2	- 8	- 5	- 2
187	"	"	"	"		+ 5	+ 4	+ 1	+ 3	+ 5
¹ 188	"	"	"	"		+ 4	+ 3	- 2	- 1	0
H26	Feb., '07	" Chemical precipitation "								
C19	June, '06									
P53	Nov., '06									
P54	Nov., '06									
P55	Nov., '06									
C12	June, '06									
C117	June, '06									
H29	Feb., '07									
P210	Mar., '07									
H28	Feb., '07									
C17	June, '06									
P52	Nov., '06									
0 1	} Paste disturbed in transit									
0 2										
P.C.N.4										
P.C.N.6										

¹ Cells which remained at the Bureau of Standards.

TABLE XIX.

Tabulation of Results.

Weston Cells—Differences in microvolts from mean of reference cells.

Cell	May 9	May 24	May 25-31 (London)	June 7	June 6-12 (Paris)	June 20-21 (Berlin)	Aug. 2	Aug. 22 ³	Aug. 22	Aug. 23	Aug. 26	Aug. 31	Sept. 17.
19	- 7		- 7			- 7		(-17)	-22	-23	-25	-30	-26
105	-19		-19			-18		(-17)	-22	-23	-25	-23	-17
P8	- 7		- 7			-10		(- 7)	-13	-12	-13	-15	- 7
P9	-10		-15			-12		(-10)	-16	-16	-16	-17	-15
P10	-10		- 8			-12		(-12)	-16	-16	-16	-17	-15
¹ P11	- 7	- 8		-10			-15		-15	-15	-15	-16	-13
¹ P12	- 9	-10		-11			-15		-15	-13	-14	-16	-12
¹ P13	- 9	-10		-13			-16		-16	-14	-16	-17	-15
181	- 2		- 8										
182	- 7		- 8						Left at National Physical Laboratory				
183	- 7		- 6			0			Left at Physikalisch-Technische Reichs-				
184	- 3		- 7			- 5			anstalt				
185	- 2		- 2						Left at the Laboratoire Central d'Electricité				
186	- 9		- 9										
187	- 1		- 4			- 5		(- 5)	- 6	- 6	- 7	- 7	- 4
¹ 188	- 5	- 7		- 7			- 9		-10	- 9	-10	-10	- 7
H26			- 5			- 7		(+ 5)	- 4	-11	-16	-17	-15
C19			- 7			+ 6		(+78)	+36	+15	- 6	-12	-15
P53			- 8			0		(+ 3)	- 4	- 5	- 8	- 7	- 5
P54			- 8			0		(+ 2)	- 3	- 3	- 6	- 7	- 4
P55			- 8			0							
C12			- 9			+ 1			Left at Physikalisch-Technische Reichsanstalt				
C117			- 8			+ 6							
H29			- 9			- 9							
P210			- 5						Left at Laboratoire Central d'Electricité				
H28			- 3										
C17			- 4										
P52			- 5										
0 1								(+338)	+246	+169	+90	+70	+47
0 2								(+ 93)	+ 86	+ 86	+78	+71	+63
P.C.M.4								(+ 8)	+ 5	+ 2	+ 4	+ 4	-10
P.C.M.6								(- 9)	- 25	- 28	-28	-30	-25

³Observations made ten minutes after cells had been put in bath. The values given represent the differences, in microvolts, between the individual cells and the mean of twelve cells of the same type taken as the basis of reference.

TABLE XX.

Tabulation of Results.

Clark Cells.—Differences in microvolts from mean of reference cells.

Cell	Date	Hg ₂ SO ₄	ZnSO ₄	Zn Amailg.	May 1, 1906	Aug. 4	Nov. 2	Jan. 10, 1907	Mar. 1	May 9
35	April 12, '06	D = 1 V = .75	Kahlbaum	10% Kahl. # 1	+ 4	+ 5	+ 5	+ 7	+ 8	+11
28a	Nov. 23, '06	D = .2 V = .5	"	"				+ 2	+ 3	+ 3
97	April 30, '07	#2 Dec. 21, '06	Baker crys-	"	May 1, 1907	May 2	May 3	May 7	May 8	May 9
98	"	1:6 H ₂ SO ₄	tals and so-	"	+ 7	+ 5	+ 4	+ 2	+ 3	+ 2
99	"	D = 5.25	lution	"	+ 5	+ 3	+ 4	+ 2	+ 2	+ 1
¹ 100	"	"	"	"	+ 9	+ 9	+10	+ 7	+ 7	+ 7
¹ 101	"	"	"	"	+ 9	+ 8	+ 6		+ 7	+ 5
103	"	"	"	"	+ 4	+14	+12	+ 8	+ 9	+ 9
104	"	"	"	"	+15	+15	+14	+16	+16	+11
105	"	"	"	"	+ 8	+10	+ 9	+10	+11	+ 9
106	"	"	"	"	+ 8	+ 8	+ 5	+ 5	+ 5	+ 2
107	"	"	"	"	+10	+10	+ 9	+11	+11	+ 9
108	"	"	"	"	+14	+14	+10	+10	+13	+10
109	"	"	"	"	+ 3	+ 3	+ 1	+ 2	+ 4	+ 2
¹ 110	"	"	"	"	+ 6	+ 9	+ 7	+ 7	+ 7	+ 8
					+13	+14	+12	+10	+11	+11

¹Cells which remained at the Bureau of Standards.

TABLE XX.

Tabulation of Results.

Clark Cells.—Differences in microvolts from mean of reference cells.

Cell	May 24	May 25-31, (London)	June 4	June 6-11 (Paris)	June 12	June 20-21 (Berlin)	July 10	Aug. 2	Aug. 22 ²	Aug. 23	Aug. 26	Aug. 31	Sept. 17
35		(-53)				+9			(-15)	+3	+2	0	-1
22a		(-56)				-2			(-45)	-3	-3	-4	-5
97		+1				+9			(+4)	+5	+8	+6	+6
98		+4				+2			(+1)	+3	+6	+5	+5
99		+7							(+1)	-2	+1	+1	+2
¹ 100	+1				+1		0	-1	+1	+1	+1	0	0
¹ 101	+3		+6		+7		+8	+7	+9	+9	+9	+9	+8
103		+3											
104		+3							} Left at National Physical Laboratory				
105		+6				-8			} Left at Physikalisches-Technische Reichsanstalt				
106		+7				-6							
107		+5							} Left at Laboratoire Central d'Electricité				
108		+2											
109		+6				+2			(-9)	+4	+7	+7	+6
¹ 110	+6		+8		+7		+8	+7	+9	+9	+9	+8	+8

² Measurements made within two hours after placing cells in bath. The values given represent the differences, in microvolts, between the individual cells and the mean of five cells of the same type taken as the basis of reference.

of the fact that the paste in both had evidently been disturbed in transit, they should not be considered as throwing any light on the question of reproducibility. The cells received from the Laboratoire Central d'Électricité show a most satisfactory agreement with the Bureau cells and with those received from the National Physical Laboratory.

The conclusions to be drawn are:

- (1) That standard cells can be set up by different investigators with different materials which agree to within a few parts in 100,000.
- (2) That standard cells can be constructed which show no appreciable change when carried considerable distances, even on ship-board, provided ordinary precautions are observed.

WASHINGTON, September 17, 1907.

THE ELECTRODE EQUILIBRIUM OF THE STANDARD CELL.

F. A. Wolff and C. E. Waters.¹

Some experiments described by Hulett² indicating the existence of a state of unstable equilibrium in the Weston cell have, on account of the importance of the subject, led to a further study of the question at the Bureau of Standards.

The equilibrium between cadmium amalgam and cadmium sulphate was first studied. Saturated solutions of the latter were shaken up in air, nitrogen, hydrogen, and *in vacuo* with cadmium amalgam, but in no case did the electromotive force of cells set up with the treated and untreated solutions differ more than 10 microvolts, even though the samples shaken in the presence of air had become cloudy from the formation of an excess of basic cadmium sulphate.

The equilibrium of the system mercury, mercurous sulphate, cadmium sulphate, and of the corresponding system of the Clark cell was then studied in special cells, so constructed that the above materials could be rotated and the effect determined without opening the cell.

This consisted of a tube about 2 cm in diameter and 12 cm long, provided at the lower end with a small bulb into which a platinum wire was sealed. The bulb was connected to the main tube by a narrow neck, so that, with sufficient mercury in the cell, the platinum terminal was not in contact with the solution, even during rotation.

¹An abstract of this paper was read at the New York meeting of the American Physical Society, December, 1906, *Phys. Rev.*, **24**, 251; 1907.

²*Phys. Rev.*, **23**, 166; 1906.

A shorter internal tube about 1 cm in diameter with several lateral openings, sealed in at the other end of the main tube, was charged with amalgam and crystals of cadmium or zinc sulphate, which were held in place by a plug of asbestos. Contact with the amalgam was made by means of a platinum wire, protected from the solution by sealing it into a glass tube, which extended through the upper seal made after the introduction of the above materials.

The mercurous sulphate and cadmium sulphate or zinc sulphate crystals were introduced over the mercury through a side tube by which the cell was exhausted and through which a saturated solution of cadmium or zinc sulphate was subsequently introduced. After filling, the side tube was sealed off, leaving only a small air bubble in the cell, thus practically eliminating any possible influence of air.

The cells were then placed in holders so arranged that one dozen could be simultaneously rotated at any desired speed, with their axes parallel to the axis of rotation and inclined at a small angle to the horizontal, thus insuring a thorough stirring of the paste and mercury with the electrolyte, even at five or six revolutions per minute. The measurements were made in an automatically controlled oil bath at 25°, immediately after stopping the rotation and raising the holder to a vertical position under the oil. Altogether 12 Weston and 5 Clark cells of this type were set up with samples of mercurous sulphate, made not only by the electrolytic method but also by several chemical methods. The mercurous sulphate was washed in a Gooch crucible three times with 1 : 6 sulphuric acid, six times with absolute alcohol, and three times with saturated zinc or cadmium sulphate solution. The data and results are given in the following tables:

TABLE I.
Weston Rotating Cells.

Cell	Date 1906	CdSO ₄	Cd Amalg.	Mercury	Hg ₂ SO ₄	
					Sample	Method of Preparation
R 1	Oct. 13	Purified by Recrystallisation.	12½% Kahl. Et. Dist. at reduced pressure.	Distilled	b ₁₃ gray	Electrolytic (D=5; V=.5)
*R 11	Oct. 30	"	"	"	b ₁₃ gray	Electrolytic (D=5; V=.5)
†R 12	Oct. 30	"	"	"	b ₁₃ gray	Electrolytic (D=5; V=.5)
R 3	Oct. 13	"	"	"	e ₃ gray	By action of HN ₂ O ₃ and H ₂ SO ₄ on Hg
R 5	Oct. 16	"	"	"	d ₂ gray	From HgNO ₃ and H ₂ SO ₄
R 6	Oct. 16	"	"	"	c gray	By action of fuming H ₂ SO ₄ on Hg
R 2	Oct. 13	"	"	"	a ₁₂ white	Electrolytic (D=.25 1:16 H ₂ SO ₄)
‡R 8	Oct. 30	"	"	"	a ₁₂ white	
R 4	Oct. 16	"	"	"	a ₉ white	
‡R 10	Oct. 30	"	"	"	a ₉ white	
R 7	Oct. 16	"	"	"	i ₂ white	By digestion of Coml. Sample with H ₂ SO ₄
‡R 9	Oct. 30	"	"	"	i ₂ white	

* Platinum terminal exposed to paste.

† Basic cadmium sulphate added to paste.

‡ Mercurous sulphate rotated with saturated CdSO₄ solution 2½ days before its introduction into the cells.TABLE II.
Clark Rotating Cells.

Cell	Date 1906	Zn SO ₄	Zn Amalg.	Mercury	Hg ₂ SO ₄	
					Sample	Method of Preparation
R 2	Nov. 5	Purified by Electrolysis	Kahl. I	Distilled at reduced pressure	b ₁₀ gray	Electrolytic (D=9.25; V=.75)
*R 3	Nov. 5	"	"	"	10 b gray	Electrolytic (D=9.25; V=.75)
R 5	Nov. 5	"	"	"	a ₁₂ white	Electrolytic (D=.25; 1:6 H ₂ SO ₄)
*R 4	Nov. 5	"	"	"	a ₁₂ white	
R 1	Nov. 5	"	"	"	a ₉ white	

* Zinc oxide added to paste.

TABLE III.

Weston Rotation Cells.

[Differences in microvolts from mean of reference cells.]

Date	R 1		R 11		R 12		R 3		R 5		R 6	
	Diff.	Time	Diff.	Time	Diff.	Time	Diff.	Time	Diff.	Time	Diff.	Time
		D. H.		D. H.		D. H.		D. H.		D. H.		D. H.
Oct. 18, '06	+72	0					+89	0	+ 83	0	+92	0
" 19	+43	0					+59	0	+ 59	0	+67	0
" 20	+11	0					+24	0	+ 34	0	+39	0
" 22	+29	0					+39	0	+ 54	0	+55	0
" 23	+44	15					+53	15	+ 57	15	+66	15
" 24	+42	18					+57	18	+ 88	18	+43	18
" 25	+45	1 9					+41	1 9	+103	1 9	+41	1 9
" 27	+63	3 1					+64	3 1	+142	3 1	+58	3 1
" 29	+88	5 1					+88	5 1	+149	5 1	+83	5 1
" 31	+61	6 21	- 4	0	-11	0	+69	6 21	+ 93	6 21	+56	6 21
Nov. 5	+73	10 14	+ 55	3 7	+37	3 7	+83	10 14	+ 90	10 14	+65	10 14
" 8	+66	12 17	+ 79	5 11	-17	5 11	+67	12 17	+104	12 17	+46	12 17
" 12	+72	16 12	+ 69	9 0	-31	9 6	+69	16 12	+ 71	16 12	+52	16 12
" 16	+64	20 8	+ 116	12 20	-54	13 2	+70	20 8	+ 50	20 8	+55	20 8
" 19	+59	23 5	+ 139	15 17	-36	15 23	+69	23 5	+ 47	23 5	+54	23 5
" 24	+50	28 4	+ 204	20 16	-26	20 22	+71	28 4	+ 41	28 4	+54	28 4
" 30	+35	28 8	+ 117	20 20	-37	21 2	+35	28 8	+ 29	28 8	+32	28 8
Dec. 5	+31	28 8	+ 93	20 20	-39	21 2	+33	28 8	+ 26	28 8	+30	28 8
" 10	+46	32 1	+ 240	24 13	- 8	24 19	+74	32 1	+ 40	32 1	+54	32 1
" 14	+50	35 16	+ 271	28 4	-25	28 10	+74	35 16	+ 45	35 16	+53	35 16
" 21			+ 348	33 21	-17	34 12	+92	41 18	+ 41	41 18	+38	41 18
Jan. 5, '07			+ 520	46 12	-32	47 3	+67	54 7	+ 41	54 7	+19	54 7
" 10			+ 567	50 6	-27	51 20	+66	59 4	+ 47	59 4	+28	59 4
" 14			+ 427	52 18	-34	54 8	+51	61 16	+ 40	61 16	+13	61 16
" 24			+ 749	61 12	-26	62 16	+66	70 0	+ 46	70 0	+33	70 0
" 30			+ 926	67 2	-36	68 6	+50	75 22	+ 42	75 22	+24	75 22
Feb. 5			+1030	71 10	-36	73 6	+49	80 21	+ 37	80 21	+32	80 21
" 8			+1073	72 21	-35	74 18	+46	82 9	+ 38	82 9	+28	82 9
Mar. 1					-43	93 14	-38	101 5	+ 38	101 5	+35	101 5
" 12					-42	102 14	-29	110 5	+ 32	110 5	+40	110 5
Apr. 5					-41	121 4	- 8	128 19	+ 90	128 19	+51	128 19
" 10					-34	124 10	-10	132 1	+ 94	132 1	+76	132 1
May 25					-38	135 4	- 7	142 19	+127	142 19	+84	142 19
June 4					-34	144 20	- 3	152 11	+135	152 11	+95	152 11
" 7					-37	147 18	+ 2	155 9	+142	155 9	+94	155 9
" 15					-37	155 6	- 3	162 21	+169	162 21	+99	162 21
Aug. 15			+ 494	72 21	-40	155 6	- 5	162 21	+ 44	162 21	+43	162 21
Sept. 27			+ 571	72 21	-41	155 6	- 6	162 21	+ 38	162 21	+23	162 21

TABLE III.

Weston Rotation Cells.

[Differences in microvolts from mean of reference cells.]

R2		R8		R4		R10		R7		R9	
Diff.	Time	Diff.	Time	Diff.	Time	Diff.	Time	Diff.	Time	Diff.	Time
	D. H.		D. H.		D. H.		D. H.		D. H.		D. H.
+148	0			+ 137	0			+ 93	0		
+115	0			+ 100	0			+ 49	0		
+ 59	0			+ 69	0			+ 10	0		
+ 98	0			+ 94	0				0		
+103	15			+ 95	15			+ 46	15		
+220	18			+ 252	18			+115	18		
+275	1 9			+ 354	1 9			+173	1 19		
+245	3 1			+ 316	3 1			+168	3 9		
+313	5 1			+ 363	5 1			+178	5 9		
+321	6 21	- 9 0		+ 441	6 21	+216	0	+111	7 5	+ 76	0
+340	10 14	+515	3 7	+ 440	10 14	+590	3 7	+187	10 22	+295	3 7
+329	12 17	+474	5 11	+ 572	12 17	+556	5 11	+141	13 2	+249	5 11
+292	16 12	+389	9 0	+ 429	16 12	+501	9 0	+ 97	16 0	+186	9 0
+279	20 8	+380	12 20	+ 810	20 8	+440	12 20	+ 90	19 20	+187	12 20
+259	23 5	+414	15 17	+ 754	23 5	+479	15 17	+ 81	22 16	+159	15 17
+237	28 4	+364	20 16	+ 754	28 4	+389	20 16	+ 61	27 16	+104	20 16
+152	28 8	+272	20 20	+ 653	28 8	+265	20 20	+ 57	27 19	+ 93	20 20
+136	28 8	+228	20 20	+ 613	28 8	+234	20 20	+ 52	27 19	+ 85	20 20
+204	32 1	+337	24 13	+ 931	32 1	+336	24 13	+ 69	31 12	+107	24 13
+194	35 16	+336	28 4	+1092	35 16	+314	28 4	+ 68	35 3	+106	28 4
+191	41 18					+248	33 21	+ 74	40 20	+ 95	33 21
+150	54 7					+207	46 12	+ 63	53 12	+ 63	46 12
+147	59 4					+207	50 6	+ 58	57 6	+ 72	50 6
+126	61 16					+195	52 18	+ 70	59 17	+ 53	52 18
+ 56	70					+196	61 12	+ 73	68 12	+ 72	61 12
+110	75 22					+146	67 2	+ 50	74 2	+ 36	67 2
+115	80 21					+135	71 10	+ 44	78 10	+ 38	71 10
+113	82 9					+139	72 21	+ 45	79 21	+ 33	72 21
+118	101 5										
+ 60	110 5										
+115	128 19										
+114	132 1										
+113	142 19										
+115	152 11										
+111	155 9										
+135	162 21										
+ 15	162 21					+ 55	72 21	+ 36	79 21	+ 12	72 21
+ 7	162 21					+ 55	72 21	+ 28	79 21	+ 13	72 21

TABLE IV.
Clark Rotation Cells.

[Differences in microvolts from mean of reference cells.]

Date	R 2		R 3		R 5		R 4		R 1	
	Diff.	Time	Diff.	Time	Diff.	Time	Diff.	Time	Diff.	Time
		D. H.		D. H.		D. H.		D. H.		D. H.
Nov. 6, '06	-25	0	-120	0	0	0	-140	0	+13	0
Nov. 7	-13	22	-130	22	+30	22	-50	22	+27	22
Nov. 8	+16	1 22	-54	1 22	+59	1 22	-26	1 22	+49	1 22
Nov. 12	+16	4 16	+45	4 16	+100	4 16	+25	4 16	+90	4 16
Nov. 16	+17	8 13	+2	8 13	+89	8 13	+2	8 13	+72	8 13
Nov. 24	+21	16 9	+4	16 9	+89	16 9	± 0	16 9	+81	16 9
Nov. 30	+36	16 12	+18	16 12	+61	16 12	-5	16 12	+46	16 12
Dec. 5	+22	16 12	-20	16 12	+45	16 12	-43	16 12	+22	16 12
Dec. 10	+22	20 6	+202	20 6	+72	20 6	-34	20 6	+22	20 6
Dec. 14	+21	23 20	+273	23 20	+113	23 20	+98	23 20	+8	23 20
Dec. 21	+26	29 22	+22	29 22	+34	29 22	+20	29 22	-36	29 22
Jan. 5, '07			-217	42 14	-136	42 14	-17	42 14	+12	42 14
Jan. 10			-243	47 10	-141	47 10	-3	47 10	+7	47 10
Jan. 14			-160	49 22	-199	49 22	-50	49 22	-99	49 22
Jan. 24			+5	58 7	-5	58 7	-47	58 7	+157	58 7
Jan. 30			+20	64 4	-159	64 4	-35	64 4	-33	64 4
Feb. 5			+16	69 4	-155	69 4	-26	69 4	-48	69 4
Feb. 8			+17	70 16	-145	70 16	-23	70 16	-4	70 16
Mar. 1			-17	89 12	-143	89 12	-37	89 12	-97	89 12
Mar. 12			-20	98 12	-182	98 12	-60	98 12	-227	98 12
Apr. 5			-88	117 2	-68	117 2	-28	117 2	-108	117 2
Apr. 10			-91	120 8	-59	120 8	-43	120 8	-145	120 8
May 25					-76	131 2	-79	131 2	+174	131 2
June 4					-91	140 18	-84	140 18	-216	140 18
June 7			-47	143 16	-79	143 16	-62	143 16		
June 15					-86	151 4	-76	151 4	+3	151 4

It will be seen that no initial low values, as observed by Hulett, were obtained. The Weston cells with gray samples of mercurous sulphate show changes which do not exceed 0.01 per cent even after continuing the rotation four months, except in one case in which the platinum terminal was purposely exposed. In this cell the terminal was inadvertently amalgamated, but was treated with aqua regia to remove the mercury before filling. The white samples generally showed considerably larger effects, but not as great as those observed by Hulett. There was also a tendency to reach a maximum value, and then, on further rotation, to approach the normal.

Cell No. 12, in which basic cadmium sulphate³ was added in excess to the cadmium sulphate solution, gave approximately normal results from the first, although the paste was quite yellow.

Although the Clark cells showed smaller effects than the Weston cells, there seem to be slight differences produced by rotation. Cells 3 and 4, in which an excess of zinc oxide was added to the zinc sulphate solution, showed only a slight difference; but after an interruption of the rotation for several days the paste caked and the results subsequently obtained were irregular. The irregularities of the remaining Clark cells subsequent to December 5 may possibly be due to the cracking of the protecting tube about the platinum wire imbedded in the amalgam. Owing to the construction employed, this can not be determined without destroying the cell.

On December 19, 1906, the Weston cells 1, 4, and 8 were opened and the paste used in setting up cells of the ordinary form. The gray sample from cell No. 1 gave, after two days, practically the same value as in the rotating cell. It has slowly decreased and at present is within 25 microvolts of the normal. The two cells set up with the white samples showed abnormally high values, 600 and 400 microvolts, respectively, both of which steadily decreased until March 25, 1907. On that date they were transferred from the bath in which they had been kept continuously at 25° to another at 20°

³Made by adding sufficient ammonia to a solution of cadmium sulphate to dissolve the precipitate first formed, filtering into a large volume of water, collecting, washing, and gently igniting the precipitate.

While only a very slight hysteresis was observed in the case of our Weston cells, particularly when finely crushed cadmium sulphate crystals were employed in the paste and over the amalgam, the two cells in question showed marked effects of this character. These persisted, with slight changes from day to day, for a long period, indicating either that a surface film was formed on the crystals during rotation or that a large excess of cadmium sulphate was employed in the paste, thus retarding the attainment of saturation equilibrium. A similar effect has been observed in a number of H cells in which the paste contained a large excess of cadmium sulphate crystals.

The results obtained show that not all samples of mercurous sulphate exhibit the behavior observed by Hulett, as gray samples, prepared by different methods, are changed by rotation for months by less than 0.01 per cent.⁴ This difference is small enough to be possibly accounted for by attrition during rotation, thus introducing the effect of size of grain, noticed by von Steinwehr.

It does not appear to the authors that the high values shown by some of the rotation cells throw any light upon the decrease below the normal electromotive force observed by Hulett in some cells of the ordinary type. According to him the first reaction which takes place when the cell is set up is a hydrolysis by which the concentration of the mercury ions is increased. Opposing this is a reaction between the mercury and the products of hydrolysis in solution, which reduces the concentration of the mercury ions. The secondary products formed in this reaction would, therefore, be responsible for the decrease observed by him in cells of the ordinary type. As it is evident that this effect can not be due directly to the formation of new mercury compounds in the presence of an excess of mercurous sulphate, since the concentration of the mercury ions would thereby be increased, it must, on this theory, be caused by other products of the reaction.

In the rotation cells the amount of these secondary products would depend upon the duration of the rotation and on the area of the mercury surface exposed to the solution. According to Hulett's

⁴ While the samples of mercurous sulphate prepared by Hulett are in practical agreement with our own, as shown by cells set up with exchange samples, his cells differ from those made at the Bureau by about one-third of the above amount.

explanation, the slight effects observed by us with gray samples of mercurous sulphate, which owe their color to the presence of finely divided mercury, would be due, not to decreased hydrolysis, but to the acceleration of the secondary reaction. It would, therefore, follow that such samples should give abnormally low values after stopping the rotation. As in no case was this result obtained, the authors conclude that the effect must be due to some other cause.

In view of the various questions which have arisen in connection with the results described in this paper, it is proposed to continue the investigation.

WASHINGTON, October 4, 1907.



A COMPARATIVE STUDY OF PLAIN AND FROSTED LAMPS.¹

By Edward P. Hyde and F. E. Cady.

Much attention has been given in recent years to the proper use of diffusing globes around incandescent lamps. These globes have the twofold object of hiding from view the brilliant filament and of producing a more economical distribution of the light. It has usually been considered that the same general results could be obtained by frosting the bulb of the lamp, but this method has been called into question² principally on account of the relatively short useful life of frosted lamps. So far as the authors know, however, no systematic study of the effects of frosting incandescent lamps has ever been undertaken. Such an investigation was therefore outlined at the Bureau of Standards several years ago, and although many observations have been made the subject is by no means exhausted.

These results of frosting may be classified under three general heads: (1) Change in Absorption; (2) Change in Distribution; (3) Change in Life.

In conjunction with the study of the change in distribution many observations were made on the mean spherical reduction factors of a number of types of filaments. In fact, owing to the importance of this question in its bearing on the commercial rating of incandescent lamps on a basis of mean spherical candlepower, much attention was given to this part of the investigation. The results will, therefore, be discussed in a separate section of the paper.

¹ Presented in abbreviated form under the title of "The Effect of Frosting Incandescent Lamps" before the National Electric Light Association, Washington, D. C., June 6, 1907.

² Cravath and Lansingh, *Electrical World*, March 17, 1906, p. 567. See also *Electrical World*, May 26, p. 1082, and June 23, p. 1304; 1906.

1. APPARATUS AND METHODS.

The principal apparatus employed in the investigation were a Reichsanstalt precision photometer bench on which the measurements of mean horizontal intensity and the determinations of distribution curves were made, and a Matthews integrating photometer on which mean spherical candlepowers were obtained. The former instrument has been described fully in a previous paper.³ The determination of mean horizontal candlepower was made in every case by rotating the lamp about its axis at a speed of 200–250 r. p. m. In the case of lamps having a pronounced flicker at that speed a single auxiliary mirror, shown diagrammatically at *M*, Fig. 1, was used. As has already been pointed out by the writers in a paper⁴ on the determination of the mean horizontal intensity of incandescent lamps by the rotating-lamp method, by combining on the photometer screen light emitted in two mutually perpendicular directions in the horizontal plane, the flicker due to the nonuniform-

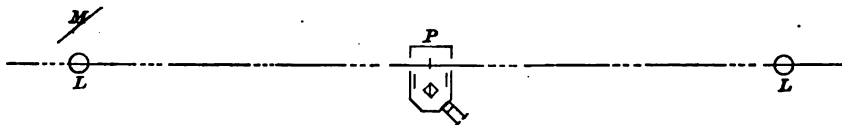


Fig. 1.—Sketch Showing Stationary Auxiliary Mirror in Position.

ity of the horizontal distribution curve of the lamp is greatly reduced, and accurate measurements can be made. Without some device of this kind it is impossible to obtain accurate results for certain types of lamps with the rotating-lamp method. In determining the mean vertical distribution curves of the lamps the speed of rotation was generally about 200–250 r. p. m. For lamps with pronounced flickers higher speeds were used, as it is impracticable to use the auxiliary mirror in these measurements.

The universal rotator which was employed in determining the vertical distribution curves is shown in Fig. 2. This instrument has recently been designed and constructed in the instrument shop of the Bureau. The authors desire to acknowledge their indebtedness to Mr. A. H. Schaaf and Mr. Oscar Lange for valuable aid in designing the instrument, and to Mr. R. Hellbach for painstaking

³ This Bulletin, 2, p. 1; 1906.

⁴ This Bulletin, 2, p. 415; 1906.

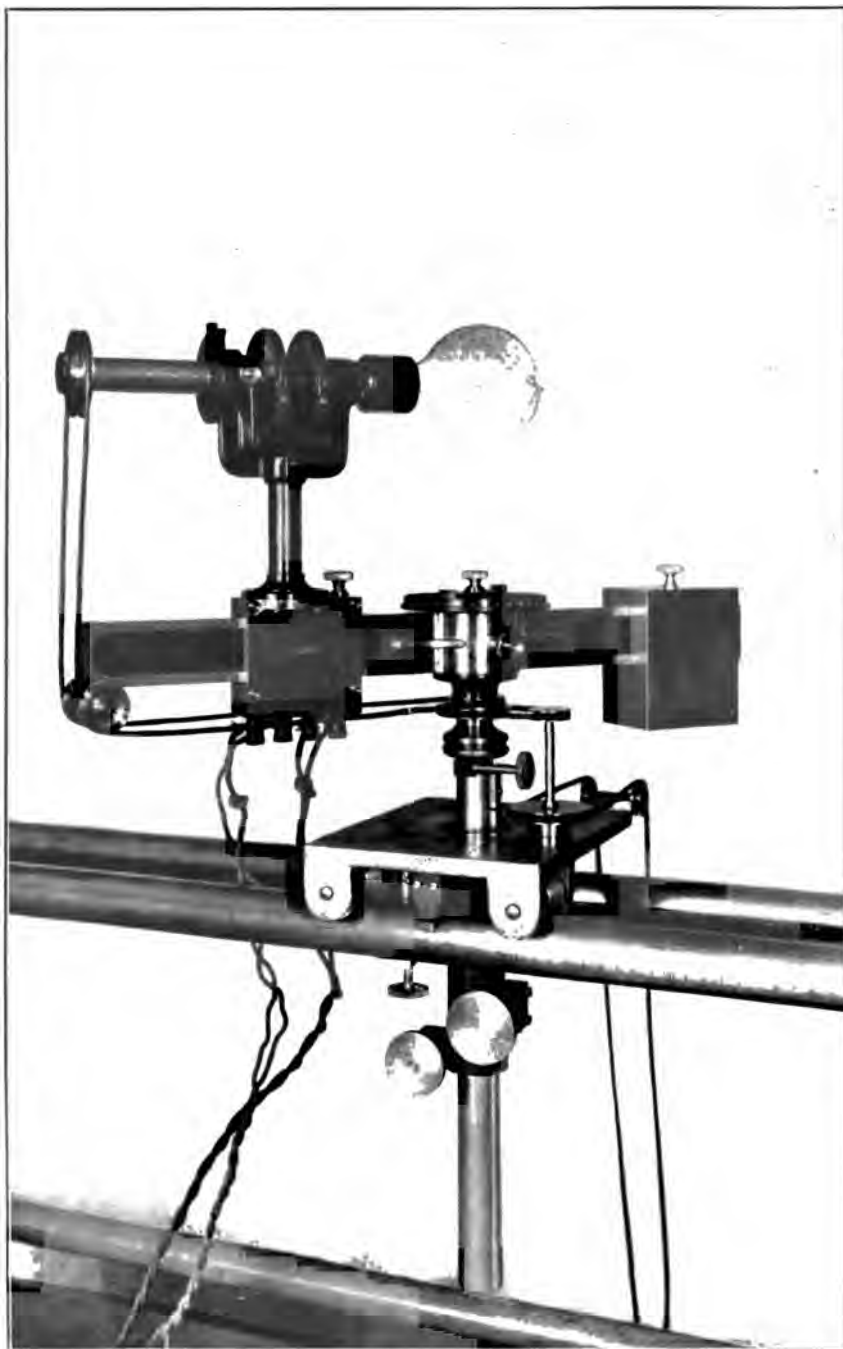


Fig. 2.—Universal Rotator.

care in constructing it. Inasmuch as there is not to be had, either in the United States or abroad, an entirely satisfactory universal rotator for laboratory purposes, it may not be amiss to describe here briefly the instrument made at the Bureau and used with entire satisfaction.⁵

This instrument is so designed that it can be mounted directly on any standard photometer bench. As shown in Fig. 2, it is mounted on a standard carriage supplied with the Schmidt and Haensch photometer. By the use of a counterpoise it is in equilibrium in all positions, and hence produces no strain on its support. The axis of rotation is horizontal at all times, permitting the use of mercury cups. If these are well made and well amalgamated the shaft can be driven at a speed of 500 or 600 revolutions per minute without throwing an appreciable quantity of mercury. Incandescent lamps of all sizes from 4 cp to 100 cp can be centered and measured, and even many of the ordinary shades and reflectors can be mounted with the lamps.

Since there is no strain on any part of the instrument, a divided circle and index with clamp screw can be used instead of a notched wheel. This permits of angular settings with an accuracy of 10 or 15 minutes for any desired angle, whether an even degree or a fraction of a degree. In most rotators on the market only angular settings at 5° intervals can be obtained, whereas it is frequently desirable to study distribution curves at much closer intervals in the neighborhood of rapidly changing curvatures.

Finally, the instrument is provided with a second graduated circle at right angles to the other, so that the instrument can be used as a universal lamp support, as well as a universal rotator. Any angle of latitude or azimuth can be obtained, so that not only the vertical distribution, but also the distribution in any latitude, can be determined.

The Matthews integrating photometer which was used throughout the investigation is shown in Fig. 3. In general principle it is similar to that originally described by Professor Matthews,⁶ but in detailed design it has been modified somewhat, both in theory and

⁵The Bureau will be glad to furnish drawings of this instrument to anyone desiring them.

⁶Proc. A. I. E. E., 20, p. 1465; 1902.

in mechanical construction. Instead of arranging the mirrors at equal angular intervals, beginning with one in the horizontal, use was made of the results of a previous theoretical investigation⁷ by one of the present authors. It was shown in that investigation by considering several simple cases of distribution curves, that the arrangement of mirrors completely satisfying Case III ($I_0 = \cos \theta$), would best satisfy all practical cases. The mirrors were therefore placed at the respective angles deduced in the previous paper. According to this arrangement there is no mirror in the horizontal, but the mirrors are situated symmetrically in the two hemispheres, so that mean hemispherical as well as mean spherical intensities can be determined.

The instrument shown in Fig. 3 was built in the instrument shop of the Bureau. It is $10\frac{1}{2}$ feet high and contains twenty pairs of mirrors. The lamp rotator, which is equipped with four mercury contacts, two for current, and two for potential leads, has two sockets, one upright and one inverted, so that lamps with soft filaments that will not support their own weight can be mounted, and even rotated at a low speed. The comparison lamp is mounted on a carriage which is moved back and forth by means of a pulley-wheel and steel tape. The latter, graduated in millimeters, passes under an index, from which the distance of the comparison lamp can be read.

In using the instrument the substitution method is employed, i. e., a lamp, or number of standard lamps, whose mean spherical candlepowers are known, are first placed in the rotator and measured, and then the test lamps are substituted and measurements are made on them. A measurement consists in determining the distance at which a lamp known as the comparison lamp must be placed in order to secure a photometric balance. From the inverse ratios of the squares of the distances of the comparison lamp, corresponding to a photometric balance on the test and standard lamps, the mean spherical candlepower values of the test lamps in terms of the mean spherical candlepower values of the standard lamps can be computed.

The adjustment of the mirrors is such that accurate ratios of mean spherical candlepower can be obtained even between lamps

⁷This Bulletin, 1, p. 255; 1905.

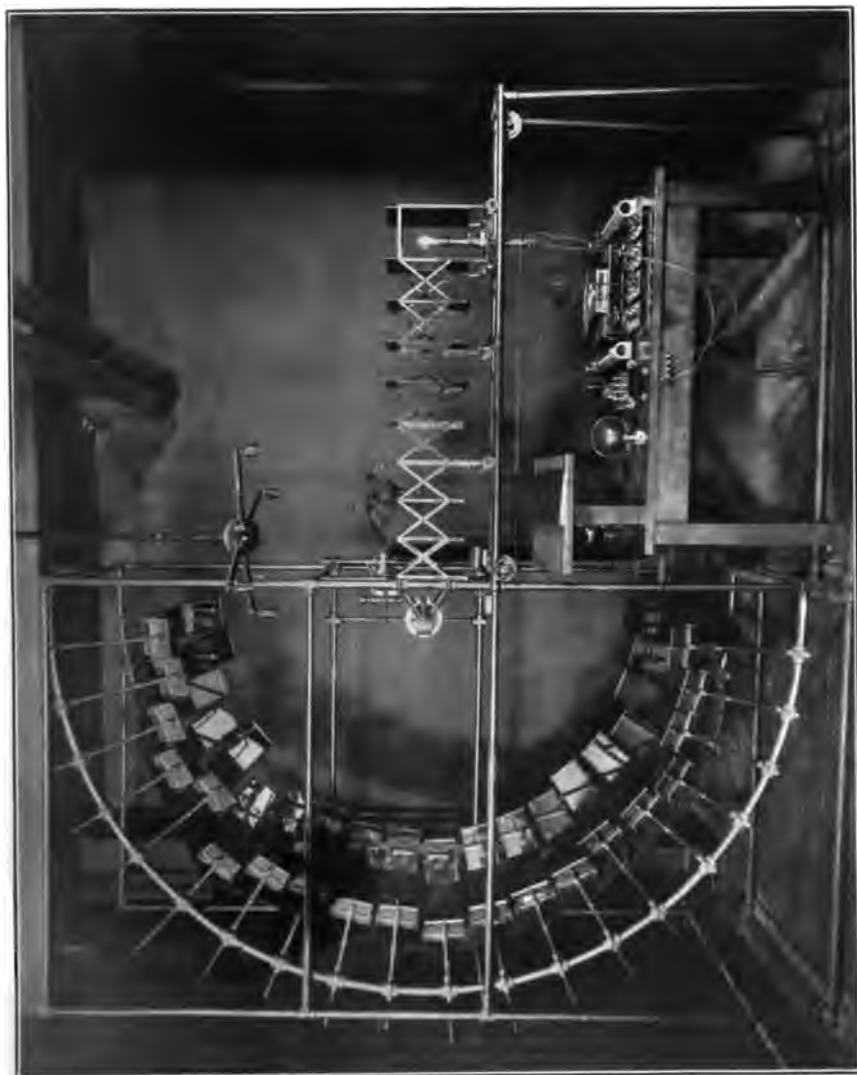


Fig. 3.—*Matthews Integrating Photometer.*



with such widely different polar-distribution curves as the double filament lamp and the downward-light lamp. However, in order to

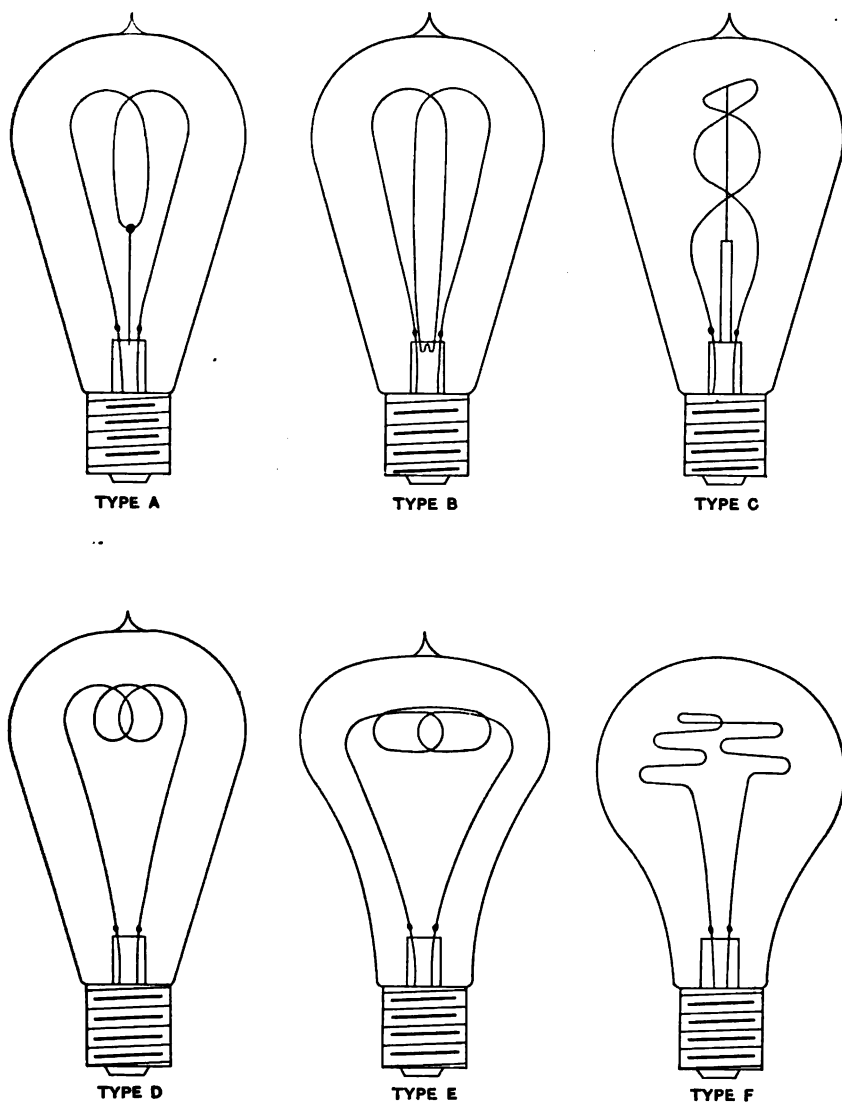


Fig. 4.—Types of Filaments Investigated.

avoid any possible error that might result if the instrument were to get out of adjustment, the substitution method is carried further.

In determining the mean spherical candlepower of lamps of any type of filament, lamps of a similar type of filament are used as standards.

2. CHANGE IN ABSORPTION.

In studying the change in absorption due to frosting, six different types of lamps were used. These are shown in Fig. 4. The six types are as follows: Oval anchored (type A in the figure), double filament (type B), spiral filament (type C), double round coil (type D), double-flattened coil (type E), and downward-light filament (type F). Twenty-five 16-candlepower, 110-volt lamps of each type were obtained directly from the factories, and all were seasoned so that changes in candlepower would not occur during the investigation. Several lamps of each type were carefully calibrated for mean horizontal and mean spherical candlepower for use as standards for that type, and all subsequent measurements on lamps of any type were made in terms of the standards of the type.

Each lot of lamps were measured for mean horizontal and mean spherical intensity, and then sent to the factory to be frosted. After being returned from the factory the lamps were again measured for mean horizontal and mean spherical candlepower. The decrease in mean spherical intensity is taken as the change in absorption due to frosting. The decrease in intensity is spoken of as a *change* in absorption rather than as the absorption itself, because there is always some absorption in the glass even when there is no carbon deposit on the inside and no frosting on the outside of the bulb. The frosting can scarcely be said to absorb light by extinction, it merely diffuses it, and in this diffusion some luminous energy which otherwise would pass through the outer surface of the bulb is compelled to traverse the glass, and whatever deposit there may be on the inside of the glass, a third and fifth time, and thus suffer further absorption. It is probably not the frosted surface that absorbs the energy, however, but the glass, and the carbon deposit on the inside of the bulb.

TABLE I.
Change in Absorption Due to Frosting.

Oval anchored		Double filament	Spiral filament	Double round coil	Double flat-ened coil	Downward light
Lot 1	Lot 2	Lot 3	Lot 4	Lot 5	Lot 6	Lot 7
(acid)	(acid)	(acid)	(acid)	(acid)	(acid)	(acid)
3%	3%	5.5%	1.5%	3%	4%	6%
4	3	5.5	2	3.5	4.5	6.5
4	3	6	3	4	4.5	6.5
4.5	3.5	6	3.5	4.5	5	6.5
4.5	3.5	6.5	4	4.5	5.5	6.5
4.5	4	6.5	4	4.5	5.5	7.5
4.5	4	6.5	4	4.5	5.5	7.5
5	4.5	7.5	4.5	4.5	5.5	7.5
5	4.5		4.5	5	5.5	8
5	5	6.2%	4.5	5	6	8
5.5			5	5.5	6	8
5.5	3.8%		5	5.5	6	8
6	(sand)	(sand)	5.5	6	6.5	8
6	5%	5.5%	5.5	6.5	6.5	8.5
6	5.5	6.5	5.5	7	7	8.5
6.5	5.5	6.5	6.5		7.5	9
6.5	5.5	6.5	6.5	4.9%	8	
7	5.5	7	6.5		9	7.5%
8	5.5	7.5	7		9	
	6	8				
5.3%	6	8.5	4.7%		6.2%	
	6					
	6	7.0%				
	5.6%					

In Table I are given the observed values of change in absorption due to frosting for the individual lamps of seven lots, obtained from different lamp factories and representing six different types of filaments, two of the lots having filaments of the oval-anchored type. As stated previously, the figures given represent the percentage decrease in mean spherical intensity due to frosting. The changes in mean horizontal intensity will be discussed later in connection with the changes in the distribution of the light due to frosting.

In the first two columns are given the values for the two lots of oval-anchored filament lamps. Half of the lamps of Lot 2 were frosted by the acid process and half by the sand-blasting process. It is interesting to note the uniformity among the values of the

lamps of each kind, and the difference between the average absorption coefficients for the acid-frosted and the sand-blasted lamps. The average value for the former is 3.8 per cent, with a range from 3 to 5 per cent for the individual lamps. For the latter the average is 5.6 per cent, with a range from 5 to 6 per cent. On the other hand, the average value for the lamps of Lot 1, all of which were frosted by the acid process, is 5.3 per cent, which is nearer to the value for the sand-blasted lamps of Lot 2 than to that for the lamps frosted by the acid process.

The only other lot of lamps of which parts were frosted by each of the two methods is Lot 3. Here again the acid-frosted lamps show a smaller absorption than the sand-blasted lamps, but the difference is not as great as that obtained with the lamps of Lot 2. This is easily accounted for by the fact that the surfaces of the sand-blasted lamps of Lot 3 were of a much finer grain than those of Lot 2. It was difficult to separate the sand-blasted lamps of Lot 3 from those frosted by the acid process.

The total range in the average absorption coefficients for the acid-frosted and sand-blasted lamps of the seven different lots is from 3.8 per cent for the acid-frosted lamps of Lot 2 (oval-anchored lamps) to 7.5 per cent for the acid-frosted lamps of Lot 7 (downward-light lamps). The mean value for the seven different lots is 5.7 per cent.

The question arises whether the figures given in Table I are to be taken as indicative of the degree of uniformity to be expected in the frosting of lamps at different factories, and of the frosting of individual lamps at any one factory. Another pertinent question which might be raised is whether the average absorption coefficient of 5.7 per cent which was found for lamps that had been *seasoned* applies equally well to new lamps. Since, in order to discuss fully the results on absorption in their bearing upon these questions it would be necessary to anticipate what will be said in regard to the change in life due to frosting, any further discussion of Table I will be deferred until the effect of frosting on life has been presented.

Before passing to the next part of the paper, however, it is well to call attention to the probable errors of observation in making the measurements from which Table I was computed. An error of 1 per cent in determining the mean spherical candlepower of an indi-

vidual lamp either before or after frosting would make a difference of 1 per cent in the observed absorption coefficient, i. e., a lamp showing an apparent change in absorption of 4 per cent might in reality have undergone a change of 3 per cent or 5 per cent. If in each of the two determinations—before and after frosting—an error of 1 per cent had been made, it is possible that the observed change in absorption might be in error by 2 per cent, although this would be quite improbable.

With the great number of observations to be made and the consequent speed with which they were conducted, it was impossible to give to each measurement the care requisite to obtain an accuracy better than 1 per cent. There is every reason to believe, however, that the observed average change in absorption is well within 1 per cent of the true value for those lamps.

Attention is called to this question of probable error, in order that in considering this and subsequent tables too much weight may not be given to small differences in individual lamps.

3. CHANGE IN DISTRIBUTION.

A second important effect of frosting incandescent lamps is the change produced in the polar distribution curves of the lamps. It would seem at first thought, perhaps, that frosting the bulb of an incandescent lamp would produce a more uniform distribution of the light; in other words, the spherical reduction factor, i. e., the ratio of the mean spherical to the mean horizontal candlepower, would approach unity. Such, however, is not the case. The shape of the lamp bulb is an important determining factor. If we supposed the frosted bulb to act as a perfect mat surface, i. e., to transmit no light directly but to scatter or diffuse the light completely, and if we assumed the form of the filament such that the whole frosted surface of the bulb were uniformly bright, then we could predict with reasonable certainty the approximate distribution of the light around the lamp. If the shape of the bulb were spherical, the intensity would be the same in all directions, and the reduction factor would be unity. If the bulb were a long, slender cylinder, the intensity would be a maximum in a direction normal to the axis of the cylinder, but would decrease as we approached the ends of the cylinder, the reduction factor being approximately 0.79.

In the actual case of frosted incandescent lamps the problem is not nearly so simple. The frosting is never so complete as to eliminate all direct transmission, and the shape of the bulb is not a simple geometrical figure, particularly if the effect of the base is considered. But the shape of the bulb has a marked influence on the distribution curve, as will be seen shortly from the experimental results.

There are two ways of showing the effect of frosting on the distribution—(1) by comparing the spherical reduction factors before and after frosting, and (2) by actually plotting the distribution curves of plain and frosted lamps. The second method is more satisfactory, but requires many more observations. We have used both methods. The average reduction factors for a number of lamps of each of the six different types of filaments described previously were determined before and after frosting. The values obtained for the individual lamps of the various types will be given below (section 4).

TABLE II.

Average Reduction Factors and Absorption Coefficients for Regular Lamps.

Lots	Types of filament	Reduction factors		Absorption coefficients	
		Plain	Frosted	Mean spherical	Mean horizontal
No. 1 (acid)	Oval anchored	0.826	0.825	5.3%	5.0%
No. 2 (acid)	Oval anchored	.824	.825	3.8	4.0
No. 2 (sand)	Oval anchored	.828	.825	5.6	5.4
No. 3 (acid)	Double filament	.806	.784	6.2	3.7
No. 3 (sand)	Double filament	.809	.797	7.0	5.9
No. 4 (acid)	Spiral filament	.914	.863	4.7	-0.9
No. 5 (acid)	Double round coil	.884	.885	4.9	5.0
No. 6 (acid)	Double flattened coil	.972	.984	6.2	7.3
No. 7 (acid)	Downward light	1.064	1.027	7.5	4.2

In Table II the average reduction factors before and after frosting are given in the third and fourth columns. The reduction factor of the oval-anchored lamps is substantially the same before and after frosting, showing no tendency to approach unity. The change in the reduction factor for all the other types of lamps is small except

for the spiral-filament and downward-light lamps. The former shows a decrease in reduction factor of 5.5 per cent and the latter a decrease of 3.5 per cent—that is, the effect of frosting is to diminish the mean spherical candlepower relatively more than the mean horizontal. This is also shown by a comparison of the spherical and horizontal absorption coefficients. In the fifth column the average spherical absorption coefficients are copied from Table I. In the sixth column the changes in horizontal absorption are given. The large decrease in reduction factor for the spiral-filament lamps is shown here as an actual increase in mean horizontal candlepower due to frosting, whereas the decrease in mean spherical intensity on frosting is over 4.5 per cent. It is evident, therefore, that the changes in mean horizontal intensity are no criterion of the actual absorption due to frosting.

A comparison of the spherical reduction factors before and after frosting, however, does not give a fair idea of the relative distribution curves under the two conditions. Thus, although the ratio of mean spherical to mean horizontal intensity is not altered appreciably by frosting the oval-anchored filament lamps, the distribution of light in the two cases is not the same. Before discussing the relative distribution curves (Figs. 5 to 13) of the plain and frosted bulb lamps of each of the six types of filament studied, some experiments made with lamps having special bulbs will be described.

In order to show experimentally that the shape of the bulb is an important factor in determining the distribution of the light around frosted lamps, we obtained, through the courtesy of the General Electric Company, a number of regular oval-anchored filaments mounted in $3\frac{1}{8}$ -inch spherical bulbs, of which some were frosted. Subsequently we also obtained, through the kindness of the National Electric Lamp Association, a number of double-flattened coil and downward-light filaments mounted in the straight sides bulb SS-19, such as is commonly used with the oval-anchored filament, but which is not supplied with the regular double-flattened coil or downward-light lamp.

These various special lamps were treated in the same way as the regular lamps, i. e., their spherical reduction factors and distribution curves were obtained. The average spherical reduction factors for each special type, both plain and frosted, are given in Table III.

TABLE III.

Average Reduction Factors for Special Lamps.

Types of filament	Shapes of bulb	Reduction factors	
		Plain	Frosted
Oval anchored.....	3½ in. round bulb	0.840	0.887
Downward light	Straight sides SS-19 bulb	.982	.938
Double-flattened coil.....	Straight sides SS-19 bulb	.995	.935

In the study of the special lamps, four plain and seven frosted oval-anchored lamps were used, four plain and four frosted double-flattened coil lamps, and five plain and two frosted downward-light lamps. Owing to the small number of lamps studied, the precise numerical values obtained are not to be insisted upon, but the general conclusions drawn from the results would seem to be perfectly definite.

The effect of the shape of the frosted bulb is evident at once in the difference between the reduction factors for the regular frosted and the special round-bulb frosted lamps with oval-anchored filaments. The value for the former is 0.825, that for the latter 0.887, a difference of over 7 per cent. Another striking illustration is afforded by the downward-light lamps. The average reduction factor for the regular frosted-bulb lamps is 1.027, whereas the reduction factors for two downward-light filaments mounted in SS-19 frosted bulbs were found to be 0.935 and 0.94. The average value 0.938 is almost 9 per cent different from the average value for the regular lamps. The differences in the shapes of the distribution curves will be discussed presently.

Another interesting result, which also becomes clearer when we consider the distribution curves, is the change in the reduction factor of plain-bulb lamps with change in shape of bulb. Only a few lamps were studied and the numerical differences found are not very large, but we are quite certain that the shape of the bulb has an appreciable influence on the reduction factor and on the distribution curve. Thus the average reduction factor for the four special round-bulb lamps is 0.840, whereas only one lamp of the forty-five regular-bulb lamps studied has a value that high, the average for

the forty-five lamps being 0.826. The effect is more noticeable with the downward-light lamps. The average reduction factor for the regular lamps is 1.064; for the five special lamps it is 0.997, a difference of almost 7 per cent. Moreover, the lowest value for any of the twenty-three regular lamps is higher than the highest value for any of the five special lamps.

The differences both for the plain and the frosted lamps are clearer when we plot the distribution curves. In Figs. 5 and 6 are shown the average vertical distribution curves for the plain and the frosted double-filament and spiral-filament lamps—the two extreme types which are supplied with the ordinary straight sides bulb SS-19. In plotting all the curves the mean horizontal intensity is taken as unity. No attempt was made to determine the exact form of the curve in the neighborhood of the base, since it depends to such an extent upon the style of rotator used, the distance at which the measurements are made, etc. The measurement nearest the base was made at 165° , or 15° from the base. The intensity at the base was assumed to be zero. As stated previously, in determining the distribution curves the speed of rotation was 200–250 r. p. m. for all lamps except those having pronounced flickers at that speed. For these lamps higher speeds were used, so that the curves obtained may be slightly different from those that would be obtained at low speeds if it were possible to make accurate readings with a badly flickering illumination of the photometer screen.

The two curves to the right in Fig. 5 are for the spiral-filament lamp, the solid curve applying to the plain-bulb lamp and the dotted curve to the frosted-bulb lamp. Similarly, the two curves to the left in Fig. 5 are for the plain and frosted bulb double-filament lamp. In Fig. 6 these same curves are reproduced in a different arrangement in order to compare the two plain-bulb curves and the two frosted-bulb curves.

It is evident, first, from Fig. 5 that frosting modifies greatly the distribution curve of the plain lamps. In particular, it is interesting to note that in the double-filament lamp the tip candlepower is increased relative to the mean horizontal intensity, whereas in the spiral-filament lamp it is decreased. From Fig. 6 it is seen that the curves of the two frosted lamps are more nearly alike than the curves of the two plain lamps, due to the fact that both types of

**DOUBLE AND SPIRAL FILAMENTS
REGULAR BULBS.**

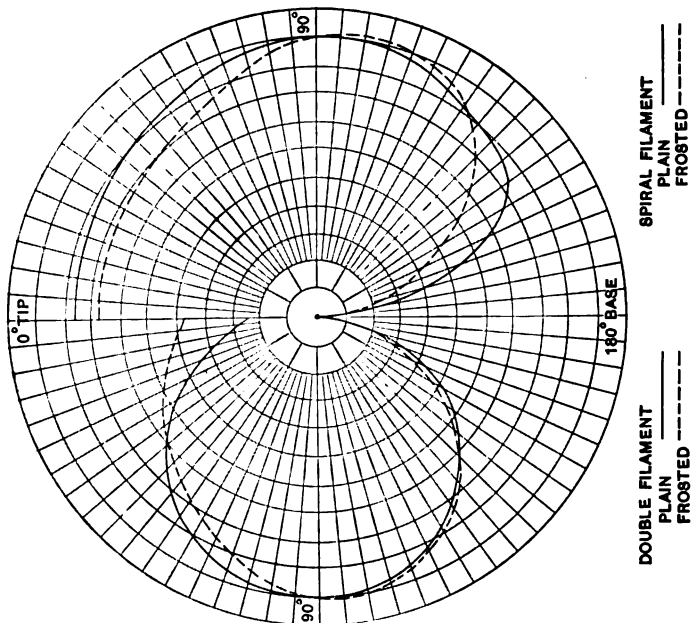


Fig. 5.

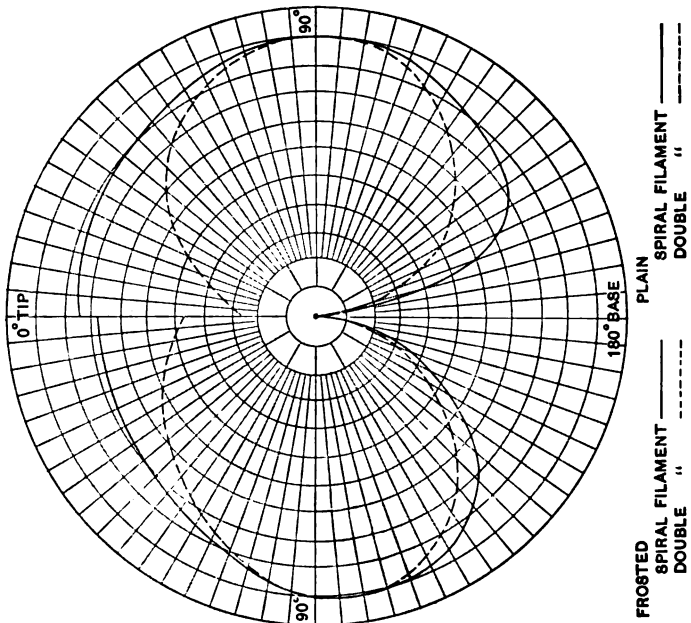


Fig. 6.

filaments are in the same bulb, and the shape of the bulb impresses its character on the distribution curves of the frosted lamps.

In Figs. 7 and 8 are shown similar curves for oval-anchored filaments in regular SS-19 and special round bulbs, plain and frosted. In Fig. 7 the two curves to the right are for the plain and frosted regular bulbs; the two to the left are for the plain and frosted special round bulbs. The effect of the shape of the frosted bulb is clearly evident from the two curves to the left in Fig. 8, of which the solid curve applies to the regular bulb and the dotted curve to the special round bulb, in both of which regular oval-anchored filaments were mounted. The two curves to the right in Fig. 8 show the effect of the shape of the plain bulb on the distribution curve.

This effect is more pronounced in the case of downward-light filaments mounted in their regular and in straight sides SS-19 bulbs, as shown by Figs. 9 and 10. These curves are analogous to those for the oval-anchored filament in Figs. 7 and 8. The effect of the shape of the bulb, both for the frosted and plain lamps, is quite noticeable.

In Figs. 11 and 12 similar curves for double-flattened filaments mounted in their regular and in straight sides SS-19 bulbs are given. The same general effects are noticed with this type of filament as with the oval-anchored and downward-light filaments. In Fig. 13 the distribution curves for the double round-coil filament in regular plain and frosted bulbs are shown.

The curves given in Figs. 5-13 are the *average* vertical distribution curves of the lamps, obtained by rotating each lamp about its axis of figure and determining the mean latitudinal intensity for different latitudes. A complete study of the effect of frosting would involve a determination of the distribution of light in each latitude. It would seem to us that too little significance is usually attached to the distribution of the light in space. In order properly to design fixtures for illuminating a room it is as necessary to know the distribution of the light in any latitude as it is to know the average distribution in the vertical plane.

4. SPHERICAL REDUCTION FACTORS.

It is generally recognized that the proper basis of comparison for lamps having different distribution curves is the ratio of the total flux of light emitted to the rate at which energy is supplied to the

OVAL ANCHORED FILAMENTS
 REGULAR AND SPECIAL BULBS.

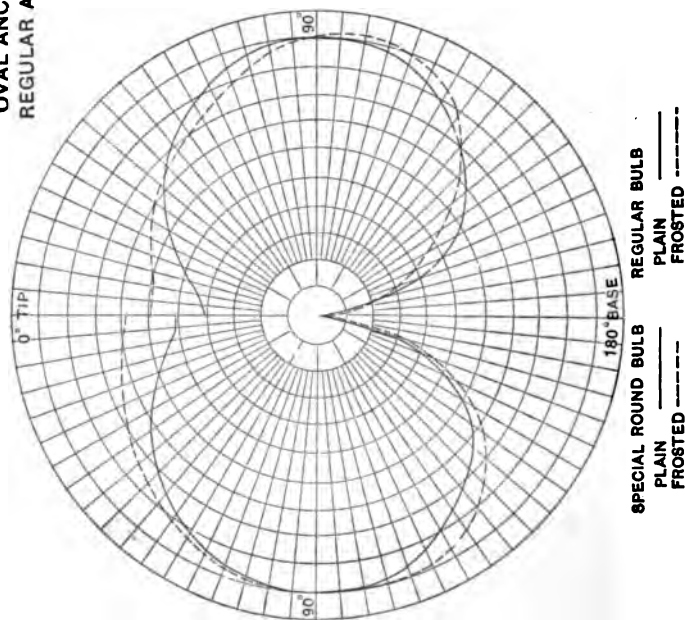


Fig. 7.

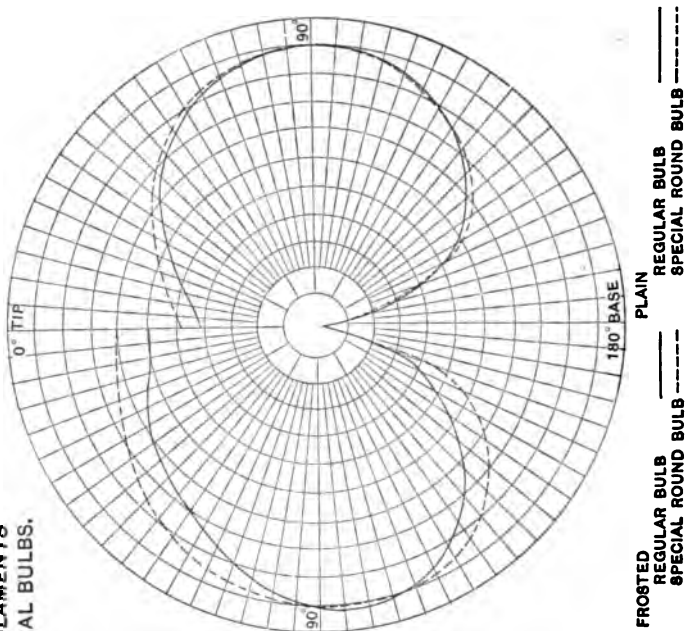


Fig. 8.

**DOWNWARD LIGHT FILAMENTS
REGULAR AND SPECIAL BULBS.**

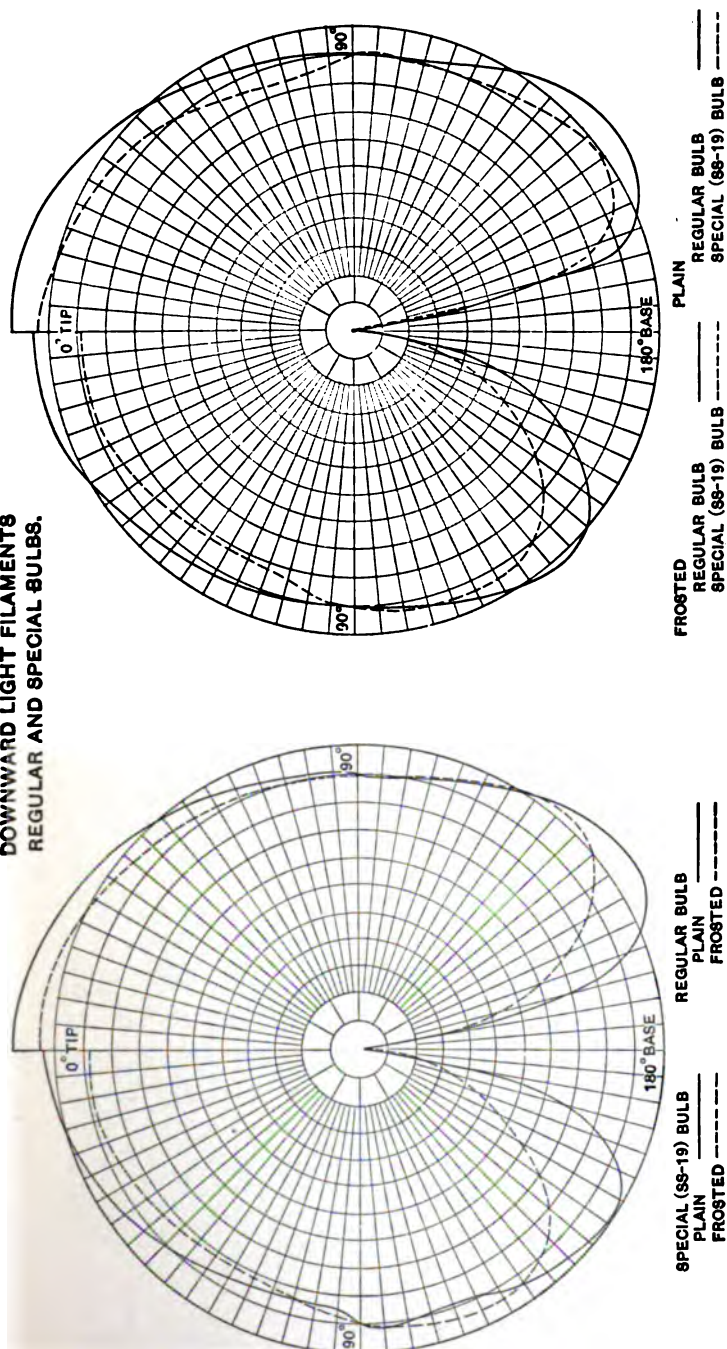


Fig. 9.

Fig. 10.

DOUBLE FLATTENED COIL FILAMENTS
REGULAR AND SPECIAL BULBS.

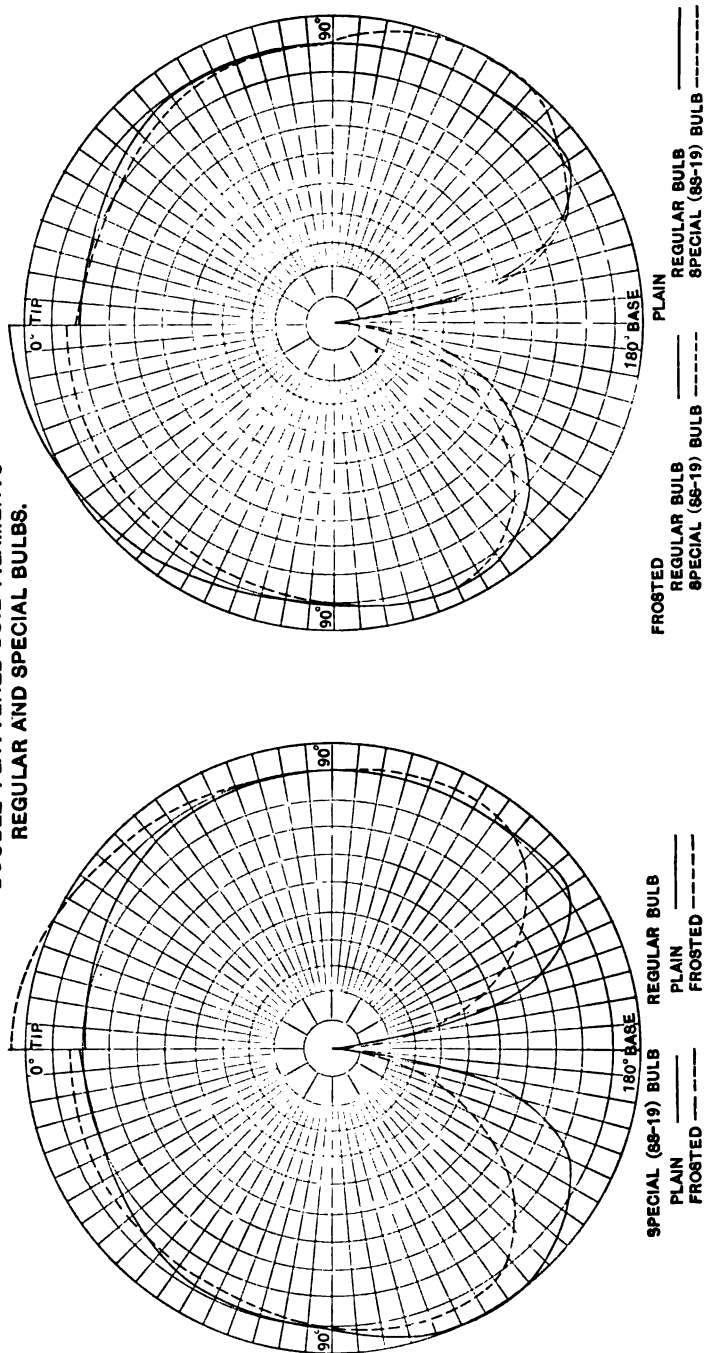


Fig. 12.

Fig. 11.

lamp. This involves a determination of the total flux of light, or, more commonly, of the mean spherical candlepower of the lamp. Although various types of integrating photometers for determining the mean spherical candlepower of incandescent lamps have been devised—some of them very excellent for laboratory purposes—there has never been developed such a photometer which could be in-

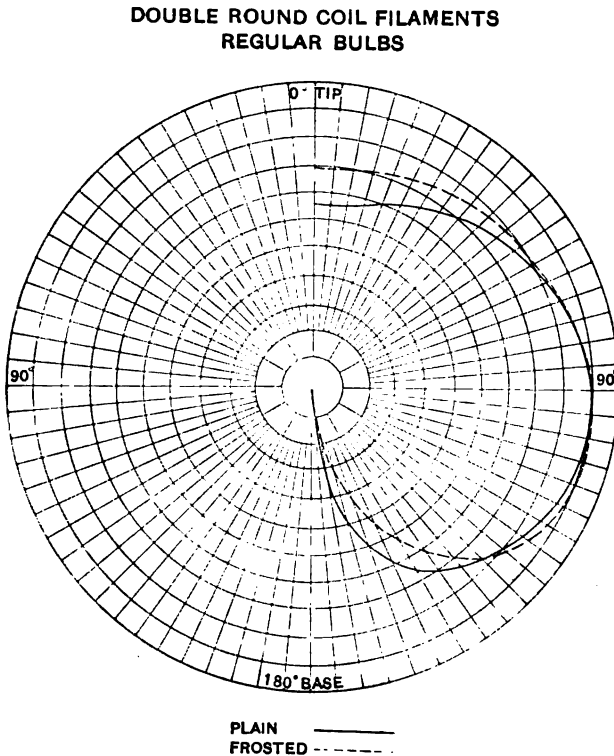


Fig. 13.

stalled and used with satisfaction in a lamp factory. As a result, very few lamps have been purchased heretofore in the United States on this basis. Until an integrating photometer suitable for use in lamp factories and testing stations is placed on the market the mean spherical rating can best be accomplished by the use of spherical reduction factors. This method has been incorporated in the lamp

specifications recently adopted for use by most of the departments of the Government.

The idea of the spherical reduction factor is an old one, and it has long been known that for any type of incandescent lamp the spherical reduction factor is approximately constant for individual lamps of that type. Many investigations of the values of spherical reduction factors for different types of incandescent lamps have been published, but owing to the recent renewed interest in the question, and the improved facilities at our disposal for measuring mean spherical candlepower, it seemed desirable to include in this investigation on the effect of frosting an auxiliary study of the spherical reduction factors of the six types of lamps used.

TABLE IV.

Spherical Reduction Factors of Regular Plain Bulb Lamps.

Oval anchored		Double fila- ment Lot 3	Spiral fila- ment Lot 4	Double round coil Lot 5	Double flat- tened coil Lot 6	Downward light Lot 7
Lot 1	Lot 2					
0.82	0.81	0.80	0.89	0.87	0.97	1.04
.82	.81	.80	.90	.87	.97	1.04
.82	.82	.80	.90	.87	.97	1.04
.82	.82	.80	.90	.88	.97	1.04
.82	.82	.80	.90	.88	.97	1.04
.82	.82	.80	.90	.88	.97	1.05
.82	.82	.80	.90	.88	.97	1.05
.82	.82	.80	.91	.88	.97	1.06
.82	.83	.80	.91	.88	.97	1.06
.82	.83	.81	.91	.88	.97	1.06
.82	.83	.81	.91	.88	.97	1.06
.83	.83	.81	.91	.88	.97	1.07
.83	.83	.81	.91	.88	.97	1.07
.83	.83	.81	.91	.89	.97	1.07
.83	.83	.81	.91	.89	.97	1.07
.83	.83	.81	.91	.89	.97	1.07
.83	.83	.81	.92	.89	.97	1.07
.83	.83	.81	.92	.90	.97	1.08
.83	.83	.81	.92	.90	.97	1.08
.83	.83	.81	.92	.90	.98	1.08
.83	.83	.81	.92		.98	1.08
.83	.83		.93		.98	1.09
	.84		.93		.98	1.09
			.93		.98	
			.93			
0.825	0.827	0.806	0.912	0.884	0.972	1.063

In Table IV are given the results obtained on the individual lamps of each type before they are frosted.^a

It is seen that for forty-five lamps of the oval-anchored type obtained from two different manufacturers the total range is from 0.81 to 0.84, the greatest observed deviation from the mean being less than 2 per cent. The agreement among the observed values for lamps of the double-filament type (Lot 3) and for those of the double flattened-coil type (Lot 6) is even better. The largest differences were observed with lamps of the downward-light type (Lot 7), where a maximum deviation from the mean of about 2.5 per cent was found. One would expect the variations among lamps of this type of filament to be larger than those among lamps of other types, because of the relatively rapid change in intensity in the neighborhood of the horizontal for downward-light filaments.

But when we contrast the 2 or 2.5 per cent maximum deviations of individual lamps of any one type from the mean value for the type with the 20 or 30 per cent differences between the various mean values for the different types, it is apparent that in the absence of a cheap, convenient, and accurate photometer for measuring mean spherical candlepower directly, the spherical-reduction factor offers an excellent substitute in the commercial rating of lamps on the basis of mean spherical efficiency.

5. CHANGE IN LIFE.

It has long been recognized that the useful life of a frosted incandescent lamp taken to 80 per cent of its initial mean horizontal candlepower is only a little more than one-half of the life of a corresponding plain-bulb lamp. The only explanation which had been advanced, so far as the authors know, previous to that suggested by the present investigation is that the temperature of the frosted lamp is higher, due to the increased absorption by the bulb, and that therefore the lamp reaches any given point in its life—e. g., the 80 per cent point—in a shorter time than that required for the corresponding plain-bulb lamp.

^aThe slight discrepancies between values given in Table II and those given in Table IV are due to the omission of the figures in the third decimal place in the values of the individual lamps in Table IV. These were taken into account in computing the means given in Table II.

Without entering at present into a discussion of the possible temperature effects, it is sufficient to notice that if the shortened useful life can be attributed to this cause, it is very probable that, due to the same cause, the total life of a frosted lamp up to the time when the filament burns out would also be very much less than that for the plain lamp. On the contrary, tests carried out at the laboratories of the National Electric Lamp Company by Mr. S. E. Doane indicated that the average total life of frosted lamps is not very different from that of corresponding plain lamps. It would seem, therefore, that some explanation other than that of the effect of temperature must be sought. A possible explanation of the phenomenon occurred to one of the authors in the course of the present investigation. An experiment devised to test it showed conclusively that it would account for at least a large part of the effect, if not for all of it. This explanation, together with a brief account of the preliminary experiments supporting it, was published in the *Electrical Review*⁹ several months ago.

The explanation, as outlined in that paper, from which we quote, is as follows: "If we consider the case of a new plain-bulb lamp, a certain small percentage of the total flux of light emitted by the incandescent filament is absorbed by the glass envelope surrounding the filament. When the lamp is frosted a large part of the light which in the plain bulb would pass through the outer surface of the bulb is diffusely reflected back through the glass. In this way a relatively large part of the total flux of light passes through the glass more than once, and so the frosted bulbs show an absorption about 5 per cent greater than that for the plain bulbs.

Now, the actual absorption coefficient of glass is quite small, so that it is readily seen that if in any way the absorption coefficient is increased appreciably, the apparent absorption due to frosting would be increased greatly. This is what happens when with increasing life the strongly absorbing film is deposited on the inside of the bulb. The effect is the same as if the absorption coefficient of the glass had been increased greatly." In the case of an old plain lamp all the light passes through the carbon film once, a small percentage passes through three times, a still smaller percentage five times, and

⁹ *Electrical Review*, April 6, 1907; p. 556. See also this Bulletin, 3, p. 341; 1907.

so on. When the lamp is frosted the percentage of light that passes through more than once is very much larger, so that the carbon film has an opportunity to absorb a much larger percentage of the total flux. According to this theory, although at any time the filament in the frosted bulb may be emitting the same total flux of light as that emitted by the filament in the plain bulb, the absorption of light in the carbon film is much greater in the one case than in the other, and so the apparent intensity of the frosted lamp at any time during life is less than that of the plain lamp, the difference in intensity increasing with the number of hours the lamps have burned.

In order to make a quantitative determination of this effect, after convincing ourselves by a few qualitative experiments that the effect was a real one, ten comparatively new lamps and twelve old lamps that had dropped to 80 per cent in candlepower were carefully measured for mean horizontal and mean spherical intensity. They were then sent to a lamp factory and frosted by the acid process, care being exercised to see that the frosting was as nearly uniform for the different lamps as it was possible to obtain. The lamps were then returned to the Bureau of Standards and measured. The new lamps were found to have decreased in mean horizontal intensity by about 4 per cent on the average, the individual lamps agreeing among themselves to within less than 2 per cent. On the other hand, the older lamps decreased in mean horizontal intensity by 18 per cent, or 14 per cent more than the new lamps. In other words, the apparent absorption of the frosting was approximately four and one-half times as great for the old lamps as for the new lamps. This means that if we were to assume no difference in the physical or mechanical properties of plain and frosted lamps, a lot of plain lamps which would decrease 20 per cent in candlepower in a definite number of hours would in the same number of hours decrease approximately 32 per cent if they were first frosted. The useful life of the frosted lamps to 80 per cent of initial candlepower would be about 60 or 70 per cent of the useful life of the plain lamps, a value in approximate agreement with that commonly accepted. This shows that whatever effects may be produced in the lamps by frosting them, the mere absorption by the deposited carbon film, as explained above, is sufficient to account for a decrease in useful life of about 30 or 40 per cent.

The new lamps decreased about the same in mean spherical as in mean horizontal candlepower, whereas the old lamps decreased several per cent more in mean spherical intensity. This is probably due to the uneven distribution of carbon on the inside of the bulb.

Several weeks after the above explanation was printed in the *Electrical Review*, Mr. Preston S. Millar published in the *Electrical World*¹⁰ an account of some experiments made upon frosted lamps at the Electrical Testing Laboratories. It is very interesting to note that the results of Mr. Millar's experiments are in very good agreement with those obtained at the Bureau of Standards and further substantiate the theory outlined above. Quite recently Dr. A. E. Kennelly has put this theory into a mathematical form in a paper published in the *Electrical World*.¹¹

In the preliminary paper printed in the *Electrical Review* an experiment was outlined to determine whether there are any other elements entering to bring about the short useful life of frosted lamps. This experiment has just been completed. Out of a number of 110-volt, 16-cp, 3.1-wpc oval-anchored filament lamps five lots of twenty-two lamps each were selected and designated as Lots A, B, C, D, and E. The nearest even voltage for each lamp was found corresponding to an initial specific consumption of 3.3 watts per mean horizontal candle. In all the subsequent photometric measurements the lamps were burned at these voltages. On the life rack, however, in order to hasten the completion of the investigation, the lamps were maintained at a voltage 12 volts higher than that at which they were measured on the photometer. This voltage corresponded to an initial specific consumption of approximately 2.3 watts per mean horizontal candle.

The five lots of lamps were measured carefully for initial mean horizontal candlepower, and then all the lamps except those of Lot A were placed on the life rack. The lamps of Lot B were burned for 9 hours, those of Lot C for 37 hours, and those of Lots D and E for 78 hours, at which time it was expected that the lamps would have reached the 80 per cent point. On measuring all the lamps for mean horizontal candlepower it was found, however, that the D

¹⁰ *Electrical World*, April 20, 1907; p. 798.

¹¹ *Electrical World*, May 18, 1907; p. 987.

and E lamps had only decreased in candlepower to 83 per cent of the initial value.

All the lamps with the exception of those of Lot E were then sent to the factory to be frosted. Upon their return they were all measured again for mean horizontal candlepower and placed upon the life rack, with the exception of those of Lot D, which already had burned for 78 hours and had decreased in candlepower (before being frosted) to 83 per cent of their average initial value. The lamps of each of the three Lots A, B and C were burned the requisite number of hours to make the total time of burning 78 hours.

In this way all the lamps were made to burn 78 hours, but the paths followed by the various lots were different. Thus, Lot A burned the total 78 hours after having been frosted; Lot B burned 9 hours while plain and 69 hours after having been frosted; Lot C burned 37 hours plain and 41 hours frosted, and Lot D burned the entire 78 hours before being frosted. The average candlepowers of each lot at the various stages are given in Table V. In the first

TABLE V.

A Comparative Study of the Changes in Candlepower of Plain and Frosted Lamps During Life.

	Mean horizontal candlepower					Mean spherical candlepower	
	A	B	C	D	E	A	D
Average initial candlepower . . .	15.31	15.29	15.15	15.26	15.29	12.61	12.62
0 hrs. Plain	100%	100%	100%	100%	100%	100%	100%
0 hrs. Frosted	96					96	
9 hrs. Plain		99					
9 hrs. Frosted		95					
37 hrs. Plain			94				
37 hrs. Frosted	86	85	87				
78 hrs. Plain				83	83		82
78 hrs. Frosted	72	72	73	73		70	70
Horizontal absorption coefficient.	2% (0 hrs)	4% (9 hrs)	7% (37 hrs)	12% (78 hrs)			

column are given the different times at which photometric measurements were made, together with a statement of the condition of the lamps at that time, whether plain or frosted. In the next five

columns are given the average mean horizontal candlepowers of the five different lots at the various stages of their life. In the first line the actual average candlepower of each lot is given, but in the succeeding lines the candlepowers are expressed in per cents of the initial value of each lot. The average candlepower of each lot determined immediately after the lamps were returned from being frosted is underscored, so that it is easy to see how many hours each lot burned before and after having been frosted.

It is interesting to note the marked agreement among the different lots upon reaching the same point in the curve by different paths. Thus, after 37 hours Lot A had decreased to 86 per cent, Lot B to 85 per cent, and Lot C (frosted) to 87 per cent, although the A lamps were frosted initially, the B lamps after 9 hours, and the C lamps after the entire 37 hours burning. Again, at the end of 78 hours the four lots A, B, C, and D had decreased in candlepower to the same value within about 1 per cent, although the A lamps burned the entire 78 hours after frosting, whereas the D lamps were frosted at the end of the 78 hours, the other two lots having been frosted at intermediate points. In other words, these tests would seem to indicate that the frosting has no appreciable effect upon the life curve of the filament, the short, useful life of frosted lamps being due entirely to the increased absorption of the light by the deposited carbon film, as explained above.

Mr. Millar, in the paper referred to previously, described one set of experiments in which the candlepower of some frosted lamps which had burned a number of hours and which were soiled, increased by about 10 per cent upon being washed. One can readily see how the candlepower of frosted lamps could be changed considerably by the presence of dirt on the surface. In order to see whether this element played any part in the investigation at the Bureau, three of the A lamps which had burned 155 hours were measured for mean spherical candlepower before and after being washed. The increase in candlepower after the lamps were cleaned was about 1 per cent on the average. Since these three lamps had been exposed double the time used in the test, the effect of dust on the bulbs was evidently entirely negligible.

Upon completing the candlepower measurements after 78 hours, all of the frosted lamps, and, in addition, the lamps of Lot E, which

had burned to 78 hours, but which had not been frosted, were placed on the life rack and kept burning, in order to see whether there is any indication of an effect of frosting on the total life of the lamps. The average total life of the lamps of each group until they burned out is as follows: Lot A, 104 hours; Lot B, 118 hours; Lot C, 133 hours; Lot D, 123 hours; Lot E, 118 hours. If frosting played an important part in determining the actual life of the filaments, the average total life of each successive group, in the order of the letters of the alphabet, would be greater than the preceding. There is no indication, however, of such a sequence. It is true that the life of the A lamps is the shortest, but the life of the E lamps, which should be the longest, is no greater than that of the B or D lamps, and is much less than that of the C lamps, which burned nearly three-fourths of their life after having been frosted. It is evident, therefore, that this test shows no marked effect of frosting on the life of the filament.

As shown in the last two columns of the table, the two extreme lots A and D were measured also for mean spherical candlepower. Here, as in the mean horizontal measurements, the average candlepower of the frosted lamps after 78 hours burning is the same, independent of whether the lamps are frosted first and burned subsequently, or vice versa. The decrease in mean spherical candlepower, however, is slightly greater than the change in mean horizontal intensity. This agrees with the results given in Table V.

In discussing the change in absorption due to frosting, the question was raised as to whether the absorption of new lamps is the same as that of seasoned lamps. The answer to this is evident from a consideration of Table V. The change in horizontal candlepower due to frosting is given in the last line of the table. The A lamps, which were frosted when new, showed an absorption of 2 per cent; the B lamps, frosted after 9 hours, an absorption of 4 per cent; the C lamps, after 37 hours, an absorption of 7 per cent; and the D lamps, after 78 hours, an absorption of 12 per cent. The average value of 5.7 per cent given earlier in the paper for seasoned lamps is therefore probably several per cent higher than the correct value for new lamps.

Since, in seasoning the lamps which were used in studying the change in absorption due to frosting, no special care was exercised

to see that they all burned at the same temperature for the same number of hours, it is not at all surprising that differences in absorption coefficients among different makes, or even among the individual lamps of any one make, occurred, since the carbon deposits may have been different.

Having shown that for the lamps studied at the Bureau and for the conditions under which the investigation was made the shortened life of frosted lamps can be accounted for entirely by the increased absorption of light by the carbon film deposited on the inner side of the bulb, it is interesting to consider briefly the question of temperature effects due to frosting. It would be very difficult to determine the temperature of the incandescent filament before and after frosting the bulb by an optical method because of the complications arising from the frosted surface. An attempt was made to determine the change of temperature of the filament by measuring the change in resistance of the filament; but though this method gave definite indications in some cases, the results were not consistent. It is probable, however, that using filaments with larger temperature coefficients and taking extra precautions in the measurements, consistent results could be obtained with this method.

Although no direct measurements of the temperature of the filament were made, Mr. H. C. Dickinson, of the Division of Thermometry, kindly undertook to determine the temperature of the bulbs of several of the A and D lamps at different periods in their life. By the use of a platinum resistance thermometer, placed around the bulb and in contact with it on the circumference of the circle of maximum diameter, the values given in Table VI were obtained.

The actual rise in temperature on lighting the lamp, as measured by this method, may not be correct to a high accuracy, but the error is small. The relative temperature changes at the different periods in the life of the lamps are more nearly correct.

It is seen from Table VI that the temperatures of the bulbs of new plain lamps are about 37° above room temperature. When a new lamp is frosted the temperature of the bulb increases by about 9° . When a lamp is burned to about 83 per cent in life, the temperature of the bulb rises only 3° due to the carbon deposit, but when an old lamp is frosted the rise due to frosting is about 15° .

TABLE VI.

A Comparative Study of the Changes in Temperature of the Bulbs of Plain and Frosted Lamps during Life.

Lamps	Time	Temperatures (centigrade)		Average rise above room temperature.
		Of bulb	Of room	
A-1 Plain	0 hrs	60°	22°	38°
A-2 "	0 "	60°	22°	
A-3 "	0 "	60°	22°	
A-1 Frosted	0 hrs	71°	23°	47°
A-2 "	0 "	69°	23°	
A-3 "	0 "	69°	23°	
A-1 Frosted	78 hrs	78°	21°	59°
A-2 "	78 "	83°	21°	
A-3 "	78 "	78°	21°	
D-1 Plain	0 hrs	58°	23°	36°
D-2 "	0 "	60°	23°	
D-1 Plain	78 hrs	59°	22°	39°
D-2 "	78 "	63°	22°	
D-1 Frosted	78 hrs	75°	22°	54°
D-2 "	78 "	78°	23°	

Of course the energy supplied to the old lamps is 4 or 5 per cent less than that supplied to the new lamps.

With the great difference in temperature between the filament and the bulb, the small variations in the temperature of the latter would scarcely be expected to change the temperature of the filament appreciably, so that although it is true that the bulbs of frosted lamps are hotter than those of plain lamps, the change in the temperature of the filament would probably not be noticeable in the life.

It is very interesting, in conclusion, to follow out the practical results suggested by an investigation of this kind. For example, it is at once evident that the useful life of frosted lamps containing filaments of some material that would deposit no absorbing film on the inner side of the bulb, would be as great as that for the corresponding plain lamps, and the efficiency would only be a few per cent less. Experiments on metal filament lamps similar to those on lamps with carbon filaments will be made to determine the effect of frosting on the life of the former.

The question of the most satisfactory size and shape of plain and frosted lamps is also very suggestive. For example, if the surfaces of bulbs of frosted lamps were doubled the useful life would be very much longer than at present. Another question of some importance in view of the present practice of using half-frosted lamps with reflector shades, is the extent to which the intensity of half-frosted lamps is diminished at different stages of their life. In other words, how does the useful life of a half-frosted lamp compare with that of a plain lamp? We hope to obtain some data on this question shortly.

WASHINGTON, July 15, 1907.

THE VARIATION OF RESISTANCES WITH ATMOSPHERIC HUMIDITY.¹

By E. B. Rosa and H. D. Babcock.

It has long been known that manganin resistances prepared in the manner developed by the Physikalisch-Technische Reichsanstalt do not remain entirely constant in value, but that appreciable differences sometimes occur among a group of apparently exactly similar coils. These variations are usually very small or inappreciable with coils of 0.1 ohm and 1.0 ohm; but with coils of 10, 100, 1,000 ohms and higher values the variations are often so considerable that when used for precision work it has been found necessary at the Bureau of Standards to determine their values by stepping up frequently from the 1-ohm standards. These variations have been found to be greater in England than in Germany, and it has been suggested that the difference may be due to some difference in the methods of preparation of the coils followed in England and in Germany.² At the Bureau of Standards the variations in the values of manganin resistances have been a source of great annoyance, and they have been found to be as great in the resistances made by the best German makers as in those made in America. These variations occur in the most carefully prepared standards of the Reichsanstalt form, as well as in the resistances of Wheatstone bridges, potentiometers, resistance boxes, etc.

SEASONAL CHANGES OF RESISTANCE.

In the course of an extended investigation³ at the Bureau of Standards on the ratio of the electromagnetic to the electrostatic unit of electricity it was found that all the resistances employed

¹ Paper read before the American Physical Society at Washington, April 21, 1907. A preliminary account of this work appeared in the *Electrical World* of June 29, 1907, and in the *London Electrician*, June 14, 1907.

² Report of Prof. R. T. Glazebrook, Director of the National Physical Laboratory, *Engineering* (London), March 22 and April 5, 1907.

³ By E. B. Rosa and N. E. Dorsey; this Bulletin, 8, p. 433, 1907.

had a higher value at the same temperature in summer than in winter, the change being a gradual drift upward from early spring to midsummer, followed by a steady drift back to the same minimum in the winter. The amplitude of the change was of course small, varying from 15 to 25 parts in 100,000, but was far too large to be neglected in precision work. Most of these resistances were kept submerged in oil all the time, but some were in air. In all cases the temperatures were taken and the resistance measurements made with great care.

The possibility of these changes being due to atmospheric humidity suggested itself, but it was not at first evident how an increase of resistance could be produced by an increase of humidity. The effect of leakage arising from moisture deposited on the coils or upon the tops of resistance boxes is of course to *decrease* the resistance, and this effect would increase with the humidity. In order to ascertain the cause of the marked change in the opposite direction, to determine its magnitude, and to find how to prevent it, we took up last November a systematic study of the question.

It was very soon found that changes such as occurred during an interval of six months in the atmosphere of the laboratory could be induced in a few days by placing the resistances in a closed case in which an atmosphere of high humidity was maintained, although it usually required several weeks for the resistance of a coil to reach a maximum, or to return to its former value when restored to the original conditions of humidity.

Two apparatus cases were employed, the atmosphere within one being kept at a nearly constant relative humidity of about 25 per cent, the other being kept at higher humidities, ranging from 40 to 100 per cent, although seldom higher than 80, and oftenest about 60 per cent. Each case was provided with thermometers and recording hygrometers, the latter being frequently calibrated by aspiration psychrometers.

The lower humidities were maintained by keeping calcium chloride in the apparatus cases, varying the quantity exposed according to the humidity desired. The higher humidities were obtained by exposing a greater or less surface of water in open vessels, and running a very small electric fan to circulate the air within the case. The records of humidities and temperatures were taken continuously through twenty-four hours, and comparatively small

fluctuations occurred except those produced by intentional changes of the conditions.

The higher resistances were measured generally to one or two parts in a million, and corrections made for small fluctuations in temperature. All measurements were finally referred to the standard of the Bureau determined by the mean value of a number of one-ohm and tenth-ohm coils. The changes in the resistances of the latter are known to be relatively small.

CHANGES ARE DUE TO MOISTURE ABSORBED BY SHELLAC.

The cause of the increase in resistance of the manganin coils, the wire of which is embedded in a heavy covering of shellac thoroughly dried out by baking, is that *the shellac absorbs moisture from the surrounding atmosphere and expands, stretching the manganin wire and thereby increasing its resistance.* The amount of moisture absorbed depends on the relative humidity of the atmosphere, the moisture in the shellac gradually coming to equilibrium with the moisture outside when any given humidity is maintained constant. When the atmosphere is drier the shellac gives up moisture and shrinks, and the wire also contracts and its resistance decreases. The resistance therefore is constantly changing with changes in atmospheric humidity, and is a constant only when the atmospheric humidity is constant or when the coil is sealed so that moisture can not get into the shellac. *Dipping the coils in melted paraffin* will seal them effectually against moisture, so that even the finest wire, such as is used in coils of 1,000 and 10,000 ohms, remains of constant resistance. *Sealing such a coil in a test tube* will keep its resistance constant, even though it had previously been exposed to a moist atmosphere. *Submerging the coils in oil does not protect the shellac from atmospheric humidity*, as the oil absorbs moisture and transmits it to the coils. Hence submerged coils increase in resistance when the atmospheric humidity increases, and decrease in resistance when the humidity decreases. The oil retards the change of resistance and decreases the amplitude somewhat, but still permits considerable changes to occur, especially in the higher resistances.

A change of resistance of 25 parts in 100,000 will be produced by an increase in length of the wire of 12.5 parts in 100,000. If the wire is wound on a spool 4 cm in diameter (the size used in a Wolff

standard of the Reichsanstalt form) the diameter must be increased by the swelling of the shellac by 0.0005 cm or 5 microns. This is of course an appreciable increase in diameter, although much more difficult to measure than the change in resistance. Coils sealed by a covering of paraffin will remain constant for months through wide fluctuations of moisture, certainly to within a few parts in a million, which indicates that the diameter has not changed during that time by as much as 0.2μ , if at all. We shall presently give evidence to show that manganin wire of all sizes is remarkably constant in resistance, and that changes that have been attributed to the manganin are due to the shellac in which it is embedded.

In Fig. 1 are given curves showing the seasonal changes of certain resistances during the past two years. These curves, which were plotted by Dr. Dorsey in the investigation referred to, prompted this study of the variation of resistances with humidity. The curves showed conclusively, what had probably never before been suspected, that the resistance of a shellac-covered manganin coil is greater in summer, at a given temperature, than in winter, at least under some conditions. These resistances were exposed only to the atmosphere of the laboratory, and in summer the humidity was kept lower than normal during 6 or 8 hours of nearly every working day by means of cold brine coils in the room, which condensed moisture upon them. The object of thus reducing the humidity was to secure more perfect insulation than could be obtained otherwise. This is especially necessary in the measurement of small capacities, where large resistances are employed and slight leakage is serious. The changes in resistances were therefore appreciably less than would have occurred normally. All resistance measurements were made in an atmosphere dry enough to give thoroughly satisfactory insulation.

It is possible that the greater changes observed in England than in Germany in manganin coils that have been shellacked are due to greater ranges in the atmospheric humidity in England. That some English makers have reported no appreciable changes in their manganin resistances is probably explained by their use of paraffin for insulation instead of shellac.

Fig. 2 shows the changes in certain manganin resistances when subjected to variations in atmospheric humidity, the values of the humidity, the dates and the variations in resistance being shown in

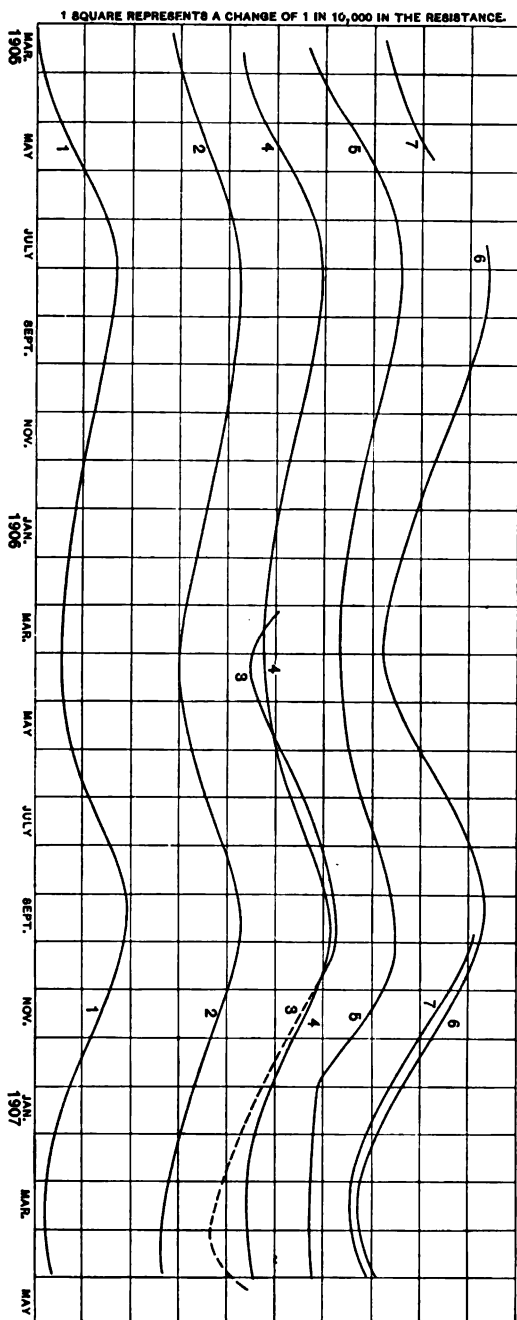


Fig. 1.—Variation of resistance due to seasonal changes.

The curves of Fig. 1 show seasonal changes of manganin resistance due to varying atmospheric humidity. Observations extended through two years and two months. Resistances always in laboratory atmosphere. All resistances were of usual construction and were made by Otto Wolf. One vertical space is 1 part in 10,000 in resistance.

Curve 1 is 1,000-ohm coil submerged in oil from resistance box No. 2470.

Curve 2 is 1,000-ohm coil submerged in oil from resistance box No. 2471.

Curve 3 is 1,000-ohm coil in air from resistance box No. 3087.

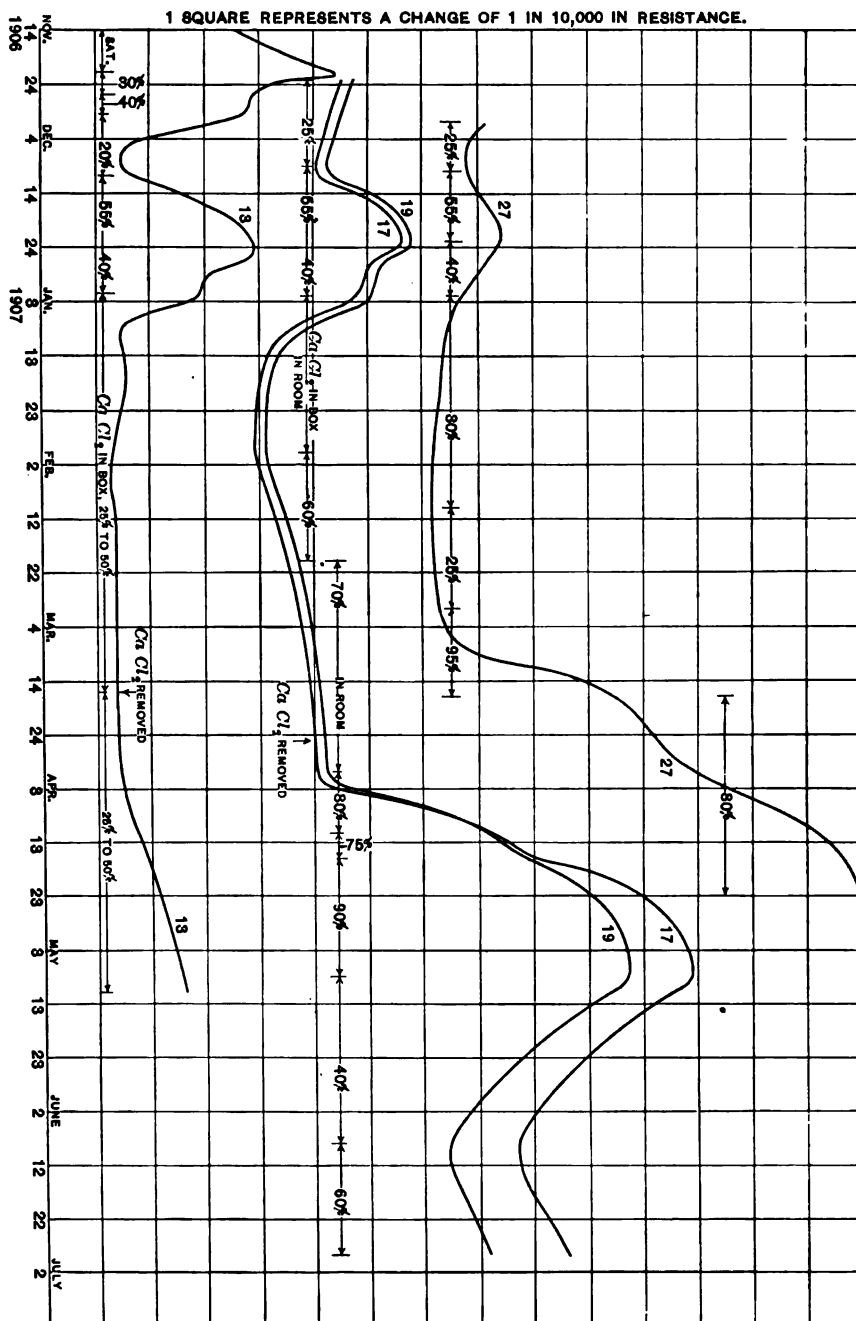
Curve 4 is 21,000-ohm coil in oil from resistance box No. 2470.

Curve 5 is 21,000-ohm coil in oil from resistance box No. 2471.

Curve 6 is mean of 6 coils each of 10,000 ohms in oil from resistance box No. 2620.

Curve 7 is mean of 9 coils each of 10,000 ohms in oil from resistance box No. 2621.

21,000 indicates 1,000 ohms made up of coils 500+200+200+100. There is no record of the values of the resistance for curve 7 between May, 1905, and September, 1906.



the curves. The 1000-ohm coil represented by curve 13 was first placed for a few days in a saturated atmosphere. The change in resistance was very rapid, but was suddenly stopped by reducing the humidity to 30 per cent. A sudden drop in resistance followed, the rate of decrease being reduced by raising the humidity for a few days to 40 per cent. The rate of decrease was again augmented by reducing the humidity to 20 per cent. After the resistance had decreased to a point below its initial value, the humidity was increased to 55 per cent, when the resistance rose as shown and then fell when the humidity was reduced to 40 per cent. The change in resistance between the maximum value of November 22 and the minimum value of December 7 was 40 parts in 100,000. On January 2 a quantity of calcium chloride was placed in the bottom of the resistance box, which was closed up as nearly air-tight as practicable without sealing. The resistance fell rapidly for a few days (about 15 parts in 100,000) and then remained nearly constant for three months, but was not entirely independent of changes in room humidity. The higher average humidity of April caused the resistance to rise appreciably.

Curves 17 and 19 show how closely the 1000 coil and the $\Sigma 1000$ (made up of coils of $500+200+100+100+\Sigma 100$) vary together as the humidity is varied. In this case also CaCl_2 was added to the box on January 2, resulting in a decrease in resistance. When the box was placed in an atmosphere of 60 and 70 per cent humidity the resistance rose appreciably in spite of the CaCl_2 . After the calcium chloride was removed the box was put in the damp case and the resistances rose rapidly, and then decreased when the humidity was reduced to 40 per cent, and then increased again with a humidity of 60 per cent.

Explanation of curves of Fig. 2, p. 126.

The curves of Fig. 2 show variations in manganin resistances due to changes in atmospheric humidity. Resistances kept in an inclosed space where the humidity was under control. Observations extended through eight months. The relative humidities are indicated on the curves. One vertical space is 1 part in 10,000 in resistance.

Curve 13 is 1,000-ohm coil in air from Wolff box No. 3087. (Same coil as No. 3 of Fig. 1.) Curve 17 is 1,000-ohm coil in air from Wolff box No. 3080. Curve 19 is $\Sigma 1,000$ -ohm coil in air from Wolff box No. 3080. Curve 27 is 1,000-ohm coil in air made by the Leeds and Northrup Company.

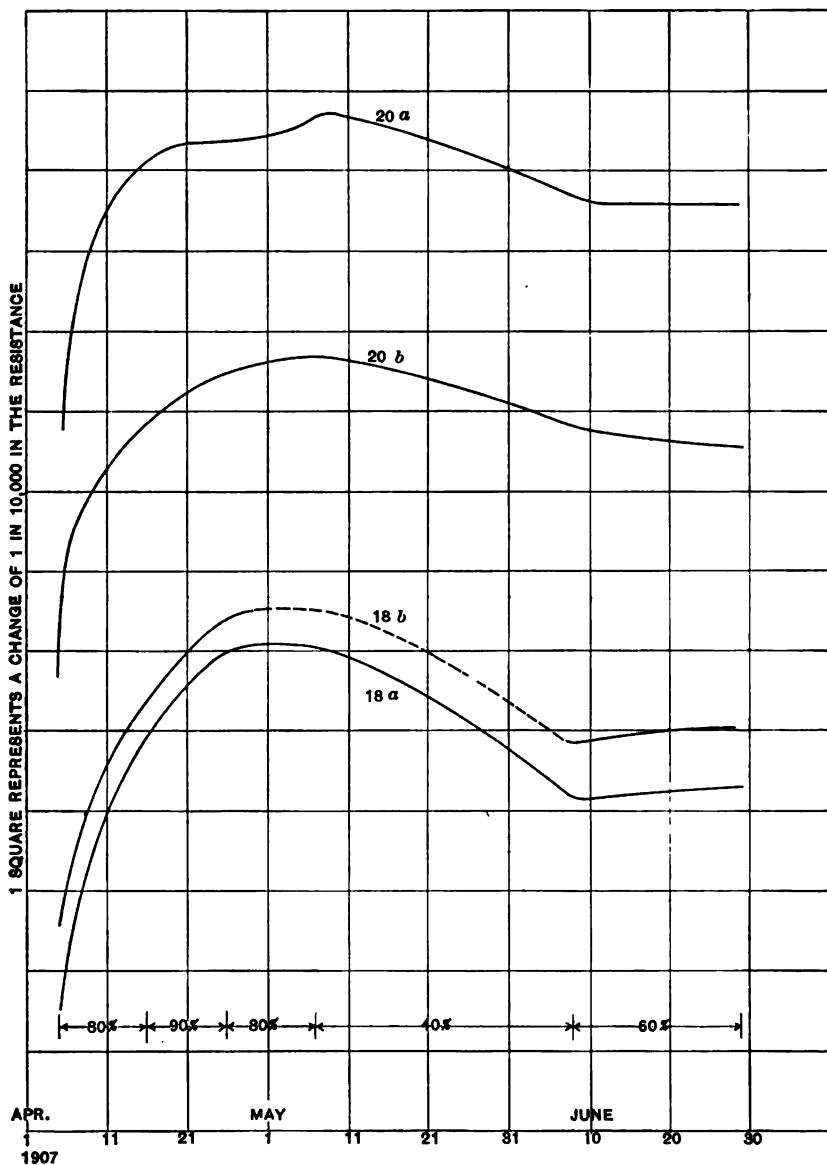


Fig. 3.—Changes of resistance of coils.

The curves of Fig. 3 show changes in resistance of manganin coils of 10 and 100 ohms.

Curves 18a and 18b are two coils of 100 ohms each in air, Wolff box No. 3080.

Curves 20a and 20b are two coils of 10 ohms each in air, Wolff box No. 3080.

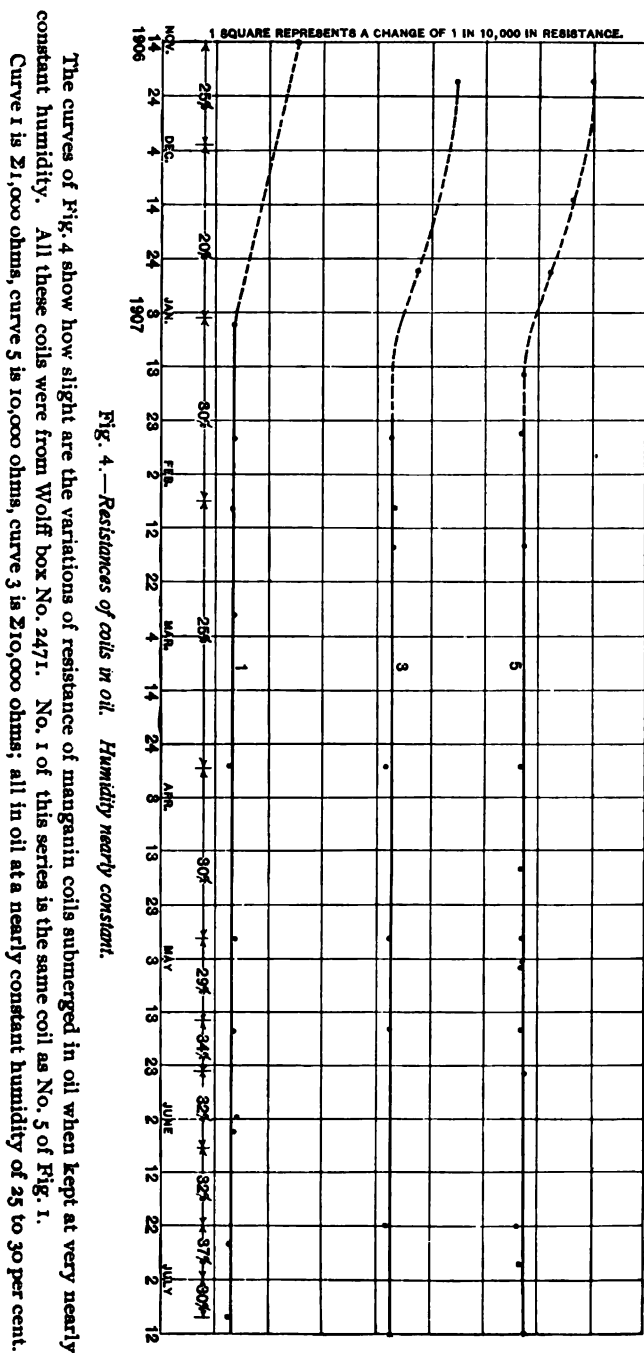


Fig. 4.—Resistances of coils in oil. Humidity nearly constant.

Curve 27 represents a shellacked manganin coil made by the Leeds and Northrup Company, showing large changes in resistance as the humidity is varied.

Fig. 3 shows that the changes in coils of heavier wire, as used in resistances of 10 and 100 ohms, are very considerable, amounting to 35 to 50 parts in 100,000, due to 80 per cent humidity.

Fig. 4 shows three resistances from Wolff box No. 2471 (all in oil), which have been kept for several months at as nearly constant humidity as possible and which have remained remarkably constant in value. Curve 1 represents the same set of coils ($\Sigma 1,000$) as curve 5 of Fig. 1. During November and December it gradually decreased in value, coming to an equilibrium value for approximately 25 per cent humidity about January 1. Between January 1 and July 9 this coil changed only a few parts in 1,000,000, having come down to a minimum value in March. The average humidity in the case has been a little higher since March, and this probably accounts for the slight increase in resistance. Curve 5 represents a single spool coil of 10,000 ohms, and curve 3 represents the $\Sigma 10,000$ coils of the same box. These three curves are practically horizontal, all showing a minimum in March a few parts in a million less than the values in January and since April, the drop in January and February being doubtless due, as stated above, to the continued drying out of the oil, and the rise in April to a slightly higher humidity in the case, due to higher humidity in the room, the case not being entirely tight.

Fig. 5 shows three curves, all representing resistance standards of the Reichsanstalt form. Curve 7 is a 1,000-ohm standard (No. 3039) kept totally submerged in oil in a space at nearly constant humidity of 25 to 30 per cent from January 4 to March 28, the resistance decreasing slowly during that period, as the oil dried out gradually. From March 28 to April 16 the coil, still submerged in the same oil, was in a case at approximately 80 per cent humidity, and the resistance rose about 17 parts in 100,000. The humidity was then reduced to about 25 per cent and the resistance fell off 11 parts in 100,000 in the same time that it rose 17 parts. This shows how great a change can take place in a coil kept submerged in pure petroleum oil, when the humidity of the atmosphere changes. Since June 25 this coil has been in the atmosphere of the laboratory, not

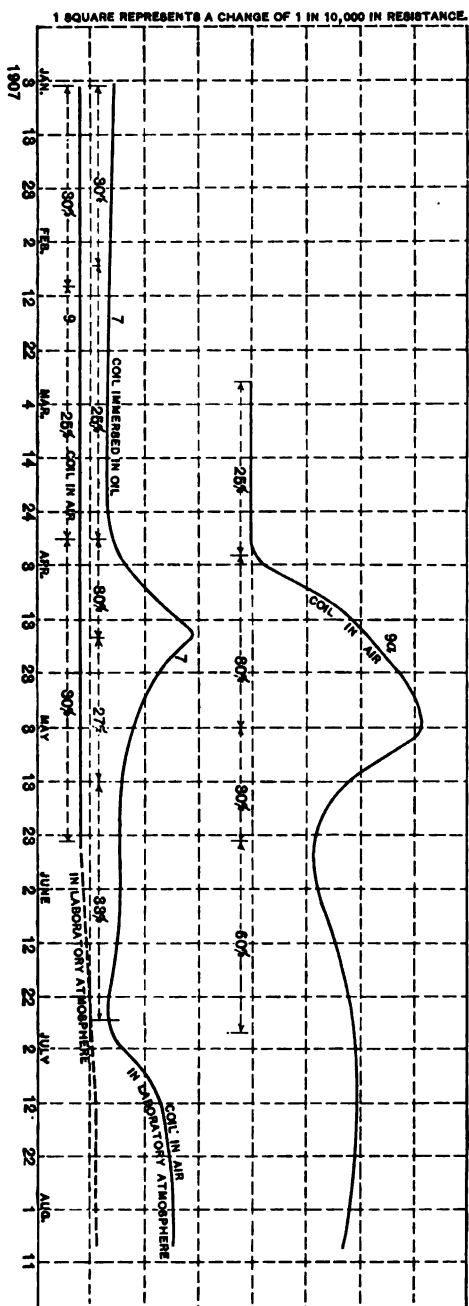


Fig. 5.—Difference in behavior of coils.

The curves of Fig. 5 show the difference in behavior between Wolf standard coils (Reichsanstalt form) when kept at constant humidity and when in an atmosphere of varying humidity.

Curve 7 is standard coil of 1,000 ohms submerged in oil, No. 3039.

Curve 9 is standard coil of 10,000 ohms in air, No. 1392.

These two coils were maintained at nearly constant humidity until March 26, when 7 was placed in an atmosphere of 80 per cent humidity until April 16, when it was restored to the drier chamber.

Curve 9a is standard coil of 10,000 ohms in air, No. 3040.

in oil, and has increased in resistance about 13 parts in 100,000. Curve 9 is a standard 10,000-ohm coil (No. 1392) kept in air at a nearly constant humidity of 25 to 30 per cent from January 4 to May 24. Its changes are very slight, showing a small decrease at first, corresponding to the coils in oil. Since May 24 this coil has been in the atmosphere of the laboratory and has gradually increased in value. Curve 9*a* represents another 10,000-ohm standard (No. 3040) in air at different humidities, showing, in marked contrast to curve 9, large changes due to variation of humidity. Curve 11, Fig. 5*a*, represents a 1,000-ohm standard (No. 3057) in air at different humidities. The fluctuations in the resistance of the latter amount to about 35 parts in 100,000 between the minimum and the

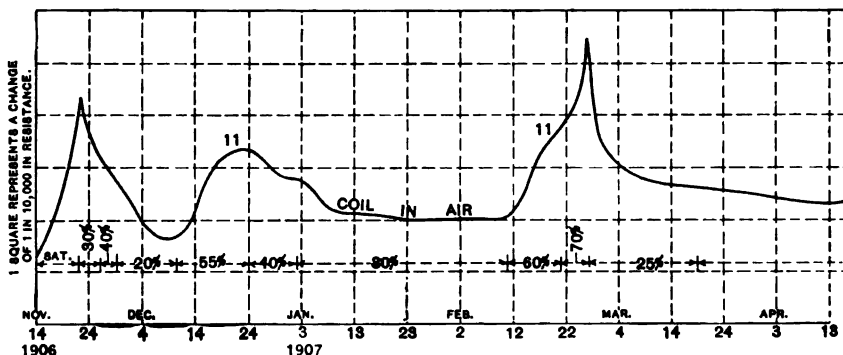


Fig. 5*a*.—Behavior of Wolff standard coil.

Curve 11 is a standard coil, No. 3057, of 1,000 ohms in air, at varying humidities.

Explanation of curves of Fig. 6, p. 133.

The curves of Fig. 6 show the differences in the behavior of resistances of manganin coils, all of which were wound on wood spools and kept in an atmosphere of different humidities, but with different methods of preparation or of mounting.

Curve 55, 1,000 ohms, wood spool thoroughly shellacked and baked, no shellac on outside of wire, no baking of wire.

Curve 57, 1,000 ohms, same kind of wire wound on similar wood spool, latter boiled in paraffin, no paraffin on wire, no heating of wire.

Curve 77, 1,000 ohms, on wood spool, wire and spool shellacked and baked and then coated with paraffin.

Curve 25, 1,000 ohms, on wood spool, wire shellacked and baked, and coil then sealed in a test tube by means of a cork and hot paraffin.

Curve 43, 1,000 ohms, on shellacked wood spool (several coats), wire shellacked and baked after winding. Unifilar winding.

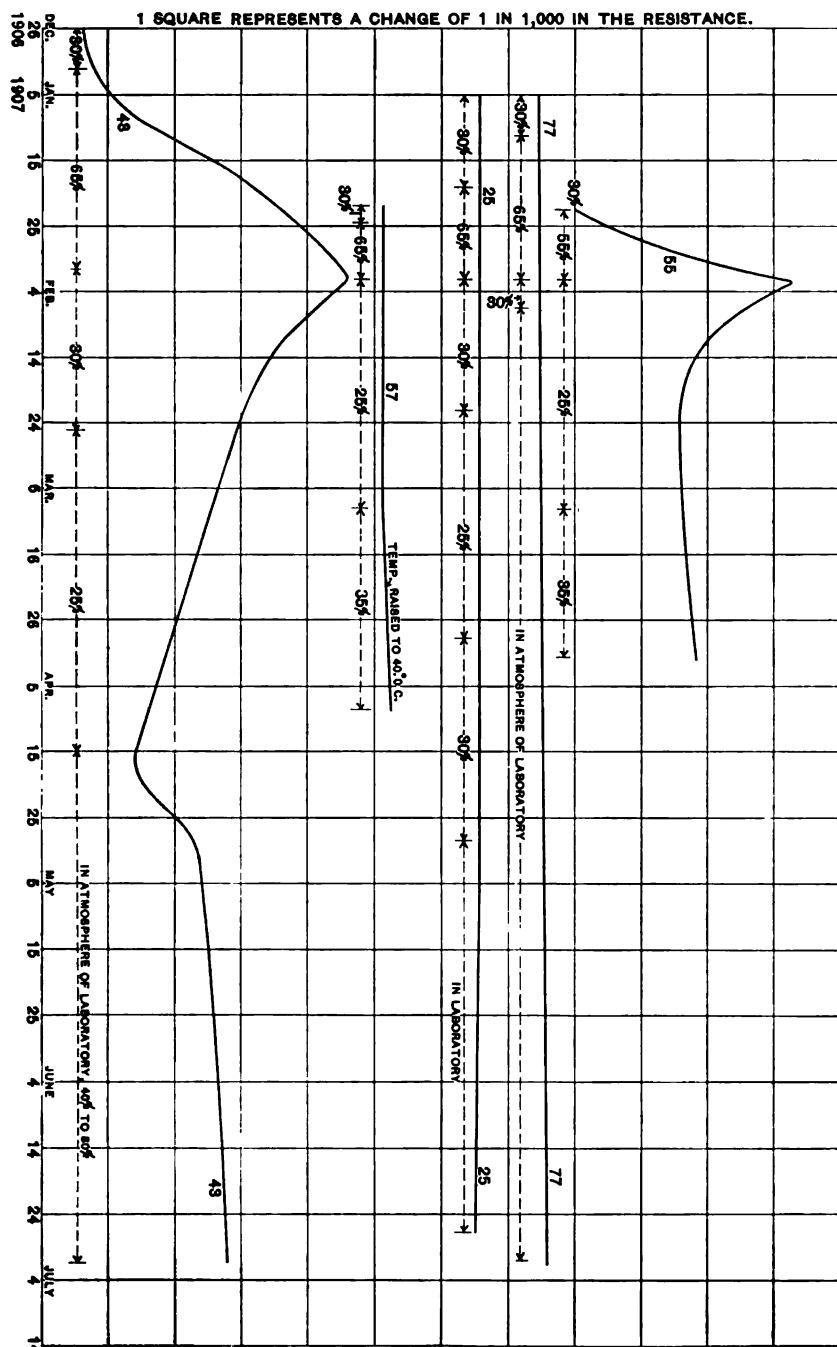


Fig. 6.—Differences in behavior of coils wound on wood spools.

(For explanation of curves see opposite page.)

maximum of February. As the humidity in the laboratory atmosphere in Washington often goes above 70 per cent in summer (unless the air is artificially dried), it will be seen that this coil would be very unreliable as a standard.

Fig. 6 represents five coils of 1,000 ohms each of 0.1-mm manganin wire, wound on wooden spools (mahogany). These curves are drawn to a different vertical scale from that used before, one space representing a change *ten times as great* as before. Curve 43 represents a coil which was given several coats of shellac and baked in the usual manner, the spool as well as the wire being thoroughly shellacked. Curve 55 represents a coil in which the spool was thoroughly shellacked, but the wire was not shellacked nor baked after winding. Curve 57 represents a coil wound on a similar wood spool that had been thoroughly paraffined, but the wire neither paraffined nor heated after winding. Curve 77 represents a coil on a shellacked wood spool, the wire shellacked and baked as usual and then dipped in melted paraffin. Curve 25 represents a coil on a wood spool, shellacked and baked after winding, and sealed in a test tube (with some calcium chloride inside the tube). The last three coils, in one of which no shellac was used and in the other two the shellac was protected from the moisture of the air, have remained remarkably constant, no change more than 1 in 100,000 having occurred in several months, except in 57, which was due to heating. The first two coils, on the contrary, changed as much as 340 and 400 parts, respectively, in 100,000.

Shellacked coils may absorb sufficient moisture in a saturated atmosphere to reduce the resistance by leakage, the coils in that case showing polarization. A shellacked coil plunged in water absorbs moisture very rapidly; in one case tried, the resistance of a coil of 1,000 ohms decreased about 50 ohms in a short time, showing polarization strongly. On the other hand, some paraffined coils submerged in water an hour did not change more than 1 part in 100,000, if at all. These coils were from a box by an English maker and had been in use for over four years. This shows that the paraffin coating had not become cracked or defective in use. Such coils kept submerged in water do indeed absorb moisture and finally become polarized, but this is under very extreme conditions. The shellacked coils of a Wheatstone bridge, potentiometer, or other

resistance apparatus may be dipped in melted paraffin and so be protected from moisture. A high grade of paraffin should be used—that is, paraffin of high melting point, free from dirt and acid. The paraffin coating, of course, slightly increases the lag of temperature of the coils when the temperature of the room is changing, but the increased error due to this would probably not be a hundredth part of the error that may be due to absorption of moisture.

Since a small and inexpensive coil sealed in a tube or covered by paraffin has a very constant resistance, one may possess a number of such working standards of resistance with very slight expense or trouble, and may have them compared at a standardizing laboratory more frequently than bulkier and more expensive standards. We do not recommend paraffining resistance standards of precision, but rather using such shellacked or varnished coils as have proved constant when sealed from the atmosphere. We are developing some new forms of sealed standards which have proved to be very constant during the few months they have been under observation. They will be described and the results obtained in measuring them given in the near future.

EFFECT OF CLIMATE.

Since the above was in type Drs. Jaeger and Lindeck have called attention⁴ to the unfavorable climate of Washington as explaining the relatively large seasonable changes observed here in shellac-covered resistances. One of us has recently published⁵ the results of an examination of the question of differences of climate between London, Berlin, and Washington, which seems to explain, in a large measure at least, why the changes observed in Washington are so much greater than those observed in Berlin.

In order to obtain a fair comparison between the atmospheric humidities of the three above-named places, reference was made to the official records of the Königlich Preussischen Meteorologischen Institut,⁶ the Royal Observatory, Greenwich,⁷ and the United States Weather Bureau.⁸ From these records the following tables were

⁴ London Electrician, August 2, 1907.

⁵ London Electrician, October, 1907.

⁶ Deutsches Meteorologisches Jahrbuch, für 1904 and 1905.

⁷ Greenwich Magnetical and Meteorological Observations, 1904 and 1905.

⁸ Report of Chief of Weather Bureau, 1904-5 and 1905-6.

compiled, showing the absolute humidities (vapor pressures in millimeters of mercury) and the relative humidities for each of the four seasons of the year for the two years 1904 and 1905, for London, Berlin, and Washington. The reason for choosing these three places was, of course, that observations on standard and precision resistances have been carried on for some years past in the national laboratories located in these cities. In Berlin no appreciable changes in resistance due to humidity have been observed; in Washington serious changes due to this cause have been found, as described above; in London appreciable changes have been reported, but we are not as yet aware whether (as seems probably the case) they have been found to be chiefly due to the effect of varying atmospheric humidity.

ABSOLUTE HUMIDITY.

Pressure of Aqueous Vapor in Millimeters of Mercury.

	London			Berlin			Washington		
	1904	1905	Mean	1904	1905	Mean	1904	1905	Mean
1. Winter—Dec., Jan., Feb.	5.4	5.4	5.4	4.5	4.5	4.5	2.9	3.0	2.95
2. Spring—Mar., Apr., May	6.5	6.3	6.4	6.2	6.3	6.25	7.0	7.9	7.45
3. Summer—Jun., Jul., Aug.	9.9	10.4	10.15	9.2	11.3	10.25	14.9	16.4	15.65
4. Autumn—Sep., Oct., Nov.	7.6	7.0	7.3	7.0	7.1	7.05	8.6	9.1	8.85
Mean for the year	7.35	7.3	7.32	6.7	7.3	7.0	8.35	9.1	8.72

The values for London are the mean of twenty-four hourly observations. For Berlin they are the mean of observations at 7 a. m., 2 p. m., and 9 p. m., which represent the twenty-four-hour average

RELATIVE HUMIDITY.

	London			Berlin			Washington		
	1904	1905	Mean	1904	1905	Mean	1904	1905	Mean
1. Winter—Dec., Jan., Feb.	87	85	86	85	84	84.5	70	67.5	68.75
2. Spring—Mar., Apr., May	78	75	76.5	70	78	74	66	68	67
3. Summer—Jun., Jul., Aug.	73	75	74	59	64	61.5	72	79	75.5
4. Autumn—Sep., Oct., Nov.	84	82	83	80	84	82	76.5	77.5	77
Mean for the year	80.5	79	79.75	73.5	77.5	75.5	71.5	72.75	72.1

very closely. For Washington they are the mean of observations at 8 a. m. and 8 p. m., which represent the twenty-four-hour average approximately, being probably a little too high.

Table I gives the values of the absolute humidities and Table II the relative humidities. It will be seen that the figures are very nearly the same for the two years.

The *mean vapor pressure* for the two years is least in Berlin, being a little higher (5 per cent) in London and considerably higher (25 per cent) in Washington than in Berlin. The vapor pressure is, however, lower in Washington in winter than in London or Berlin.

The *mean relative humidity* for the two years is *lower* in Washington than in Berlin or London, due, of course, to its higher mean temperature. In summer the relative humidity is higher in Washington than Berlin, and a very little higher than in London. During the other three seasons it is appreciably lower than in Berlin or London. This is especially noticeable in winter, when both absolute and relative humidities are lower in Washington than in London or Berlin.

But as the resistances are kept in the laboratories, whereas the above measurements were made out of doors, it is necessary to consider what differences in the relative humidities existed, on the average, between the laboratories and the outside atmosphere.

In the laboratories of the Bureau of Standards the rooms are heated by a mixture of hot and tempered air, forced through a double-duct system by large blowers, the temperature of the rooms being automatically regulated (usually to about 20° or 21° C. in winter) by thermostats. In the colder weather the blowers are kept going until midnight, and started very early in the morning, so that the temperature of the laboratories is kept at about 20° C. most of the time and seldom falls below 15° at night. The result is that the relative humidity of the atmosphere in the laboratories during the three winter months probably averages not over 30 per cent (allowing for the increase of humidity due to respiration and evaporation within the laboratories). The buildings of the Reichsanstalt are probably not heated to so high a temperature, nor heated for so many hours at night, so that the mean temperature for twenty-four hours during the three winter months would be considerably lower,

whereas the absolute humidity is higher. Hence the relative humidity (for twenty-four hours) possibly averages as high as 60 per cent during the winter months (allowing as before for some evaporation within the laboratories). In the summer a laboratory is cooler in the daytime and warmer at night than the outside atmosphere, having practically the same average temperature as the outside. Hence the mean relative humidity would be nearly the same as out of doors, or between 60 and 65 per cent in Berlin. In the spring and autumn months, due to heating the buildings part of the time, the indoor humidity would be somewhat less than outside. Hence it would seem probable (always considering the average for twenty-four hours) that the mean relative humidity in the laboratories of the Reichsanstalt varies comparatively little from season to season.

In Washington, on the other hand, the average humidity in summer (even with some artificial drying during part of each day) is more than double that in winter. Thus the differences of climate and differences in the methods of heating the laboratory buildings combine to make any effects of varying moisture much greater in Washington than in Berlin, the difference in climate being as much due to the drier winter of Washington as to the damper summer.

But Berlin is probably no more a representative city in this respect than Washington; resistances are used in all kinds of climates. The value of a resistance standard or a precision box or potentiometer should, of course, not depend on the climate, nor require a climate of relatively uniform humidity to enable it to remain constant. If a resistance standard is calibrated in Berlin or Washington at 60 per cent humidity for example, its value elsewhere will be more or less according to the humidity, if the resistance is not independent of the humidity.

Drs. Jaeger and Lindeck give results of recent measurements, which show very slight variations, indeed, in a number of standard coils of various denominations up to 10,000 ohms, confirming their previous work in this respect. Whether these coils were originally selected from a larger number because of their exceptional constancy, or whether they were differently prepared from those we have been getting in recent years, or whether it is partly accidental that the humidity varied during May and June of this year so as to leave the

coils with almost exactly the same values at the end of June that they had at the end of April, we do not know.

The 1-ohm standard coils (of the Reichsanstalt form) of the Bureau of Standards have fluctuated very little in comparison with the changes in the coils of higher values, but they do vary among themselves under natural conditions by an appreciable amount, due to variations in atmospheric humidity, and these variations have been a source of serious concern to us.

The sealed standards which we have recently devised and have been studying for several months differ mainly from those now generally used (and known as the Reichsanstalt form) in being of smaller size, having closed cases which are hermetically sealed, and in having the cases permanently filled with pure oil. The resistances are thus oil immersed, and the temperature can be accurately obtained, and yet no change due to varying humidity can occur, as the thoroughly dried resistances and oil are perfectly protected from the atmosphere.

Our work tends to confirm the work of Drs. Jaeger and Lindeck, as they very properly observe, not only that manganin is very well adapted for resistance standards, but that the method of mounting the resistances and protecting them from the oxidation of the air or oil by shellac or varnish is very satisfactory, *provided they are also protected from the effects of atmospheric humidity*. This effect is certainly too great in the higher resistances in general to be safely ignored in precision work, whether in standards or in resistance boxes. If by some modification in the construction of the standards, as Drs. Jaeger and Lindeck suggest, or by sealing them from the atmosphere as we have done, the danger of change due to humidity is obviated, then they would seem to be all that could be expected of wire standards. With resistance boxes that are not to be submerged in oil, we have found a coating of paraffin over the shellac or varnish to be of value. The temperature coefficient of manganin is so small that a slight uncertainty of temperature in many cases is not material. With boxes that are to be oil immersed the coils (or at least the higher valued coils) may be separately sealed in metal tubes, or better the entire box, if properly designed, may be sealed.

It has been a mystery to us at the Bureau of Standards why the manganin standards of the Reichsanstalt remained so much more constant than those in Washington. We think the facts given above as to the probable differences in the variation of the relative humidity in the laboratories of the Reichsanstalt and the Bureau of Standards go far toward explaining the difference in the behavior of the resistances. It is also possible that, although we have supposed the resistance coils to be made exactly alike, it is partly due to some difference in their preparation.

WASHINGTON, October 4, 1907.

THE SELF-INDUCTANCE OF A TOROIDAL COIL OF RECTANGULAR SECTION.

By Edward B. Rosa.

One of the most carefully constructed standards of self-inductance ever made is that described some years ago by Prof. Fröhlich, of Budapest.¹ It consists of a large marble ring of rectangular section, very accurately ground and measured, and wound with a single layer of fine silk-covered wire. Each turn of the wire lies in a plane passing through the axis of the ring, the wires being wound as closely as possible on the inner surface of the ring and uniformly spaced on the outer surface. Taking account of the magnetic field within the space occupied by the wire as well as the space within the marble core, Prof. Fröhlich obtained the following expression for the self-inductance of the coil:

$$L = 2\pi^2 h \left[\log \frac{r_2}{r_1} - 0.28721 \rho \left(\frac{1}{r_1} + \frac{1}{r_2} + \frac{2}{h} \log \frac{r_2}{r_1} \right) \right] \quad (1)$$

in which r_1 and r_2 , Fig. 1, are the inner and outer radii of the ring

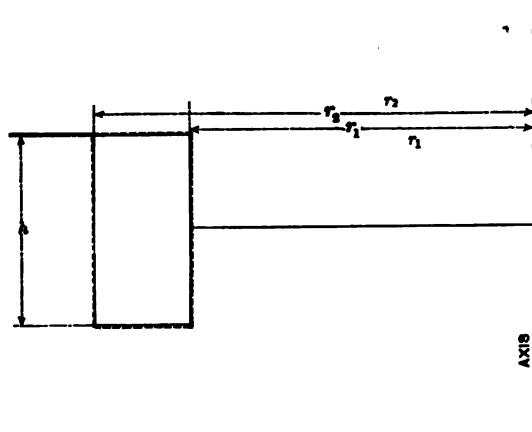


Fig. 1.

respectively, measuring to the axis of the wire in each case, ρ is the

radius of the bare wire, h is the height of the section (center to center of wire), and n is the whole number of turns of wire on the coil.

These quantities as determined by Fröhlich's were as follows:

$$r_2 = 35.05377 \text{ cm}$$

$$r_1 = 24.97478 \text{ cm}$$

$$h = 20.08455 \text{ cm}$$

$$\rho = 0.011147 \text{ cm}$$

$$n = 2,738$$

The value found by substituting these quantities in equation (1) was

$$\begin{aligned} L &= (0.10208928 - .00009865) 10^9 \text{ cm} \\ &= 0.10199063 \text{ henrys.} \end{aligned}$$

It was assumed in deriving formula (1) above that for a winding of fine wire one could consider the current as distributed in a thick current sheet, the thickness of the latter being 2ρ . That is, it was tacitly assumed that the flux of magnetic force, and therefore the self-inductance, were substantially the same as though the winding consisted of n turns of wire of rectangular section, the insulation being of infinitesimal thickness and the winding, therefore, completely enveloping the marble core. The wire in such a case would have a radial thickness of 2ρ and a breadth on the inner surface of the ring of $\frac{2\pi r_1}{n}$, on the outer surface of $\frac{2\pi r_2}{n}$, and on the upper and

lower surfaces the breadth would increase gradually from the former to the latter value. Such a winding is of course impracticable, but was assumed as equivalent to the actual winding of round, insulated wire. I have elsewhere^{*} pointed out that one can not assume a winding of round wires, no matter how fine, to be equivalent to a current sheet; and I wish now to show how serious is the error in the above case of a circular solenoid arising from this assumption.

The expression for the self-inductance of the circular solenoid of rectangular section enveloped by a *thin* current sheet is

^{*} This Bulletin, 2, p. 161; 1906.

$$L_0 = 2n^2 h \log \frac{r_2}{r_1} \quad (2)$$

The correction term ΔL as obtained by Fröhlich was *negative*, and *proportional to ρ , the radius of the wire*. For extremely fine wire ΔL would therefore become vanishingly small.

A thin current sheet would be realized by a winding of infinitely thin tape, covering the core completely. The problem is to find the difference ΔL in the self-inductance of such a winding and the actual winding of round insulated wire. I have done that² for a straight solenoid and the results obtained are approximately correct in this case. It may be worth while, however, to derive the correction formulæ anew for this particular case.

The self-inductance of a square of round wire is

$$L = 2 \left[l \log_e \frac{l + \sqrt{l^2 + R^2}}{R} - \sqrt{l^2 + R^2} + R \right]$$

where l is the length of wire in the square ($= 4a$) and R is the geometrical mean distance of the section of the wire. For wires of which the diameter is small in comparison with the side of the square this is very approximately

$$L_w = 2l \left[\log \frac{2l}{R_w} - 1 \right]$$

For a narrow tape bent into a square the expression for L is

$$L_t = 2l \left[\log \frac{2l}{R_t} - 1 \right]$$

where R_t is the geometric mean distance of the section of the tape. The difference is

$$L_t - L_w = 2l \left[\log_e \frac{R_w}{R_t} \right]$$

For a wire of circular section $R_w = 0.3894d$, d being the diameter. For a tape of infinitesimal thickness and width D , $R_t = 0.22313 D$.

Thus $\frac{R_w}{R_t} = 1.745 \frac{d}{D}$ and

$$L_t - L_w = 2l \log_e \left(1.745 \frac{d}{D} \right)$$

This is the excess of the self-inductance of one turn of thin tape over the self-inductance of one turn of round wire. For the entire winding of n turns we should therefore have

$$\Delta L_1 = 2nl \cdot \log_e \left(1.745 \frac{d}{D} \right)$$

This is equivalent to the expression previously found* for the case of a winding on a circular cylinder, $4\pi na$ being twice the whole length of the wire as $2nl$ is here. The same result would of course follow for a rectangular section not a square.

In addition to the correction for the difference in the self-inductances of the single turns is the correction for the differences in the mutual inductances upon each turn of all the others.

The mutual inductances of two parallel squares³ is

$$M = 8 \left[a \log_e \left(\frac{a + \sqrt{a^2 + d^2}}{a + \sqrt{2a^2 + d^2}} \cdot \frac{\sqrt{a^2 + d^2}}{d} \right) \right] + 8 \left[\sqrt{2a^2 + d^2} - 2\sqrt{a^2 + d^2} + d \right]$$

where a is the side of either square and d is the perpendicular distance between them. If each square is made up of round wire, d is the distance between the centers or axes of the wires. If, however, the squares are each made up of flat strip, d is the geometric mean distance of the two parallel strips, which may be written kd , where k is always a little less than unity,⁴ but nearer to unity as the distance d is greater. Putting M_w for the mutual inductance of the two rectangles (or squares) of round wire and M_t for that of the rectangles (or squares) of thin tape, which will be the same expression with d replaced by kd , it can easily be shown that, neglecting small quantities of the second order,

$$\Delta M = M_t - M_w = 8a \log_e \frac{1}{k} = 2l\delta,$$

which is equivalent to the expression (23) previously found* for parallel circles, $4\pi a$ being twice the length of the wire, as is $8a$

*This Bulletin, 2, p. 161; 1906.

³Webster, *Elect. and Mag.*, p. 456.

⁴This Bulletin, 2, p. 175; 1906.

here. As before, it is necessary to compute the value of this correction term ΔM for all pairs of wires for which it is appreciable. It is greatest for two adjacent wires, and rapidly becomes inappreciable as the distance increases, so that the fact that the several turns are not quite parallel can not make any appreciable difference. It is only the small differences in the mutual inductances we have to consider, and these would not be changed appreciably by inclining two rectangles at a small angle.

For two adjacent wires we have

$$\delta M_1 = 8a \log \frac{1}{k_1} = 2l\delta_1$$

For two wires separated by one turn

$$\delta M_2 = 8a \log \frac{1}{k_2} = 2l\delta_2$$

and so for each succeeding pair. In an endless ring every wire is affected by the same corrections as every other, so that the summation will be

$$\begin{aligned} \Delta M &= 2n[\delta M_1 + \delta M_2 + \delta M_3 + \dots] \\ &= 2nl[2\Sigma\delta]. \end{aligned}$$

The values of the δ s are given in the first column of Table IV.⁵ $\Sigma\delta$ is about 0.332 for 24 terms, being a little greater than in the case of a straight solenoid of 25 turns of wire. We may neglect the effect beyond 25 turns, and use Table VII for the values of the constant A and take B as equal to +0.332 in the expression

$$\Delta L = -(\Delta L_1 + \Delta M) = -2nl(A + B)$$

corresponding to (27) in the previous article. This expression for ΔL is always to be added; when $(A+B)$ is negative ΔL will be positive and will therefore tend to increase the self-inductance.

In the case of Fröhlich's circular solenoid the diameter of the bare wire is

⁵ This Bulletin, 2, p. 178; 1906.

$$d = 0.0223.$$

The mean distance between centers of successive turns is

$$D = 2\pi \left(\frac{r_1 + r_2}{2\pi} \right) = 0.0689$$

$$\therefore \frac{d}{D} = 0.324$$

From Table VII, $A = -0.57$

$$B = +0.33$$

$$A + B = -0.24$$

$$2nl = 2 \times 2738 \times 60.327 = 330300 \text{ cm}$$

$$\Delta L = +79300 \text{ cm}$$

$$= +0.0000793 \text{ henry}$$

$$L_0 = 0.1020893 \quad "$$

$$L = L_0 + \Delta L = 0.1021686 \quad "$$

instead of 0.1019906 as found by Fröhlich.

The value of A is positive when the insulation is thin, that is when d is nearly equal to D . When

$$1.745 \frac{d}{D} = 1,$$

the correction term A is zero. That is, for $\frac{d}{D} = 0.57$ approximately.

For $\frac{d}{D}$ less than 0.57, A is negative, and the corresponding correction tends to increase L_0 . The term B , being positive, always tends to decrease L_0 . Hence when A is negative and greater than 0.33 the combined correction increases L_0 , as in the present case.

The meaning of this is that the self-inductance of a circle or rectangle of wire is less than that of a flat strip of width equal to the diameter of the wire. As the diameter of the wire is reduced, however, its self-inductance increases, and becomes greater than that of the strip. So if the solenoid is wound with round wire of very thin insulation its self-inductance is less than L_0 , the current sheet value; but if the wire is finer and covered with thick insulation (its outer diameter remaining the same), the total self-inductance increases and may be more than L_0 .

To further show the marked effect of spacing the wire, suppose the above solenoid has been wound with a double winding of 1,369 turns each, so that either could be employed, or both, in series or in parallel. If both were in parallel and half the current flowing in each, the self-inductance would be one-fourth of that just calculated, or $L = 0.0255223 + .0000198 = .0255421$.

By Fröhlich's formula it would make no difference whether one winding or both in parallel were employed as to the value of L . But if one winding only is used D is twice as great as before, and therefore $\frac{d}{D} = 0.162$.

$$\begin{aligned}
 \text{By Table VII,} \quad A &= -1.42 \text{ approx.} \\
 B &= +0.33 \\
 \hline
 A + B &= -1.09 \\
 -2nl(A+B) &= 180,013. \\
 \Delta L &= +0.0001801 \text{ henry} \\
 L_0 &= 0.0255223 \quad " \\
 \hline
 L &= 0.0257024 \quad "
 \end{aligned}$$

This is greater than the preceding value by *nearly two-thirds of 1 per cent*. The meaning of this is that when only one winding is used there are a great many lines of force immediately surrounding the wires, which, carrying the whole current and having more space between the wires, give a larger correction term than when the current is divided between the two wires.

If the wires were very fine indeed (the whole number of wires remaining constant), the correction ΔL would be very great, easily *several per cent* of the value of L_0 , whereas by Fröhlich's formula the correction ΔL would be approaching zero and the self-inductance would be simply L_0 .

I have given these examples rather fully to emphasize the serious error that may result from assuming a winding of fine wires to be equivalent to a current sheet, and to show that for a given spacing of the wires *the finer they are (beyond a certain point) the greater is the error* arising from this assumption.

The straight solenoid offers many advantages over the circular solenoid as a standard. It is easier to construct and measure the core and immensely easier to wind, and the calculation of L from the dimensions is just as easy and just as certain.

WASHINGTON, August 10, 1907.

ON THE SELF-INDUCTANCE OF CIRCLES.

By Edward B. Rosa and Louis Cohen.

Various formulæ have been given by different authors for the self-inductance of circles; that is, for closed rings of circular cross section. Some of these formulæ are at once convenient and accurate, while others are both inconvenient and unreliable, and should be avoided in numerical calculations. We therefore propose in this paper to critically examine and test these various formulæ, and to show which of them are trustworthy and which are wrong. This seems the more necessary inasmuch as some of the latter have been given by writers of reputation and they have been quoted and used in the belief that they were correct.

The formula for the self-inductance of a circle was first given by Kirchhoff¹ in the following form:

$$L = 2l \left\{ \log \frac{l}{\rho} - 1.508 \right\} \quad (1)$$

where l is the circumference of the circular conductor and ρ is the radius of its cross section. This is equivalent to the following:

$$L = 4\pi a \left\{ \log \frac{8a}{\rho} - 1.75 \right\} \quad (2)$$

a being the radius of the circle. These formulæ are approximate, being more nearly correct as the ratio ρ/a is smaller.

A more accurate expression can be obtained from Maxwell's principle of the geometrical mean distance. The mutual inductance of two equal parallel circles near each other is, to a close approximation,

¹ Pogg. Annalen, 121, p. 551; 1864.

$$M = 4\pi a \left\{ \left(1 + \frac{3}{16} \frac{b^2}{a^2} \right) \log \frac{8a}{b} - \left(2 + \frac{b^2}{16a^2} \right) \right\} \quad (3)$$

where a is the common radius of the circles and b their distance apart. The self-inductance of a single circular ring is equal to the mutual inductance of two equal and parallel circles whose distance apart is equal to the geometrical mean distance R of the cross section of the ring. Hence

$$L = 4\pi a \left\{ \left(1 + \frac{3R^2}{16a^2} \right) \log \frac{8a}{R} - \left(2 + \frac{R^2}{16a^2} \right) \right\} \quad (4)$$

Substituting in this equation the value of the geometrical mean distance for a circular area, $R = \rho e^{-1/4} = .7788\rho$, we obtain

$$L = 4\pi a \left\{ \left(1 + 0.1137 \frac{\rho^2}{a^2} \right) \log \frac{8a}{\rho} - .0095 \frac{\rho^2}{a^2} - 1.75 \right\} \quad (5)$$

This is a very accurate formula for circles in which the radius of section ρ is very small in comparison with the radius a of the circle. The geometrical mean distance R has, however, been computed on the supposition of a linear conductor, and can only be applied to circles of relatively small value of ρ/a . We must therefore expect an appreciable error in formula (5) when the ratio ρ/a is not very small. Formulæ 1, 2, and 5 have been deduced on the supposition of a uniform distribution of the current over the cross section of the ring.

If the ring is a hollow circular thin tube, or if the current in the ring is alternating and of extremely high frequency, so that it can be regarded as flowing on the surface of the ring, the geometrical mean distance for the section would be the radius ρ , and we should derive from (4) by substituting $R = \rho$,

$$L = 4\pi a \left\{ \left(1 + \frac{3}{16} \frac{\rho^2}{a^2} \right) \log \frac{8a}{\rho} - \frac{\rho^2}{16a^2} - 2 \right\} \quad (6)$$

In the case of solid rings carrying alternating currents of moderate frequency the value of L would be somewhere between the values given by (5) and (6).

WIEN'S FORMULÆ.

Max Wien² has given what is probably the most accurate formula yet derived for the self-inductance of a circle.

If we consider the ring of radius a and radius of section ρ , Fig. 1, to be made up of an indefinite number of elementary circular filaments, the self-inductance of the ring is equal to the mean value of the sum of the mutual inductances on each filament of all the others. If, therefore, we express the mutual inductance of an element at P on a second element at Q and integrate this over the entire area of the section, we obtain the mutual inductance of the single filament P on the entire ring. Integrating again over the ring we obtain the self-inductance of the ring. Wien's result is as follows:

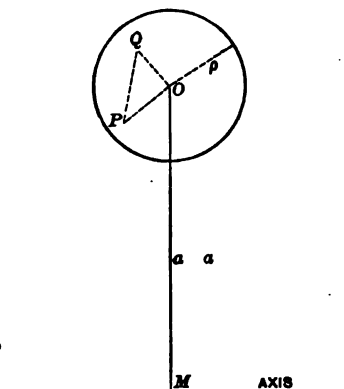


Fig. 1.

$$L = 4\pi a \left\{ \left(1 + \frac{1}{8} \frac{\rho^2}{a^2} \right) \log \frac{8a}{\rho} - .0083 \frac{\rho^2}{a^2} - 1.75 \right\} \quad (7)$$

It will be noticed that the formula differs very slightly from the preceding (5). Neglecting the terms in ρ^2/a^2 we get from either (5) or (7) Kirchhoff's approximate formula.

If the current be not distributed uniformly over the section of the wire, but the current density at any point is proportional to the distance from the axis of the ring, Wien's formula for the self-inductance is

$$L = 4\pi a \left\{ \left(1 + \frac{3}{8} \frac{\rho^2}{a^2} \right) \log \frac{8a}{\rho} - .092 \frac{\rho^2}{a^2} - 1.75 \right\} \quad (8)$$

which differs very slightly from (7).

This would apply to the case of a ring revolving about a diameter in a uniform magnetic field.

As would be expected, (8) gives a greater value than (7).

Rayleigh and Niven gave³ the following formula for a circular coil of n turns and of circular section:⁴

²Wied. Annalen, 53, p. 928; 1894.

³Rayleigh's Collected Papers, II, p. 15.

⁴Neglecting the correction for effect of insulation and shape of section of the separate wires.

$$L = 4\pi n^2 a \left\{ \left(1 + \frac{\rho^2}{8a^2} \right) \log \frac{8a}{\rho} + \frac{\rho^2}{24a^2} - 1.75 \right\} \quad (10)$$

When $n=1$, this will be the self-inductance of a single circular ring. It agrees with Wien's, except as to one term, which is

$$+ \frac{\rho^2}{24a^2} \text{ instead of } -0.0083 \frac{\rho^2}{a^2}.$$

If used for a coil of more than one turn, the expression for L (whether obtained from (10) or from one of the preceding more accurate expressions) must be corrected for the space occupied by the insulation between the wires and for the shape of the section.⁵

TESTS OF THE FOREGOING FORMULÆ FOR CIRCLES.

For a circle of radius $a=25$ cm and $\rho=0.05$ cm we obtain from the foregoing formulæ the following values of L :

By Wien's formula (7)	$L=654.40537 \pi$ cm
By Maxwell's formula (5)	$L=654.40533 \pi$ cm
By Rayleigh and Niven's (10)	$L=654.40548 \pi$ cm
By Kirchhoff's formula (2)	$L=654.40496 \pi$ cm
By Wien's second formula (8)	$L=654.40617 \pi$ cm

Thus, for so small a value of ρ/a as $\frac{1}{500}$ any of these formulæ is sufficiently accurate, the greatest difference being less than 1 in a million, except in the case of formula (8).

Take for further tests a circle for which $a=25$, $\rho=0.5$ cm, ρ/a being $\frac{1}{50}$, and another with $a=10$, $\rho=1.0$, ρ/a being $\frac{1}{10}$

	$\rho/a = \frac{1}{50}$	$\rho/a = \frac{1}{10}$
By Wien's formula	$L=424.1761 \pi$	105.497π
By Maxwell's formula	$L=424.1734 \pi$	105.476π
By Rayleigh and Niven's formula	$L=424.1781 \pi$	105.517π
By Kirchhoff's formula	$L=424.1464 \pi$	105.281π
By Wien's second formula	$L=424.2326 \pi$	105.902π

It will be seen that for the smallest ring of radius 10 cm and diameter of section 2 cm Maxwell's formula gives a result 1 part in 5,000 too small and Rayleigh and Niven's a value as much too large, while the simple approximate formula of Kirchhoff is in error by 1 in 500. For the larger ring the differences are much smaller.

Wien's second formula gives appreciably larger values, as it should do.

⁵ See Rosa, this Bulletin, 3, p. 1; 1907.

RUSSELL'S FORMULAE.

In his recent paper in the Philosophical Magazine, Russell* derives the approximate expression (2) for the self-inductance of a circle by an original method. Assuming that the flux of magnetic force through the aperture of a ring is the same as though the current in

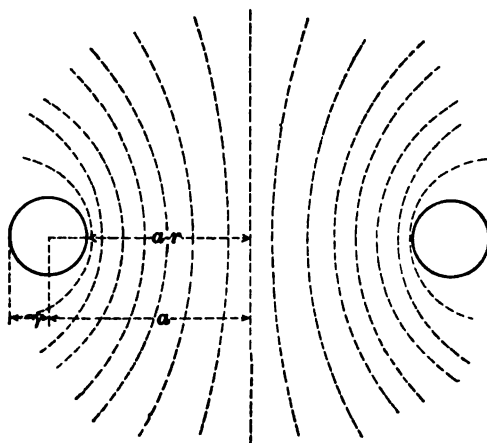


Fig. 2.

the ring were concentrated in the circular axis of the wire, he writes down the expression for the mutual inductance between the two co-planar circles, whose radii are a and $a-r$, which is the following:

$$M = \frac{8\pi a \sqrt{a(a-r)}}{\sqrt{k'}} (F' - E') = 8\pi a (F - E)$$

where the modulus is $\frac{a-r}{a}$.

This will be *approximately* that part of the self-inductance of a ring due to the flux through the aperture.

He then derives the second part of the flux, namely, that inside the section of the ring, which he gives as

$$\phi_1 = 2\pi \int_0^r (a-\xi) Z\left(\frac{\xi}{r}\right) d\xi$$

* Phil. Mag. 18, p. 428; 1907.

$$\text{where } Z = \frac{2i'}{\xi} + \frac{i'}{a} \log \frac{8a}{\xi}.$$

The integral of this expression for ϕ_s is πa *approximately* (neglecting the second term in Z). The total flux inside the section of the ring is, however, $2\pi a$ nearly. The expression πa results from the consideration that these $2\pi a$ tubes cut only part of the section of the conductor. Hence, multiplying by the factor $\frac{2}{\pi}$ the quantity πa results, which is half the flux and is the second term of the self-inductance. Adding the two terms, Russell obtains his approximate expression for L , which he shows can be put in the form of (2). This is an interesting variation in the method of obtaining the approximate value of L , but in itself gives no indication of the degree of the approximation, as do Maxwell's and Wien's methods.

MINCHIN'S FORMULÆ.

Prof. Minchin⁷ has undertaken to derive the self-inductance of a ring by finding the magnetic flux through the aperture of the ring

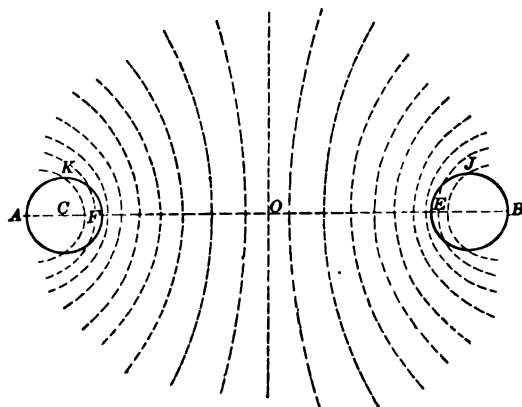


Fig. 3.

and adding the lines emanating from one side of the ring. Thus, suppose in Fig. 3, N_1 lines pass through the aperture FE and N_2 lines emerge from the upper surface EJB (taken throughout the circumference of the ring), then Minchin's statement is that $N_1 + N_2$

⁷Calculation of the coefficient of self-induction of a circular current of given aperture and cross section. Phil. Mag. 87, p. 300; 1894.

is the total number of lines of force linked with and emanating from the current, and the number divided by i , the strength of the current, is the "coefficient of self-induction." *As this neglects entirely all the lines of force wholly within the ring, which also contribute to the self-inductance*, Minchin's expression for L is necessarily wrong. The large value found in the single example given by Minchin is mainly due to an error in calculation, as his formula gives *too small a value for L* . His expression (5) for the flux through the aperture is substantially correct, but his expression (8) which contains the factor c is very small and does not include the considerable number of lines wholly within the section of the wire. Hence the sum of the two (9) which he gives as the coefficient of self-induction is wrong. Minchin's expression for L slightly rearranged is as follows, where c means the same as ρ above,

$$L = 4\pi a \left\{ \left(1 + \frac{c}{2a} - \frac{c^2}{32a^2} \right) \log \frac{8a}{c} - \left(2 + \frac{5c}{8a} + \frac{19c^2}{64a^2} \right) \right\} \quad (11)$$

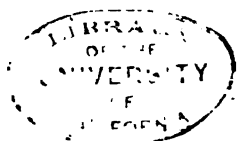
This expression is derived on the assumption that the current is inversely proportional to the distance from the axis. When the wire is very small this reduces to

$$L = 4\pi a \left(\log \frac{8a}{c} - 2 \right) \text{ approximately,} \quad (12)$$

whereas we have seen above that the second term should be 1.75. The difference is due, as just stated, to neglecting the lines of force within the section of the wire; but changing 2 to 1.75 does not make (11) correct.

Minchin also finds the expression for the self-inductance for a superficial current in the circular ring. This is given in his expression (10) and is somewhat *greater* than the other. Of course the self-inductance is *less* for a superficial current than for a distribution through the section of the wire (as we have seen above), whether the latter is uniform or inversely as the distance from the axis of the ring, as assumed by Minchin.

Prof. Minchin says that Maxwell gives the approximate value of L for a circle the same as (12) above, agreeing with his result. This is, however, a mistake. Maxwell gives



$$L = 4\pi a \left(\log \frac{8a}{R} - 2 \right)$$

but R is not the same as c above. R is the geometrical mean distance of the section of the wire, not the radius of section. As already shown, this leads to 1.75 for the absolute term.

HICKS'S FORMULÆ.

Prof. W. M. Hicks³ has discussed this question from a different standpoint and has derived expressions for the self-inductance of a ring both for the cases of uniform distribution of current, and for current density inversely proportional to the distance from the axis. He also has misinterpreted Maxwell's approximate expression for the self-inductance of a ring, not noticing that r (as he writes it, R as Maxwell wrote it) is the geometrical mean distance of the section and not the radius of section.

Hicks derives two formulæ, one for uniform current density and the other for current density inversely as r , corresponding to (7) and (9) above. Hicks derived his formulæ by the use of toroidal functions and obtained the following expression:

$$L_u = 4\pi a \left\{ -\frac{7}{6} - \frac{2}{3} \frac{\cos^3 a + 3 \cos^2 a + 4 \cos^4 a}{(1 + \cos a)^3} \right. \\ \left. + \frac{8 \cos^3 a}{3(1 + \cos a)^4} \left[6 \log \frac{8a}{\rho} + \frac{1}{2} + 39 \left(\log \frac{8a}{\rho} - 5 \right) k^4 \left(\frac{2115}{16} \log \frac{8a}{\rho} - \frac{981}{64} \right) \right] \right\} \quad (13)$$

where $\sin a = \frac{\rho}{a}$, $k^2 = \frac{1 - \cos a}{1 + \cos a}$; a and ρ are as before the radii of the ring and of its section respectively.

When the current density is inversely as the radial distance from the axis,

$$L_v = 4\pi a \left\{ \frac{4 \cos^2 a}{(1 + \cos a)^2} \left(\left(1 + \frac{7}{2} k^2 \right) \log \frac{4}{k} - \frac{3}{2} - \frac{9}{2} k^2 \right) - \frac{1 + 2 \cos a}{12} \right\} \quad (14)$$

For ρ/a very small, the terms in k^2 may be neglected and $\cos a = 1$ approximately, and we have as before for the approximate value of L

³ Phil. Mag. 38, p. 456; 1894.

$$L = 4\pi a \left(\log \frac{8a}{\rho} - 1.75 \right)$$

Taking the three circles previously used to test the formulæ of Kirchhoff, Maxwell, Wien, and Rayleigh and Niven we have—

	$\rho/a = \frac{1}{3.14}$	$\rho/a = \frac{1}{8}$	$\rho/a = \frac{1}{16}$
Hicks (13) . . $L_u =$	654.3898π	423.802π	102.7900π
Hicks (14) . . $L_v =$	654.4048π	424.130π	105.2388π

It will be seen by comparing the above results by Hicks's formulæ with those previously given, that these values are in every case less than given by Wien's two formulæ, and in the case of uniform current density less than the values given by any of the formulæ, *even less than by Kirchhoff's approximate formula*. But the correction terms must always increase the value of the inductance. Hence, it appears that Hicks's formula for uniform density at least, and probably also for variable density, *is entirely untrustworthy, the correction terms making the error greater rather than less*. The approximate formula gives a result too small by $\frac{1}{3.14}$ in the case of the third ring, where $\rho/a = \frac{1}{16}$, while Hicks's elaborate formula gives a result too small by over 2.5% for this case.

It may be asked how we know the formulæ of Wien and Maxwell to be correct. The answer is that Maxwell's for large rings is derived directly from the expression for the mutual inductance of two parallel circles using the expression for the geometric mean distance of the circular section of the wire, which for a straight wire is an absolute expression, not an approximation. Hence, because they agree we know that for large circles (ρ/a small) both Maxwell's and Wien's expressions are correct to a very high degree, and since for the third circle they agree to 1 in 5000, we may safely assume they are quite accurate for that case also. The correction factor is positive for large circles and must always be positive.

Hicks's formulæ were derived by a very elaborate process, which involved successive approximations. It is evident that the errors occurring in these approximations exceeded the total value of the small correction terms which it was the object of the investigation to determine.

BLÁTHY'S FORMULÆ.

Bláthy^{*} gave an expression for the self-inductance of a circle which he supposed to be exact, and also expanded it into a more convenient form for calculation, the latter being presumably accurate to a very high degree.

The following are Bláthy's formulæ, the first being the so-called "exact expression:"

$$L = 4\pi a \left\{ \log_e \frac{4a - \rho + \sqrt{16a^2 - 8a\rho}}{\rho} + \frac{16a^2}{15\rho^2} - \left(\frac{16a^2}{15\rho^2} + \frac{4a}{15\rho} + \frac{8}{5} \right) \sqrt{1 - \frac{\rho}{2a}} \right\} \quad (15)$$

$$L = 4\pi a \left\{ 0.57944 + \log_e \frac{a}{\rho} - \frac{2\rho}{a} - \frac{\rho^2}{24a^2} - \frac{\rho^3}{48a^3} - \dots \right\} \quad (16)$$

Calculating the self-inductance of the above circles by the first of these formulæ, we have,

For largest circle,	$a = 25$	$\rho = 0.05$	$L = 676.2056\pi$
For second circle,	$a = 25$	$\rho = 0.5$	$L = 449.4835\pi$
For smallest circle,	$a = 10$	$\rho = 1.0$	$L = 115.9656\pi$

Comparing these results with the values by the formulæ of Maxwell, Wien, and Kirchhoff we see that the first is in error by 3.5%, the second by 6%, and the third by 9%. The second formula gives substantially the same results.

Neglecting the three smallest terms in the second formula it may be written

$$L = 4\pi a \left(\log \frac{8a}{\rho} - 1.50 \right)$$

The absolute term should be 1.75 instead of 1.50, and this accounts for the principal part of the error in the results by Bláthy's formulæ.

Examining his method, we see that two assumptions have been made that are not permissible. The first is that one may integrate continuously up to the center of the wire in finding the total flux

^{*} London Electrician, 24, p. 630; April 25, 1890.

through the ring, and the second in assuming that all the lines within the ring cut the entire area of the section of the ring. This necessarily gives too large a value for L , and makes Bláthy's formula entirely unreliable.

Bláthy's formula is often given,¹⁰ apparently because it is a simple formula and was supposed to be very exact. But Kirchhoff's is much simpler, and, as the three examples given show, is amply accurate for most cases.

We thus see that Kirchhoff's simple approximate formula (2) is not only very convenient, but for many cases amply accurate; that the formulæ (5) and (6) derived by means of Maxwell's principle of the geometrical mean distance, and formulæ (7), (8), and (9), derived by direct integration, are very accurate for all cases except where the cross section of the ring is very large in comparison with the radius a ; and that the more complex formulæ of Minchin, Hicks, and Bláthy are wrong as well as inconvenient, and should be avoided.

WASHINGTON, August 10, 1907.

¹⁰ Heydweiller (*Elektrische Messungen*) gives only Bláthy's formula for circles.

THE INFLUENCE OF FREQUENCY ON THE RESISTANCE AND INDUCTANCE OF SOLENOIDAL COILS.

By Louis Cohen.

The resistance and inductance of a conductor depend on the distribution of the current within the conductor. When the current is continuous the distribution is uniform throughout the section of the conductor, and the problem of finding the resistance and inductance is a very simple one. If, however, the current is varying with respect to time, eddy currents with their accompanying magnetic fields will be generated within the conductor, which will alter the distribution of the current and consequently change the resistance and inductance of the conductor. The problem of finding the law of variation of resistance and inductance with the frequency is of considerable importance, and for the case of a straight cylindrical wire the problem has been satisfactorily worked out. The results, however, as obtained for the case of a straight wire, will not hold when the same wire is wound into a coil. It has been found experimentally that the change in resistance of a coil for any given frequency is much larger than if the winding of the coil were drawn out into a straight conductor. Dolezalek,¹ in his experiments with high-frequency currents, was the first to call attention to the fact that the change of resistance is larger than that calculated by the theory for straight conductors. Battelli and Maggi have also carried out a series of experiments along that line and found the same results.

Owing to the importance of the problem in its application to wireless telegraphy, several investigators have sought to obtain solutions to this problem. Their efforts were not attended by any

¹Dolezalek, *Annalen der Physik*, 12; 1903.

high degree of success, inasmuch as the results obtained do not agree with experimental facts.

Max Wien,³ who first investigated the problem, obtained a solution in a form which is rather a slow convergent series, and the calculation of the various terms is very laborious. Independently of this inconvenience, which Wien himself pointed out, his theory is not correct, as was shown by Battelli, since he implicitly assumes that the distribution of the current in the section of the wire is the same at all points at the same distance from the axis. Sommerfeld,⁴ who made the next attempt to get a solution to the problem, also failed to obtain an agreement between his theoretical conclusions and experiment. According to Sommerfeld's theory, the change of resistance of a coil will be independent of the width of the winding—that is, if we replace one turn of 10 mm in width by ten turns of 1 mm in width, but of the same thickness, then the change in resistance will, in accordance with Sommerfeld's theory, be the same. This appears to the author to be incorrect; for apriori we can say that the change of resistance must be a function of the section of the winding. Recently another solution of the problem was given by G. Picciati,⁵ but unfortunately his theoretical results fail also to agree with experimental results.

The table on page 170 shows to what extent the values calculated by the various formulas, for the per cent change in resistance, differs from the experimental values. From the table it can readily be seen that none of the formulas derived by the various investigators agree with experimental results.

The question of change of self-inductance is not of such practical importance as the change in resistance, and there are no experimental results on record by which we could verify any theory. The problem was discussed by Wien, but the same criticism which was offered to his work on the change of resistance will also apply to his work on the change of inductance. Doctor Coffin⁶ has recently contributed two papers on the change of inductance of coils with frequency. He has not, however, developed any new theory,

³ M. Wien, *Annalen der Physik*, **14**, p. 1; 1904.

⁴ Sommerfeld, *Annalen der Physik*, **15**, p. 673; 1904.

⁵ G. Picciati, *Il Nuovo Cimento* (5), **2**, p. 351.

⁶ This Bulletin, **2**, p. 275; *Physical Review*, **22**, p. 365.

reproducing Sommerfeld's work with such modifications as to obtain the change in inductance in place of resistance.

In view of the above considerations, it appeared to the author desirable to investigate the problem anew, and it was very gratifying to find that the results obtained agree with experiment. My conclusions are that the change in resistance of a coil is a function of the frequency, the section of the winding, and the pitch of the winding.

Suppose we have a very long solenoid, and the windings very close together, so that we may assume the magnetic field within the solenoid to be uniform all the way through, even close to the winding, and parallel to the axis of Z ; the field outside the solenoid will be zero. Let us suppose the section of the winding to be square, and consider now the electric and magnetic distribution in a single

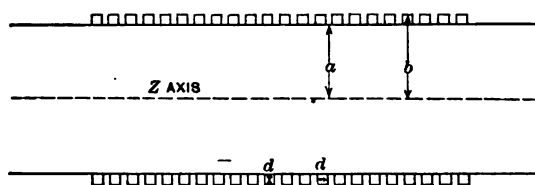


Fig. 1.

winding. The electric force will be circular, centered on the axis of Z , while the magnetic force will have two components, one radial and the other longitudinal, parallel to the axis of Z .

Let E denote the electric force.

- M " the longitudinal component of the magnetic force.
- L " the radial component of the magnetic force.
- I_1 " the current at any point in conductor due to variation of magnetic field within conductor.
- I " the total current in conductor.
- σ " the conductivity.
- d " width or breadth of section of winding.

The connection between electric and magnetic forces are as follows:

Curl (magnetic force) = $4\pi \times$ current.

Curl (electric force) = rate of variation of magnetic force
with respect to time.

In this particular case these relations can be expressed in the following form:

$$\left. \begin{aligned} \frac{dM}{dr} - \frac{dL}{dz} &= 4\pi\sigma E \\ -\frac{dE}{dz} &= \frac{dL}{dt} \\ \frac{dE}{dr} + \frac{E}{r} &= \frac{dM}{dt} \end{aligned} \right\} \quad (1)$$

From the above three equations we can easily deduce the following equation of propagation:

$$\frac{d^2 E}{dr^2} + \frac{1}{r} \frac{dE}{dr} - \frac{E}{r^2} + \frac{d^2 E}{dz^2} = 4\pi\sigma E \quad (2)$$

If the impressed force is simple harmonic with respect to the time, we can put

$$E = u \cos mze^{ipt}$$

where u is function of r only, and m is any arbitrary constant.

Introducing the value of E into equation (2) we get the following:

$$\frac{d^2 u}{dr^2} + \frac{1}{r} \frac{du}{dr} + \left(-k^2 - \frac{1}{r^2} \right) u = 0 \quad (3)$$

$$k^2 = m^2 + 4\pi\sigma ip$$

The solution of equation (3) will be

$$u = AJ_1(ikr) + BY_1(ikr)$$

J_1 and Y_1 are the Bessel functions of the first and second kind, and of the first order. The complete solution of equation (2) will be

$$E = \left\{ AJ_1(ikr) + BY_1(ikr) \right\} \cos mze^{ipt} \quad (4)$$

The values of L and M may be obtained from equation (4) by the aid of the relations between the electric and magnetic forces as given by equations (1), thus:

$$L = \frac{m}{ip} \left\{ A J_1(ikr) + B Y_1(ikr) \right\} \sin mze^{ipt} \quad (5)$$

$$\begin{aligned} \frac{dM}{dr} &= \frac{dL}{dz} + 4\pi\sigma E = \left(\frac{m^2}{ip} + 4\pi\sigma \right) \left\{ A J_1(ikr) + B Y_1(ikr) \right\} \cos mze^{ipt}. \\ &= \frac{k^2}{ip} \left\{ A J_1(ikr) + B Y_1(ikr) \right\} \cos mze^{ipt} \\ M &= \frac{k}{p} \left\{ A J_0(ikr) + B Y_0(ikr) \right\} \cos mze^{ipt} \end{aligned} \quad (6)$$

J_0 and Y_0 are the Bessel functions of the zero order.

Since m is an arbitrary constant, we can choose the value of m so as to suit the conditions of the problem.

Let us put

$$m = \frac{n\pi}{d} \quad (n = 1, 3, 5, 7, \dots)$$

and since k is a function of m , k will also have an infinite number of values. If we now put

$$A = \frac{4}{n\pi} (i)^{n-1} A', \quad B = \frac{4}{n\pi} (i)^{n-1} B'$$

then the values of E , L , and M will be given by the following expressions:

$$\left. \begin{aligned} E &= \sum_{n=1}^{\infty} \left\{ A'_n J_1(ik_n r) + B'_n Y_1(ik_n r) \right\} \frac{4}{n\pi} (i)^{n-1} \cos \frac{n\pi z}{d} e^{ipt} \\ L &= \sum_{n=1}^{\infty} \frac{m_n}{ip} \left\{ A'_n J_1(ik_n r) + B'_n Y_1(ik_n r) \right\} \frac{4}{n\pi} (i)^{n-1} \sin \frac{n\pi z}{d} e^{ipt} \\ M &= \sum_{n=1}^{\infty} \frac{k_n}{ip} \left\{ A'_n J_0(ik_n r) + B'_n Y_0(ik_n r) \right\} \frac{4}{n\pi} (i)^{n-1} \cos \frac{n\pi z}{d} e^{ipt} \end{aligned} \right\} \quad (7)$$

If the solenoid is of considerable length the magnetic field within the solenoid will be constant and equal to the magnetizing force, which is $4\pi SI$ where I is the total current and S the number of turns per unit length, and the field outside the solenoid will be zero.

At the surfaces of the solenoid, M , the longitudinal component of the magnetic force within winding, will be the only one acting;

and to satisfy the condition of continuity we must have the following relations satisfied:

$$\left. \begin{aligned} \Sigma \frac{k_n}{\rho} \left\{ A'_n J_0(ik_n a) + B'_n Y_0(ik_n a) \right\} \frac{4}{n\pi} (i)^{n-1} \cos \frac{n\pi z}{d} e^{i\rho t} &= 4\pi s I e^{i\rho t} \\ \Sigma \frac{k_n}{\rho} \left\{ A'_n J_0(ik_n b) + B'_n Y_0(ik_n b) \right\} \frac{4}{n\pi} (i)^{n-1} \cos \frac{n\pi z}{d} e^{i\rho t} &= 0 \end{aligned} \right\} \quad (8)$$

(a is the internal and b the external radius of the solenoid)

These two equations will be satisfied if we determine every set of A' and B' from the following equations:

$$\left. \begin{aligned} k_n \rho \{ A'_n J_0(ik_n a) + B'_n Y_0(ik_n a) \} &= 4\pi s I \\ \{ A'_n J_0(ik_n b) + B'_n Y_0(ik_n b) \} &= 0 \end{aligned} \right\} \quad (9)$$

Then the first equation of (8) will reduce to

$$4\pi s I \left\{ \cos \pi z - \frac{1}{3} \cos \frac{3\pi z}{d} + \frac{1}{5} \cos \frac{5\pi z}{d} - \dots \right\} = 4\pi s I \quad (10)$$

Now since the series in the brackets for the interval $-\frac{d}{2}$ to $+\frac{d}{2}$ is $\frac{\pi}{4}$, equation (10) is an identity, and consequently the conditions as expressed by equation (8) will be satisfied.

The constants A' and B' , as determined by equation (9), are as follows:

$$\left. \begin{aligned} A'_n &= \frac{4\pi s I \rho}{k_n} \frac{Y_0(ik_n b)}{J_0(ik_n a) Y_0(ik_n b) - J_0(ik_n b) Y_0(ik_n a)} \\ B'_n &= \frac{4\pi s I \rho}{k_n} \frac{J_0(ik_n b)}{J_0(ik_n a) Y_0(ik_n b) - J_0(ik_n b) Y_0(ik_n a)} \end{aligned} \right\} \quad (11)$$

If we denote by I_1 the current at any point in the conductor, then

$$I_1 = \sigma E = \sigma \sum_{n=1}^{n=\infty} \left\{ A'_n J_1(ik_n r) + B'_n Y_1(ik_n r) \right\} \frac{4}{n\pi} (i)^{n-1} \cos \frac{n\pi z}{d} e^{i\rho t}$$

The heating effect produced by the current I_1 in a given volume will be

$$\iiint \frac{I_1^2 dv}{\sigma}$$

and it must be remembered that this heating effect is produced by the eddy currents only. In addition to that there is of course a heating effect due to the current flow in the circuit.

Considering, however, only the eddy currents, we may look upon the heating effect as if we had a resistance R and a current I passing through it that is

$$\iiint \frac{I_1^2 dv}{\sigma} = R' I^2$$

$$R' = \frac{1}{I^2} \iiint \frac{I_1^2 dv}{\sigma}$$

Considering our volume as that of a single turn, we have

$$R' = \int_0^{2\pi} \int_a^b \int_{-\frac{d}{2}}^{\frac{d}{2}} \frac{I_1^2 r dr dz d\theta}{\sigma}$$

Introducing the value I_1^2 , we get

$$R' = 16\pi^2 \sigma p^2 s^2 \int_0^{2\pi} \int_a^b \int_{-\frac{d}{2}}^{\frac{d}{2}} \left[\sum_{n=1}^{n=\infty} \frac{1}{k_n} \right.$$

$$\left. \frac{Y_0(ik_n b) J_1(ik_n r) - J_0(ik_n b) Y_0(ik_n r)}{J_0(ik_n a) Y_0(ik_n b) - J_0(ik_n b) Y_0(ik_n a)} \frac{4}{n\pi} (i)^{n-1} \cos \frac{n\pi z}{d} \right]^2 r dr d\theta dz \quad (12)$$

Since, however,

$$\int_{-\frac{d}{2}}^{+\frac{d}{2}} \cos p \frac{\pi z}{d} \cos q \frac{\pi z}{d} dz = 0$$

Where p and q are two distinct odd numbers, therefore in calculating (12) we must only consider the square of each term and neglect the product terms, and therefore

$$R' = 256 s^2 p^2 \sigma 2\pi \int_a^b \int_{-\frac{d}{2}}^{\frac{d}{2}} \sum_{n=1}^{n=\infty} \frac{1}{n^2 k_n^2}$$

$$\left[\frac{Y_0(ik_n b) J_0(ik_n r) - J_0(ik_n b) Y_0(ik_n r)}{J_0(ik_n a) Y_0(ik_n b) - J_0(ik_n b) Y_0(ik_n a)} \right]^2 (i)^{2n-2} \cos^2 \frac{n\pi z}{d} r dr dz$$

$$= 256 \, s^3 p^3 \sigma \pi d \int_a^b \sum_{n=1}^{\infty} \frac{1}{n^3 k_n^3} \left\{ \frac{Y_0(ik_n b) J_0(ik_n r) - J_0(ik_n b) Y_0(ik_n r)}{J_0(ik_n a) Y_0(ik_n b) - J_0(ik_n b) Y_0(ik_n a)} \right\}^2 (i)^{n-1} r dr \quad (13)$$

Now,

$$\begin{aligned} k^2 &= m^2 + 4\pi\sigma i p = (a + i\beta) \text{ say} \\ \therefore a^2 - \beta^2 &= m^2 \\ 2a\beta &= 4\pi\sigma p \end{aligned}$$

and, therefore,

$$\begin{aligned} a &= \sqrt{m^2 + \sqrt{\frac{m^4 + 16\pi^2 \sigma^2 p^2}{2}}} \\ \beta &= \sqrt{-m^2 + \sqrt{\frac{m^4 + 16\pi^2 \sigma^2 p^2}{2}}} \end{aligned}$$

Even if we shall neglect m^4 as compared with $16\pi^2 \sigma^2 p^2$, then the real part of k will be

$$a = \sqrt{2\pi\sigma} p = \sqrt{4\pi^2} f$$

where f is the frequency.

If the winding is of copper, then $\sigma = 5.9 \times 10^{-4}$, approximately, and if we assume f to be 2,000, then,

$$a = 2\sqrt{5.9 \times 10^{-4} \times 10 \times 2 \times 10^3} = 7, \text{ approximately.}$$

If $r = 5$ cm, say, then the real part of kr will be a large quantity, and we may therefore substitute for the Bessel's functions their approximate values, which are as follows:⁶

$$\begin{aligned} J_0(ikr) &= \frac{e^{kr}}{\sqrt{2\pi kr}}, & J_1(ikr) &= \frac{i e^{kr}}{\sqrt{2\pi kr}} \\ Y_0(ikr) &= \frac{e^{-kr} \sqrt{\pi}}{\sqrt{2kr}}, & Y_1(ikr) &= \frac{-i e^{-kr} \sqrt{\pi}}{\sqrt{2kr}}. \end{aligned}$$

⁶ See Gray and Mathews Treatise on Bessel Functions, p. 156.

Introducing these values in equation (13) we obtain the following:

$$\begin{aligned}
 R' &= 256 \ s^2 p^2 \pi d \sigma \int_a^b \sum_{n=1}^{\infty} \frac{1}{n^2 k^2} \left\{ \frac{e^{k_n(b-r)} + e^{-k_n(b-r)}}{e^{-k_n d} - e^{+k_n d}} \right\}^2 (i)^{2n-2} r \frac{a}{r} dr \\
 &\quad (b-a=d) \\
 &= 256 \ s^2 p^2 \pi d a \sigma \int_a^b \sum_{n=1}^{\infty} \frac{1}{n^2 (a_n + i\beta_n)^2} \left\{ \frac{e^{(a_n + i\beta_n)(b-r)} + e^{-(a_n + i\beta_n)(b-r)}}{e^{-(a_n + i\beta_n)d} - e^{(a_n + i\beta_n)d}} \right\}^2 dr
 \end{aligned}$$

In working, however, with complex quantities the real part of the squares of a complex quantity can be obtained by multiplying the quantity by its conjugate. In this case the conjugate is obtained by substituting $-i$ for i everywhere in the expression, and this will give

$$\begin{aligned}
 R' &= 256 \ s^2 p^2 \pi d a \sigma \int_a^b \sum_{n=1}^{\infty} \frac{1}{n^2 (a_n^2 + \beta_n^2)} \frac{\cosh 2a_n(b-r) + \cos 2\beta_n(b-r)}{\cosh 2a_n d - \cos 2\beta_n d} dr \\
 &= 128 \ s^2 p^2 \pi d a \sigma \sum_{n=1}^{\infty} \frac{1}{n^2 (a_n^2 + \beta_n^2)} \frac{\frac{1}{a_n} \sinh 2a_n d + \frac{1}{\beta_n} \sin 2\beta_n d}{\cosh 2a_n d - \cos 2\beta_n d} \quad (14)
 \end{aligned}$$

When d is small (1 or 2 mm), and the frequency is fairly high, then to a very high degree of approximation we can neglect $\sin 2\beta_n d$ and $\cos 2\beta_n d$ as compared with $\sinh 2a_n d$ and $\cosh 2a_n d$, and under these conditions $\sinh 2a_n d$ will be very nearly equal to $\cosh 2a_n d$, and therefore equation (14) will reduce to

$$R' = 128 \ s^2 p^2 \pi a d \sigma \sum_{n=1}^{\infty} \frac{1}{n^2 (a_n^2 + \beta_n^2) a_n} \quad (15)$$

The value of R' , as given by equation (15), represents the increase in resistance due to the variation of the electric and magnetic fields within the material of conductor. In addition to this we have of course the ordinary resistance of the wire which it offers to direct current, and if we denote this resistance by R_0 , then the total resistance will be

$$R = R' + R_0.$$

The following table gives the values of (per cent change in resistance) for two different coils and three distinct frequencies. Column one gives the experimental value of $\frac{R'}{R}$ as determined by Wien,⁷ and the other columns gives the values of $\frac{R'}{R}$ as calculated by various formulæ. R is the direct current resistance of coil, and R' is the change in the resistance due to frequency.

TABLE.

Coil I. $r=0.0485$ cm. $s=8.75$.

Frequency	$\frac{R'}{R}$ measured	$\frac{R'}{R}$ by Wien's formula	$\frac{R'}{R}$ by Sommerfeld's form	$\frac{R'}{R}$ by Picciatti formula	$\frac{R'}{R}$ by formula (15)
$2\pi \times 4050$	0.021	0.023	0.042	0.032	0.023
$2\pi \times 5680$	0.045	0.045	0.080	0.062	0.044
$2\pi \times 8310$	0.089	0.100	0.165	0.125	0.090

Coil II. $r=0.1$ cm. $s=4.57$.

$2\pi \times 4050$	0.34	0.521	0.58	0.45	0.365
$2\pi \times 5680$	0.60	1.045	0.94	0.75	0.59
$2\pi \times 8310$	0.89	2.053	1.37	1.11	0.89

The agreement between the observed values and those calculated by my formula is exceedingly good, which shows that the deduction of the formula on the assumption that the winding is of square section leads to accurate results. The formula (15) is very convenient

⁷ Loc. cit.

for calculation, the series is very rapidly converging, in fact for all ordinary cases where the wire is fairly fine and the frequency is anywhere above 2,000, the first term will suffice. An examination of equation (15) will show that the change in resistance is a function of the frequency, of the diameter of winding, and also of the number of turns per unit length or the pitch of the winding.

When the frequency is very high, say 10^6 , and the diameter of the wire is not exceedingly small, then the value of R as given by equation (15) will reduce to a still simpler form, for in that case we

may neglect $\frac{n^4 \pi^4}{d^4}$, as compared with $16 \pi^2 \sigma^2 p^2$, and the value of a and

β will be as follows:

$$a = \beta = \sqrt{2\pi\sigma p}$$

Introducing these values of a and β into equation (15) we get

$$\begin{aligned} R' &= \frac{128 s^2 p^2 \pi d a \sigma}{4 n \sigma p \sqrt{2 n \sigma p}} \sum_{n=1}^{n=\infty} \frac{1}{n^3} \\ &= \frac{32 s^2 d a \sqrt{p}}{\sqrt{2 \pi \sigma}} \sum_{n=1}^{n=\infty} \frac{1}{n^3} \end{aligned}$$

The resistance per unit length is

$$R' = \frac{16 s^2 \sqrt{f} d}{\pi \sqrt{\sigma}} \sum_{n=1}^{n=\infty} \frac{1}{n^3}$$

The total resistance will be

$$\begin{aligned} R &= R' + R_0 = R_0 + \frac{16 s^2 \sqrt{f} d}{\pi \sqrt{\sigma}} \sum_{n=1}^{n=\infty} \frac{1}{n^3} \\ &= R_0 \left\{ 1 + 4 s^2 \sqrt{f \sigma} d^2 \sum_{n=1}^{n=\infty} \frac{1}{n^3} \right\} \end{aligned} \quad (16)$$

It is interesting to compare the resistance of a solenoid at such high frequency with that of a straight wire of same length. The

resistance of a straight cylindrical conductor per unit length at a very high frequency is

$$R = \sqrt{\frac{1}{2} \rho R_0} = R_0 \sqrt{\frac{\frac{1}{2} \sigma d^2 \pi p}{4}} \\ = R_0 \frac{1}{2} \pi d \sqrt{\sigma f} \quad (17)$$

Now, if the frequency is very high, we can neglect unity as compared with the remaining term in (16), and therefore the ratio of the resistance of coil to that of straight conductor will be

$$\frac{R_c}{R_w} = \frac{4s^2 \sqrt{f \sigma} d^2 \sum_{n=1}^{\infty} \frac{1}{n^2}}{\frac{1}{2} \pi d \sqrt{f \sigma} \sum_{n=1}^{\infty} \frac{1}{n^2}} = \frac{8s^2 d^2 \sum_{n=1}^{\infty} \frac{1}{n^2}}{\pi \sum_{n=1}^{\infty} \frac{1}{n^2}} \quad (18)$$

Equation (18) shows that for high frequencies the ratio $\frac{R_c}{R_w}$ is independent of frequency, the ratio increases as the square of diameter of wire and inversely as the square of the pitch. The same conclusions were arrived at by Battelli by an entirely different method, and were verified by Blake experimentally.

EXPERIMENTAL RESULTS.

Though my theory agrees so closely with the experimental results of Wien, yet it was thought desirable to verify it further by additional experiments. Professor Pupin kindly consented to let me use his high-frequency machine, and the experiments were therefore conducted in his laboratory. The method adopted in measuring the resistance was the ordinary bridge method. To balance for the inductance I used a torroidal coil, which Professor Pupin had in his laboratory. The coil is wound with stranded wire of very small diameter, the whole diameter of the wire not being more than 0.2 cm, and this is made up of 30 wires, so that we can safely neglect the change in resistance in the balancing coil. As a detecting instrument I used a tuned telephone which Professor Pupin employs in some of his investigations. It is probably 50 times as sensitive as the ordinary telephone receiver. The method of procedure was to

determine the resistance of the coil by direct current, and then to measure its resistance in the bridge for alternating current. Two solenoids wound with wires of different diameters were employed. The frequencies were maintained constant by the aid of a tuning fork.

The dimensions of the coil are as follows:

Coil I:

Internal radius of coil	=	4.961 cm
diameter of wire	=	0.29 cm
Number of turns per cm	=	3.1433
total number of turns	=	476
resistance of coil	=	0.441 ohms

Coil II:

Internal diameter of coil	=	4.961 cm
diameter of wire	=	0.206 cm
Number of turns per cm	=	4.319
total number of turns	=	580
resistance of coil	=	1.028 ohms

The results obtained are given in the following table:

Coil I.

Frequency	$\frac{R'}{R}$ measured	$\frac{R'}{R}$ calculated
$2\pi \times 1280$	0.0159	0.160
$2\pi \times 896$	0.091	0.084

Coil II.

$2\pi \times 1280$	0.052	0.044
$2\pi \times 896$	0.027	0.022

The experimental values of the resistances could not be relied upon within .005 ohms, and hence the values may be modified slightly, which may either improve the agreement between the calculated and measured values or make it worse. The small difference between the measured and calculated values may be accounted for

if we bear in mind that, to get the final expression for the change in resistance, we have replaced the Bessel functions by exponential functions which is more nearly true as the argument gets large. Now, since the argument is a function of the frequency and of the radius of coil it is evident that for higher frequencies the approximation will be closer than for lower frequencies and this may account for the small discrepancy at the lower frequency. By final formula (15) therefore as it stands will hold absolutely for all cases where the argument (kr) is fairly large, say, 20 or more, which implies a fairly high frequency, perhaps 1,500 or more, and the radius of the coil must not be very small. We can of course vary the two factors, making one very large and one very small, but so long as the product kr is not small the formula will hold. If we have a low frequency and the radius of the coil is also small, then the final expression (15) will not be strictly true, and it will approximate the true value as kr gets larger.

CHANGE IN INDUCTANCE.

The change of inductance can be obtained by a method similar to that employed in obtaining the change in resistance. In this case we make use of the equality of electro-magnetic energy and electro-kinetic energy,

$$\frac{1}{8\pi} \int H^2 dv = \frac{1}{2} \mathcal{L} I^2$$

or

$$\mathcal{L} = \frac{1}{4\pi I^2} \int H^2 dv \quad (19)$$

Now

$$H^2 = L^2 + M^2$$

The values of L and M are given by equations (7)

$$L = \sum_{n=1}^{\infty} \frac{m_n}{ip} \left\{ A'_n J_1(ik_n r) + B'_n Y_1(ik_n r) \right\} \frac{4}{n\pi} (i)^{n-1} \sin \frac{n\pi z}{d} e^{ip\theta}$$

$$M = \sum_{n=1}^{\infty} \frac{k_n}{p} \left\{ A'_n J_0(ik_n r) + B'_n Y_0(ik_n r) \right\} \frac{4}{n\pi} (i)^{n-1} \cos \frac{n\pi z}{d} e^{ip\theta}$$

Introducing the values of the constants A' and B' from equations (11) and using the approximate values of the Bessel's functions we get:

$$L = \sum_{n=1}^{\infty} \frac{16 m_n s I}{k_n n} (i)^{n-1} \left\{ \frac{e^{k_n(b-r)} + e^{-k_n(b-r)}}{e^{-k_n d} - e^{k_n d}} \right\} \frac{\sqrt{a}}{\sqrt{r}} \sin \frac{n\pi z}{d}$$

$$M = \sum_{n=1}^{\infty} \frac{16 m_n s I}{n} (i)^{n-1} \left\{ \frac{e^{k_n(b-r)} - e^{-k_n(b-r)}}{e^{-k_n d} - e^{k_n d}} \right\} \frac{\sqrt{a}}{\sqrt{r}} \cos \frac{n\pi z}{d}$$

Now

$$H^2 = L^2 + M^2$$

and since H , L , and M are complex quantities, the square of the amplitude can be obtained by multiplying the quantity by its conjugate, which in this case will be merely putting $(-i)$ everywhere in place of (i) , and this will give us

$$H^2 = \sum_{n=1}^{\infty} \frac{256 s^2 I^2}{n^2 (a_n^2 + \beta_n^2)} \left\{ \frac{e^{2a_n(b-r)} - e^{-2a_n(b-r)} + e^{2i\beta_n(b-r)} + e^{-2i\beta_n(b-r)}}{e^{2a_n d} + e^{-2a_n d} - e^{-2i\beta_n d} - e^{2i\beta_n d}} \right\} \frac{a}{r} \sin^2 \frac{n\pi z}{d}$$

$$+ \sum_{n=1}^{\infty} \frac{256 s^2 I^2}{n^2} \left\{ \frac{e^{2a_n(b-r)} - e^{-2a_n(b-r)} - e^{-2i\beta_n(b-r)} - e^{2i\beta_n(b-r)}}{e^{2a_n d} - e^{-2a_n d} - e^{-2i\beta_n d} - e^{2i\beta_n d}} \right\} \frac{a}{r} \cos^2 \frac{n\pi z}{d}$$

$$= \sum_{n=1}^{\infty} \frac{256 s^2 I^2}{n^2 (a_n^2 + \beta_n^2)} \left\{ \frac{\cosh 2a_n(b-r) - \cos 2\beta_n(b-r)}{\cosh 2a_n d - \cos 2\beta_n d} \right\} \frac{a}{r} \sin^2 \frac{n\pi z}{d} \quad (21)$$

$$+ \sum_{n=1}^{\infty} \frac{256 s^2 I^2}{n^2} \left\{ \frac{\cosh 2a_n(b-r) - \cos 2\beta_n(b-r)}{\cosh 2a_n d - \cos 2\beta_n d} \right\} \frac{a}{r} \cos^2 \frac{n\pi z}{d}$$

For one single turn,

$$\int H^2 d\tau = \int_0^{2\pi} \int_a^b \int_{-\frac{d}{2}}^{+\frac{d}{2}} H^2 r dr dz d\theta = 2\pi \int_a^b \int_{-\frac{d}{2}}^{+\frac{d}{2}} H^2 r dr dz$$

and

$$\mathcal{L} = \frac{1}{4\pi I^2} \int H^2 dv = \frac{1}{2I^2} \int_a^b \int_{-\frac{d}{2}}^{+\frac{d}{2}} H^2 r dr dz$$

Introducing value of H^2 from equation (21) we get:

$$\begin{aligned} \mathcal{L} &= 128s^2a \left[\sum_{n=1}^{\infty} \int_a^b \int_{-\frac{d}{2}}^{+\frac{d}{2}} \left\{ \frac{1}{a_n^2 + \beta_n^2} \frac{\cosh 2a_n(b-r) - \cos 2\beta_n(b-r)}{\cosh 2a_nd - \cos 2\beta_nd} \right. \right. \\ &\quad \left. \left. \sin \frac{n\pi z}{d} + \frac{\cosh 2a_n(b-r) - \cos 2\beta_n(b-r)}{\cosh 2a_nd - \cos 2\beta_nd} \cos \frac{n\pi z}{d} \right\} dr dz \right] \\ &= 128s^2ad \sum_{n=1}^{\infty} \frac{1}{n^2} \left(\frac{1}{a_n^2 + \beta_n^2} + 1 \right) \frac{\frac{1}{2a_n} \sinh 2a_nd - \frac{1}{2\beta_n} \sin 2\beta_nd}{\cosh 2a_nd - \cos 2\beta_nd} \end{aligned} \quad (22)$$

When the frequency is very high and the diameter of wire is not very large, a will be a large quantity and equation (22) will reduce to the following:

$$\mathcal{L} = 64s^2ad \sum_{n=1}^{\infty} \left(\frac{1}{a_n^2 + \beta_n^2} + 1 \right) \frac{1}{a_n^3 n^3} \quad (23)$$

This is the inductance due to the magnetic field within the material of winding of a single turn, and the inductance due to the total number of turns will be

$$\mathcal{L} = 64s^2dal \sum_{n=1}^{\infty} \frac{1}{a_n n^3} \left(\frac{1}{a_n^2 + \beta_n^2} + 1 \right) \quad (24)$$

Where l is the length of the solenoid. The change in inductance will be the difference between the inductance, due to magnetic field within the material of the coil, when the current is continuous and alternating. For continuous current the inductance due to magnetic field in the material of the winding alone is⁸

$$\mathcal{L} = \frac{1}{6} 4\pi^2 s^2 l d (4a + d). \quad (25)$$

⁸ Heaviside collected papers, 1, p. 356.

Hence the change in inductance $\mathcal{L}'$ will be

$$\mathcal{L}' = s^2 dl \left\{ \frac{2}{3} \pi^2 (4a + d) - 64sa \sum_{n=1}^{\infty} \frac{1}{a_n^2 n^2} \left(\frac{1}{a_n^2} + \beta_n^2 + 1 \right) \right\} \quad (26)$$

The per cent change in inductance will be the ratio of change in inductance to the total inductance of solenoid. Now the total inductance of solenoid consists of two parts; one due to magnetic field within the solenoid, and the other due to magnetic field within material of the windings, thus:

$$\mathcal{L} = 4\pi^2 s^2 l a^2 + \frac{1}{6} 4\pi^2 s^2 l d (4a + d).$$

If a is not very small, then we may put

$$\mathcal{L} = 4\pi^2 s^2 l a^2.$$

If we denote the per cent change in inductance by $\mathcal{L}_s$, then we have

$$\mathcal{L}_s = \frac{s^2 dl \left\{ \frac{2}{3} \pi^2 (4a + d) - 64sa \sum_{n=1}^{\infty} \left(\frac{1}{a_n^2} + \beta_n^2 + 1 \right) \right\}}{4\pi^2 s^2 l^2}$$

$$= \frac{d \left\{ \frac{2}{3} \pi^2 - 16s \sum_{n=1}^{\infty} \frac{1}{a_n^2 n^2} \left(\frac{1}{a_n^2} + \beta_n^2 + 1 \right) \right\}}{\pi^2 a}$$

As an example, suppose $a = 5$ cm, $d = 0.2$ cm, $s = 4$

and

$$p = 2\pi \times 10^6 \quad a^2 = \beta^2 = 23.6 \times 10^3, \quad a = 154$$

and

$$\mathcal{L}_s = \frac{0.2 \left\{ \frac{2}{3} \times 10 - 16 \times 4 \frac{1}{154} \left(1 + \frac{1}{9} + \dots \right) \right\}}{50} = 0.025 \text{ approximately.}$$

The change in inductance for this particular case is about 2.5 per cent.

Résumé.—To sum up the results of this investigation we can say that the theory developed in this paper for the change in resistance and inductance of solenoids due to frequency is fully corroborated by experimental facts. The final expressions given by equations (15) and (26) are very simple and convenient for calculation, and will give accurate results in all cases except when the frequency is exceedingly low. For convenience of reference I reproduce here equations (15) and (26), indicating also the meaning of the factors involved.

$$R' = 128 \ s^2 p^2 \pi d a \sigma \sum_{n=1}^{n=\infty} \frac{1}{n^2 (a_n^2 + \beta_n^2)} a_n \quad (15)$$

$$\mathcal{L}' = s^2 d l \left\{ \frac{2}{3} \pi^2 (4a + d) - 64 s a \sum_{n=1}^{n=\infty} \frac{1}{a_n^2 n^2} \left(\frac{1}{a_n^2 + \beta_n^2} + 1 \right) \right\} \quad (26)$$

R' is the change in resistance due to frequency.

$\mathcal{L}'$ " " " " inductance " " "

$$a^2 = \frac{m^2 + \sqrt{m^4 + 16\pi^2 \sigma^2 p^2}}{2}$$

$$\beta^2 = \frac{-m^2 + \sqrt{m^4 + 16\pi^2 \sigma^2 p^2}}{2}$$

$$m = \frac{n\pi}{d} \quad (n = 1, 3, 5, 7, 9 \dots \dots \dots)$$

d denotes diameter of wire.

σ " conductivity.

p " 2π frequency.

s " number of turns per unit length.

a " internal radius of solenoid.

l " length of solenoid.

In conclusion, I wish to express my thanks to Prof. M. I. Pupin, of the Columbia University, for his kindness in permitting me to use his private laboratory, where the experimental part of the work was carried out, and I also wish to thank his assistant, Mr. W. E. Cushman, for his kind assistance in the experimental work.

WASHINGTON, August 16, 1907.

THE ATOMIC WEIGHT OF HYDROGEN.

By William A. Noyes.

Some years ago the writer¹ carried out a series of determinations of the quantitative composition of water, from which the value 15.896 was calculated for the atomic weight of oxygen on the hydrogen basis. The opinion was expressed, in the paper, that the true value was probably within one part in a thousand of this number and that it was rather below than above the value reported. Several years later Morley² published an account of his elaborate determinations of the densities of hydrogen and oxygen and of the composition of water by weight, which led him to the value 15.879 for oxygen, or 1.00762 for hydrogen ($O = 16$). Meanwhile Richards³ had shown that copper oxide retains occluded gases obstinately, a fact which was partly known to me at the time of my work, but which was not as carefully considered as it should have been. There was obtained in each of my experiments, at the end, a small amount of gas which was assumed to be nitrogen on the basis of one or two rather imperfect analyses of the earlier samples. The weight of this gas, calculated as nitrogen, was subtracted from the weight of the hydrogen, giving, on the average, a correction of about one part in a thousand. Several years after the publication of Morley's results it occurred to me that this gas came, very probably, from the copper oxide, and that its weight should be subtracted from the weight of the oxygen instead of from that of the hydrogen. On applying the correction in this manner the value 15.879 (or 1.00765) was found.

¹ Am. Chem. J., 12, p. 441; 1890.

² Smithsonian Contributions to Knowledge, 1895. Am. Chem. J., 17, p. 267 and p. 396; 1895. Z. phys. Chem., 17, p. 87; 1895.

³ Proc. Am. Acad., 26, p. 276; 1891.

While this agreement with Morley's value was gratifying, and the fact was stated to a few personal friends, it did not seem proper to publish a statement about the matter until the surmise could be confirmed by new experimental evidence.

In addition to the desire of confirming or refuting the above explanation of my earlier results, several reasons have made it seem worth while to undertake a new determination of this constant. Hitherto our accurate knowledge of the composition of water has rested on Morley's work alone. He secured so high a degree of concordance in his results, and he exercised such an extraordinary degree of care at every point that the results obtained by all other observers must be considered as having only confirmatory value, in comparison.⁴ It seemed worth while, if possible, to secure a similar order of accuracy by a different method.

Richards and Wells⁵ have recently given us a very accurate determination of the ratio between silver and chlorine, but the exact value for the atomic weight of chlorine is still in doubt, because the ratio between silver and oxygen is not satisfactorily known.⁶ A determination of the ratio between hydrogen and chlorine has been carried out at this Bureau, and by using hydrogen generated in the same manner for that determination and for the determination of the composition of water a very direct comparison between oxygen and chlorine has been secured. As the atomic weights of more than forty elements have been determined in their relation to silver or the halogens, or both, the fundamental importance of the ratios of chlorine and silver to oxygen is apparent.

Finally, it seemed possible so to carry out the work as to secure some evidence with regard to the question of change of weight in a chemical reaction in which a large amount of energy is dissipated. This phase of the problem was suggested to me while considering the claim made by Professor S. M. Babcock some years ago that ice loses weight when it melts. He proposed the theory that the loss of weight was intimately connected with the gain of energy by the ice as it melts. If this were true, hydrogen and oxygen should gain weight when they combine to form water. It may be said, at

⁴This will be discussed further in a later paper.

⁵J. Am. Chem. Soc., 27, p. 459; 1905.

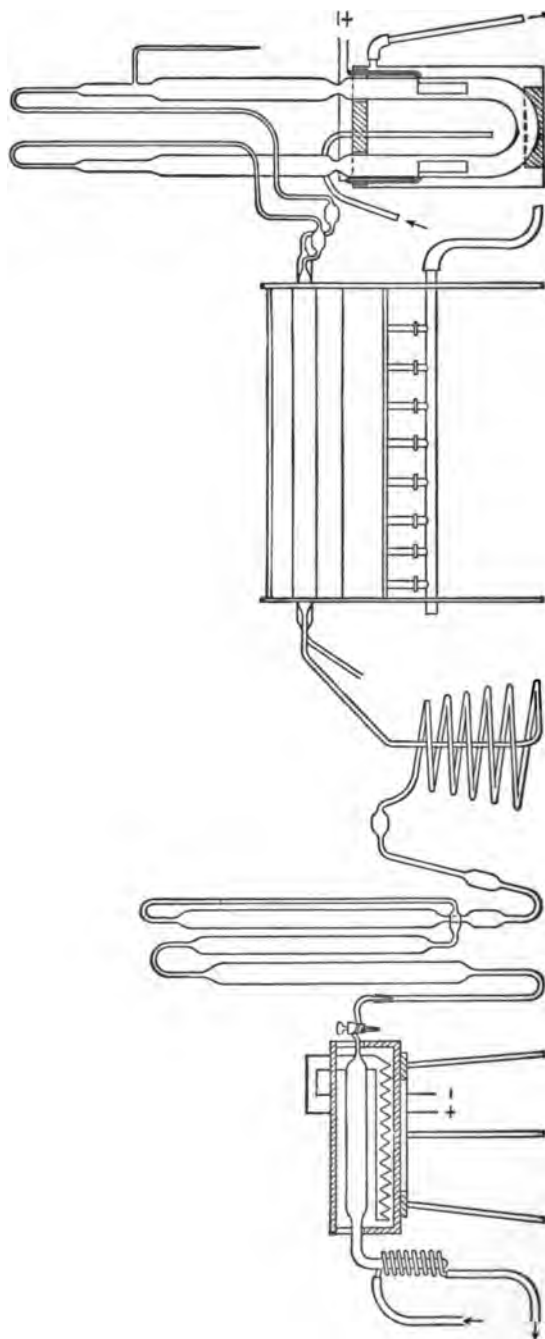
⁶Guye and Ter Gazarian, Compt. Rendus, 143, p. 411; 1906.

once, that the results to be given are not conclusive on this point. So far as they have any bearing on the question they indicate a loss rather than a gain in weight when hydrogen is converted into water.

Apparatus.—The apparatus used in the work was, in part, very similar to that used in the previous investigation.¹ Five series of determinations have been carried out. In the first four of these the hydrogen and oxygen were obtained by the electrolysis of dilute (15 per cent) sulphuric acid. In the last series the gases were generated by the electrolysis of a 6 to 8 per cent solution of barium hydroxide.

The purifying train will be easily understood from the figure. In the first three series of experiments, and in all but the last three determinations of the fourth series, the hydrogen was passed at first through a tube of common glass filled partly with platinized asbestos and partly with copper gauze, and the oxygen was passed through a similar tube filled partly with platinized asbestos, partly with asbestos mixed with lead chromate. These tubes were heated to 300°–350°. In the last three experiments of the fourth series hard glass tubes were substituted for the common glass, so that a higher temperature could be used, and in the fifth series hard glass tubes filled with platinized quartz were used. The platinized quartz was prepared by moistening the quartz, which had been heated and quenched in water, with a solution of chlorplatinic acid, and reducing the latter in a current of hydrogen. Each gas then passed through a serpentine tube about 4 meters in length and of 8 mm internal diameter. These tubes contain a 15 per cent solution of potassium hydroxide in which was dissolved a small amount of lead oxide. Sulphur compounds present in the hydrogen were reduced to hydrogen sulphide as the mixture passed over the platinized asbestos and copper, and the sulphur of the hydrogen sulphide was retained by the lead as lead sulphide. Only a very small amount of lead sulphide was deposited during the passage of more than 1,000 liters of hydrogen through the asbestos. When the barium hydroxide was used as an electrolyte, the serpentine tubes were replaced by bulbs to collect the condensed water and the solution formed by the deliquescence of the potassium hydroxide. The union between the hard and soft glass was made by grinding the

¹Am. Chem. J., 12, p. 441; 1890.



joint and then melting in it a very small amount of Khotinsky cement. Ample evidence is given below that these joints were perfect.

The gases passed next through the tubes containing potassium hydroxide in the form of sticks, and then through two or three tubes 15 mm in diameter and 25 cm long filled with phosphorus pentoxide which had been sublimed in a current of oxygen. When not in use these tubes were always sealed, and great care was exercised to avoid the entrance of moist air from the exits where the gases were delivered for use. If the phosphorus pentoxide were to become moist on that side there would be danger that the gases might carry some of the moisture away with them.

The electrolytic apparatus containing the sulphuric acid or solution of barium hydroxide consisted of a U-tube with platinum electrodes. The limbs of the tube were about 35 mm in diameter and 55 cm long. It was filled about two-thirds full at first, and when the solution became too concentrated, water recently boiled and cooled out of contact with the air was introduced through the small tube shown on the oxygen side of the apparatus. It was easy to do this without allowing any air to enter, but after each filling a considerable amount of the gases were generated before further use of them for a determination. The water used for preparing the original solution and for subsequent dilutions was prepared by redistilling distilled water with the addition of an alkaline solution of potassium permanganate and collecting the portion that was free from ammonia.

The resistance of the sulphuric acid in the apparatus was 4 to 5 ohms, while that of the solution of barium hydroxide was considerably greater. It was necessary to cool the electrolytic apparatus with a pretty rapid current of water through the jar containing it. The electric current used had a pressure of 120 volts and was reduced by means of a rheostat placed in series with the electrolytic cell. With the sulphuric acid electrolyte a current up to 15 amperes could be used, giving 6 liters of hydrogen per hour. With the barium hydroxide it was not considered safe to go beyond 4 liters per hour, because of the greater resistance and consequent heating of the solution.

Copper Oxide.—In the first three series of experiments copper oxide was used to convert the hydrogen into water. In the first three experiments copper oxide prepared by precipitating copper sulphate with a hot solution of sodium hydroxide was used. The precipitate was washed by decantation until it became colloidal and then washed thoroughly on a Buchner funnel. This oxide retained a trace of sulphate which was reduced by the hydrogen to sulphur dioxide, and the latter was found in small amount in the water obtained. Only two determinations were made with this oxide. Seven determinations were made with a sample of copper oxide purchased as pure from Kahlbaum. It contained arsenic and other impurities. As these determinations formed a part of the first series, the results of which are rejected for reasons to be given below, they need not be discussed further here. The copper oxide for the remainder of the first and for the second and third series was prepared by precipitation of a hot solution of copper sulphate, Rochelle salt, and sodium hydroxide with glucose. The cuprous oxide obtained could be washed indefinitely, and all sulphates were very completely removed. After ignition in a current of oxygen the oxide retained a very small amount of carbon dioxide; but as this appeared as a gas at the end of the experiment and the amount could be determined, no serious error can have arisen from this source. The same copper oxide was used repeatedly, and after several determinations the carbon dioxide nearly disappeared.

Balance and weights.—The balance used was made by Ruprecht, of Vienna, and was designed to carry a maximum load of 1 kilogram on each pan. The air of the balance case was dried by means of a rapid current of air blown through two wash bottles containing concentrated sulphuric acid and a little chromic anhydride. The spray of sulphuric acid carried by the current was removed by passing through two tubes, the first containing glass wool and the second cotton wool. The current of air was stopped ten to twenty minutes before the weight was determined. During the last series of experiments the balance case was inclosed in a larger glass case and the air within the latter was dried by means of large dishes of calcium chloride set on top of the inner case. For determinations in which the weight of the hydrogen was involved the weight was always determined twice at an interval of one-half hour or more and with

the current of air passing into the case during a part of the interval. In 100 pairs of weights taken from the notebook at random the average difference was 0.05 mg and the maximum difference 0.12 mg.

Weights.—The weights used were carefully calibrated by Mr. Pienkowsky, of this Bureau, at the beginning of the work, a second time after they had been in use for ten months, and a third time at the completion of the investigation, a little more than two years after it was begun. The largest change observed in any of the weights used was 0.04 mg, while none of the platinum weights changed more than 0.01 mg. The aluminum rider gained 0.014 mg. A recalculation of the amounts of hydrogen in the last series showed that the use of the new corrections caused no change exceeding 0.14 mg in the weight of the hydrogen for any experiment of that series, while the algebraic sum of the changes for the five experiments of the series was 0.09 mg or 1 part in 250,000 of the weight of hydrogen used.

The calibration of the weights was of course to a vacuum standard. Since the substances to be weighed were always inclosed in a glass apparatus whose volume remained constant, and which was counterpoised by another glass apparatus of very nearly the same volume and weight, no vacuum correction in the ordinary sense was required. For our present purpose it is most convenient to assume the brass weights in air as standard. If we do this, the platinum weights will appear too heavy because they displace less air in proportion to their weight. This requires a correction of 0.087 mg, which must be added for each gram of platinum weights used. For convenience a table was prepared which included this correction for the platinum weights with the other corrections, as determined in the comparisons of the weights.

PURITY OF THE GASES USED.

As with all atomic weight determinations, the value of the present investigation depends in very large measure upon the purity of the substances weighed—in this case, hydrogen, oxygen, and water. The impurities which might be present were: In the hydrogen, nitrogen, oxygen, water, phosphorus pentoxide, and compounds of sulphur; in the oxygen there might be the same impuri-

ties with hydrogen in place of oxygen; in the water, sulphur dioxide or carbon dioxide.

Nitrogen.—In the former series of determinations there was obtained at the end of each experiment a small amount of gas which, when calculated as nitrogen, corresponded to about one one-thousandth of the weight of hydrogen used. It was found quite early in the present investigation that when the water is removed from the apparatus containing reduced copper it carries with it a small quantity of hydrogen, probably hydrogen which has been occluded by the copper. The gases were analyzed in a narrow eudiometer of such length that when the amount of gas was small the column of mercury in the eudiometer was 600 to 650 mm in length, so that the gas was under a pressure of only about one-seventh of an atmosphere. In this way quantities of nitrogen as small as 0.02 mg could be measured.

As will be seen below, it was possible to secure copper oxide so pure that only very small amounts of nitrogen were found at the close of experiments in which it was used. But the best evidence of the freedom of the hydrogen from nitrogen was obtained in the last series. From 22.6 g of hydrogen and the equivalent amount of oxygen there was obtained only 0.59 mg of nitrogen. This includes all of the nitrogen in the oxygen, as well, and all of the leakage of the stopcock during seven weeks. If we assume that two-thirds of this nitrogen came from the hydrogen, it is only 1 part in 57,000 by weight, or 1 part in 800,000 by volume. On the basis of this evidence it is assumed that all of the hydrogen used was free from nitrogen, and no correction has been made for the small quantities found in each experiment. It is not believed that the error from this source can be so great as 1 part in 20,000.

Incidentally these experiments have demonstrated the possibility of using, without leakage, ordinary, well-ground, and carefully lubricated stopcocks, under such conditions that they are subjected to atmospheric pressure with a vacuum inside of the apparatus for days together. The well-known rubber lubricant was used. It was made by heating a mixture of 16 parts of vaseline, 8 parts of pure rubber, and 1 part of paraffin to a temperature of 350° to 400° for several hours until it was thoroughly homogeneous. For warm

weather, or for a stopcock liable to become warm, a little more paraffin was added.

Oxygen.—After the investigation had progressed so far that it was evident that the results differed from Morley's by a much larger amount than can be accounted for by accidental errors, it was feared that the hydrogen contained a trace of oxygen which escaped conversion into water as the gas passed over the platinized asbestos and copper gauze, especially as the latter were heated to only about 350° . To test this possibility the purified and dried hydrogen was passed through a hard glass tube containing platinized asbestos heated to dull redness and then through a phosphorous pentoxide tube. The latter was closed with stopcocks and was weighed each time filled with hydrogen and with the use of a counterpoise. The earlier experiments gave appreciable amounts of water (1 mg from 1 g of hydrogen), which was in part, at least, from the asbestos, and even after the asbestos had been heated almost daily for six weeks some water was still obtained (1.1 mg from 2.74 g of hydrogen).⁸ But when some of the same hydrogen was passed over strips of palladium heated to 360° in an electrically heated air bath 3.78 g of hydrogen gave a change of weight of -0.08 mg. It still seemed possible that a trace of oxygen mixed with *dry* hydrogen might pass the palladium without change. To test this a small T tube was introduced between the hydrogen generator and the palladium tube in such a manner that the lower arm could be filled with dry oxygen. This would then find its way into the hydrogen by slow diffusion. The amount of oxygen retained in the arm of the T tube was found in two experiments to be 1.68 and 1.94 mg, respectively, while in three experiments in which similar amounts of oxygen were allowed to diffuse into the hydrogen before it entered the palladium tube there were found 2.29, 2.21, and 2.58 mg of water. While the conditions were such that an exact quantitative agreement could not be expected the experiments demonstrated that minute quantities of oxygen can be readily detected by this method.

⁸ It seems possible that this water may have passed through the walls of the red-hot tube during the five hours of the experiment. It seems not improbable that glass, as a viscous liquid, may dissolve a small amount of water, and that such dissolved water might slowly diffuse through to the inner surface. This question is worthy of further study.

After the close of the last series of experiments the hydrogen was tested by passing it through the tube containing strips of palladium heated to 360° and then through the pentoxide tube. In these experiments there were passed 2.74, 2.92, and 2.92 g of hydrogen, respectively, while the changes in weight of the phosphorus pentoxide tubes were -0.44 , $+0.14$, and $+0.04$ mg, quantities scarcely beyond the errors of weighing and insignificant in proportion to the weight of the hydrogen.

Water.—The experiments just described demonstrate the absence of water in the hydrogen used in the last series. After the close of the third series hydrogen from the apparatus was passed through the phosphorus pentoxide tube at the rate of 6 liters an hour for five successive days, 12.6 g of hydrogen in all. The changes in weight of the pentoxide tube were -0.01 , $+0.53$, -1.06 , -0.33 , $+0.14$ mg; in all a net loss of 0.76 mg, or one part in 17,000. At the close of the fourth series 3.78 g of hydrogen were passed through the pentoxide tube at the rate of 6 liters an hour with a change in weight of -0.04 mg, and a similar amount passed through the tubes containing heated palladium gave a change of weight of -0.08 mg.

It seems, therefore, that the phosphorus pentoxide used to dry the hydrogen retained its efficiency to the end. The first of these tubes showed considerable deliquescence caused by the water still remaining in the hydrogen after it passed the sticks of caustic potash, but the last tube showed no sign that it was affected by the moisture.

Phosphorus Pentoxide.—Morley⁹ has shown that 1 liter of a gas at ordinary temperatures carries with it only 0.00002 mg of vapor of phosphorus pentoxide. This would correspond to about 0.0002 mg in 1 g of hydrogen, an amount without any significance in comparison with the degree of accuracy now possible in determinations of atomic weights. It is believed that the crystalline phosphorus pentoxide obtained by sublimation, adhering as it did closely to the walls of the tube and fibers of glass wool on which it was deposited, was much less likely to be carried on mechanically by the current of gas than the finely pulverulent pentoxide in its usual form would have been.

Compounds of Sulphur.—After the close of the third series a careful test for compounds of sulphur was made by bringing together

⁹J. Am. Chem. Soc. 26, p. 1171; 1904.

the hydrogen and oxygen delivered by the apparatus in such a manner that they could be burned and the water collected. The hydrogen was evolved at the rate of 6 liters an hour, the most rapid rate used in the atomic weight determinations. The water obtained weighed 112.4 g. A little bromine water and 0.1 g of sodium carbonate were added to it and it was evaporated to dryness in a platinum dish on an electric plate. The residue was acidified with hydrochloric acid, filtered from silica which came from the glass apparatus in which the gases were burned, and the filtrate was tested with a solution of barium chloride. A trifling turbidity appeared slowly, estimated to be less than that occasioned by 0.1 mg of sulphuric acid in a solution of similar volume and character. This would correspond to 1 part of sulphur dioxide, by weight, in 250,000 parts of the hydrogen.

Purity of the Oxygen.—The experiment just described demonstrates the practical absence of sulphur in the oxygen. Tests for moisture also showed that no amount which could be of significance was present.

Purity of the Water.—In the two experiments referred to above, in which copper oxide prepared by the precipitation of copper sulphate with sodium hydroxide was used, some sulphur dioxide was found. When pure copper oxide was used no evidence of its presence was ever obtained. In all experiments in which the sulphuric acid electrolyte was used the water was tested with a dilute solution of potassium permanganate (1 cc = 0.05 mg available O) and with N/10 barium or sodium hydroxide, with phenol phthalein as an indicator. The amounts required never exceeded 0.1 cc of the permanganate and 0.03 cc of the N/10 alkali, to produce a coloration in from 20 to 40 cc of the water. The water of the last two series, especially, was frequently tested for hydrogen peroxide, but none was found.

FIRST SERIES.

This series consisted of twenty experiments. In the earlier experiments the weight of the hydrogen was determined by the gain in weight of a piece of apparatus containing copper oxide, the water formed by the oxidation of hydrogen being condensed within the same apparatus. The method used will be clear from Fig. 1.

About 160 g of copper oxide were placed in the part of the apparatus represented as lying in the electrical air bath. After sealing the apparatus at the end designed to collect the water, its volume was determined by hydrostatic weighing, and a tube of almost the same volume and weight was prepared. The apparatus was then placed in the air bath and heated to 400° while it was connected with a Sprengel air pump and exhausted. A phosphorus pentoxide tube, sealed to the pump, was interposed between the pump and the apparatus and also a small McLeod gauge.¹⁰ The exhaustion was carried to the hundred thousandth of an atmosphere, or further, and the heating continued for some time.

For connection with the pump and for other similar connections used throughout the work the end of the apparatus was drawn out to fit a glass socket connected with the pump. The joint was completed by melting a little Khotinsky cement between the parts in contact. Such a joint, if properly made, will hold for an indefinite length of time against atmospheric pressure without any leakage that can be measured. The parts are easily separated by gentle warming and cleaned by warming and wiping with cotton, followed by a cloth moistened with alcohol.

After cooling, the apparatus was rinsed with distilled water, wiped with a clean cloth, and placed on the balance. The tare and weights were placed on the other pan, the current of dry air started through the balance case and the weight completed, with the use of the rider, on the following day.

The apparatus was then connected with the source of hydrogen, the copper oxide heated to 300° to 350° and hydrogen passed in for four to six hours. A coil of small tin pipe conveying cold water was placed around the part of the apparatus outside of the air bath to condense the water. By means of the rheostat connected with the electrolytic apparatus it was easy to regulate the supply of hydrogen to correspond with the rate at which it was oxidized by the copper oxide. The operation was always stopped before the

¹⁰ The bulb of the gauge had a capacity of about 16.5 cc. To avoid contaminating the mercury by contact with india rubber the gauge was sealed below to an upright glass tube about 12 mm in diameter. In this was a light, loosely fitting glass plunger. By pressing the plunger down the mercury could be easily forced up into the gauge for the measurement.

copper oxide was all reduced, and by heating for a short time after closing the stopcock the conversion of the hydrogen to water was completed.

After cooling and standing overnight, as before, the weight of the hydrogen which had been introduced was determined. The apparatus was then connected at A with the apparatus shown in Fig. 2, the other end, E, of the apparatus being connected with the Sprengel pump. This apparatus was filled with phosphorus pentoxide from C to E. The end at C is designed to collect the phosphoric acid resulting from the deliquescence of the pentoxide, while it is quite necessary that the part C D should be directed up and not down, as otherwise the sirupy phosphoric acid runs down into the unchanged phosphorus pentoxide and seals the tube. The apparatus was, of course, previously exhausted and weighed.

After exhausting the connecting tubes by means of the pump the bulb F was placed in a freezing mixture of ice and sulphuric acid, the stopcock B was closed and the stopcock of the copper oxide apparatus opened. By passing hot water through the coil of tin pipe surrounding the part of the apparatus containing the water the latter was easily distilled into F in the course of two to four hours. By opening B occasionally any permanent gas brought over with the water vapor was allowed to pass on to the pump and transferred by means of the latter to the eudiometer, in which the gas was analyzed. When permanent gases were allowed to accumulate in F the passage of the water vapor was checked and condensation occurred in the connecting tubes. This difficulty occurred especially in the last two series when considerable hydrogen was present, but was rarely experienced in the first three series. After the water had been transferred to F and the bulb containing the copper had been heated to 400° for a short time, the apparatus containing the

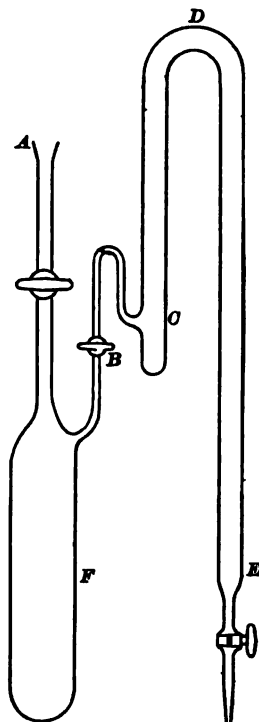


Fig. 2.

water was removed and replaced by a tube containing phosphorus pentoxide and the heating of the apparatus continued for some time longer. Only a few milligrams of water were obtained in this manner. The apparatus containing the reduced copper was then cooled and weighed next morning, the loss of weight as compared with the original weight giving the weight of the main portion of the oxygen.

The reduced copper still retains, however, at 400° , either water or hydrogen which can not be removed by the process described. To obtain this the apparatus was again placed in the electrical air bath, heated to 400° , and the copper oxidized as far as possible by means of oxygen from the electrolytic apparatus. The water formed was then removed by connection with the phosphorus pentoxide tube and heating as before. After cooling and weighing, the apparatus was now ready for a second experiment.

In seven determinations of this series the hydrogen was absorbed in palladium, and its weight was determined twice, first by the loss in weight of the palladium tube and second by the gain in weight of the copper oxide tube. The palladium tube was given the form shown at A in Fig. 3. The palladium foil was furnished by Heraeus. It was 0.05 mm in thickness. Three hundred and sixty grams of it were cut into strips 4 cm wide, and these were wound in three rolls, which were placed in the tube A. Before use the rolls were placed in a hard glass tube and heated to redness in a current of oxygen. After placing it in A it was heated for some time in a current of hydrogen till thoroughly dry. It was then cooled and charged with hydrogen, and when saturated a current of hydrogen was passed through the tube for a short time. The tube was weighed with a counterpoise and after standing overnight, in the same manner as described for the copper oxide tube.

The arrangement employed for the transfer of the hydrogen to the copper oxide tube is shown in Fig. 3. For greater simplicity the air baths in which A and B were placed and supported are not shown. The tube C was connected with the Sprengel pump. After exhaustion of the connecting tubes the connection with the pump was cut off by means of a stopcock situated close to the connection with C. The palladium tube was then slowly heated while the copper oxide tube was heated to 350° . The pressure in the

system could be easily followed by means of the shortened manometer D E. This contained a small amount of air at D but not enough to carry the mercury beyond the bend at E when the pressure in F fell to zero. The heating of A was so regulated that the pressure was usually a little below that of the atmosphere, the reaction in B proceeding most rapidly under slightly reduced pressure. The 360 g of palladium absorbed 2.3 g of hydrogen, and of this 2 g could be easily expelled at 150° to 160° . This amount

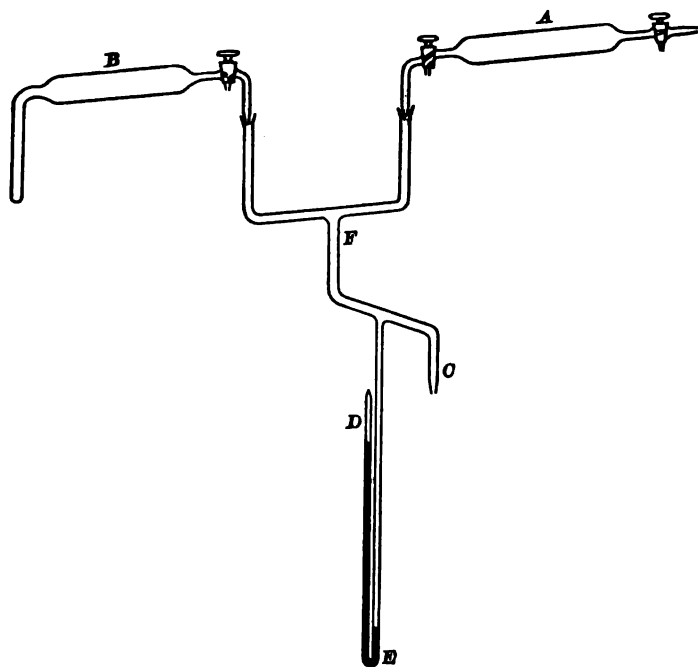


Fig. 3.

could be transferred from A to B in three to five hours. The rate of transfer could be followed by the amount of water condensed in B. When enough water had collected the stopcock of the copper oxide tube was closed and the palladium was allowed to cool till most of the hydrogen in the connecting tubes had been reabsorbed. The stopcock of the palladium tube was then closed and the hydrogen in the connecting tubes pumped out into a eudiometer and measured. This hydrogen varied from 0.02 to 0.14 mg according to the time allowed for the palladium to cool.

The twenty experiments of the first series gave for the atomic weight of hydrogen, as calculated from the weight of the hydrogen and the weight of the oxygen, 1.00819 ± 0.00010 , and from the weight of the hydrogen and water 1.00821 ± 0.00010 . This is a smaller probable error than has been obtained by any previous observer except Keiser and Morley; but it was found at the close of the series that it is subject to a constant error which is apparently, about three times the "probable error."

It was known at the beginning of the investigation that copper oxide at 400° would retain a small amount of water. It was hoped, however, that this might be made very small and also that by making the end of one experiment the beginning of the next the total amount of water retained would be so nearly constant that it would not affect, appreciably, the result of any experiment in the series, after the first. At the close of the last experiment of this series it was thought wise to test this point by transferring the copper oxide to a hard glass tube and heating it in a current of oxygen. To my surprise there was obtained 37.7 mg of water, although the previous oxidation and heating at 400° had given only 3.3 mg. If we assume this water to have come from the last six experiments of the series which formed a continuous set in which the end of one experiment was the beginning of the next, the value for the six determinations becomes 1.00782 as calculated from the hydrogen and oxygen or 1.00791 as calculated from the hydrogen and water. It was at once decided to reject the determinations thus far made, and which had taken about a year's time, and to begin a new series in which this source or error should be eliminated, if possible.

SECOND SERIES. HYDROGEN FROM SULPHURIC ACID, WEIGHED TWICE.

In this series the hydrogen was weighed in palladium and also after transfer to the copper oxide tube as described above. In order to increase the quantity of hydrogen and reduce the error, after the water had been removed and the copper reoxidized, a second quantity, and in one case a third quantity, of hydrogen was introduced and the water removed. Finally the reduced copper was transferred to a hard glass tube and oxidized in a current of oxygen, and the amount of water formed was determined. The oxygen used was partly from the S. S. White Dental Manufacturing Company and

was purified by passing it over red-hot copper oxide and through wash bottles containing a solution of sodium hydroxide and over phosphorus pentoxide. In some of the experiments electrolytic oxygen was used.

Several experiments were made to determine the amount of water taken up by the copper oxide in the transfer to the hard glass tube. In a similar manipulation with dry copper oxide there were obtained 2.63, 1.30, 5.32, and 2.45 mg of water. This would correspond to an average error of about 1 part in 10,000. It has not been thought best to apply any correction for this error, partly because of the uncertainty in its amount but chiefly because it is probably balanced by small errors in the opposite direction, due to the retention of water by the copper oxide even after several hours at a red heat in a current of oxygen, and to the retention of a trace of water by the walls or lubricant of the copper oxide apparatus. There is good reason, however, for believing that the error from each of these sources is very small.

Seven determinations were made in this series; but in one of them the weight of the oxygen was lost. The results were as follows:

Weights of Hydrogen. Second Series.

	Grams H CuO tube	Grams H Pd tube	Mg H in T tube	Mg H from Cu	Cor. H CuO	Cor. H Pd	Average H
1	3.72469	3.72689	0.15	0.07	3.72462	3.72667	3.72565
2	3.80383	3.80280	0.17	0.05	3.80378	3.80258	3.80318
3	3.75898	3.75886	0.18	0.10	3.75888	3.75858	3.75873
4	2.96309	2.96357	0.05	0.03	2.96306	2.96349	2.96328
5	2.11437	2.11366	0.08	0.03	2.11434	2.11355	2.11395
6	3.53126	3.53173	0.19	0.04	3.53122	3.53150	3.53136
7	3.53963	3.53982	0.19	0.04	3.53959	3.53959	3.53959

Weights of Oxygen and Water. Second Series.

	Grams O from loss of CuO	Mg N	Mg CO ₂	Mg water by reoxida- tion	Grams O corrected	Grams water
1	29.56469	0.76	0.28	15.26	29.57891	33.30408
2	30.16645	0.37	2.33	20.25	30.18400	33.98748
3	29.81141	0.32	0.79	23.28	29.83358	33.59127
4	23.49099	0.13	0.60	29.61	23.51987	26.48379
5		0.20	1.16	27.65		18.89214
6	27.99401	0.38	0.89	36.36	28.02910	31.56024
7	28.05652	0.24	0.22	40.13	28.09619	31.63554

Atomic Weight of Hydrogen. Second Series.

	From H : O	From H : H ₂ O
1	1.00765	1.00767
2	1.00800	1.00799
3	1.00792	1.00795
4	1.00792	1.00790
5		1.00795
6	1.00791	1.00792
7	1.00785	1.00786
Mean,	1.00787 ± 0.00003	1.00789 ± 0.00003

Mean of all.....1.00788 ± 0.00002

THIRD SERIES. HYDROGEN FROM SULPHURIC ACID WEIGHED AFTER CONVERSION TO WATER BY COPPER OXIDE.

In this series hydrogen directly from the electrolytic apparatus was passed into the copper oxide bulb and converted into water. The purpose of the series was more especially to determine whether the purity of the hydrogen had been increased by absorption in the palladium. The results of the series are not as concordant as those of the second, but they indicate that the hydrogen directly from the electrolytic apparatus is, if anything, a little lighter, and, presumably, a little more pure than after it has been absorbed in palladium. The results were as follows:

Weights of Hydrogen and Oxygen. Third Series.

	Grams H by gain of CuO	H from Cu	Grams H corrected	Grams O by loss of CuO	Mg N	Mg CO ₂	Mg water by reoxidation
1	2.44282	0.03	2.44279	19.34104	0.16	0.10	56.79
2	2.18741	0.02	2.18739	17.33722	0.10	0.05	25.98
3	2.75132	0.03	2.75129	21.80908	0.18	0.05	34.60
4	4.00068	0.06	4.00062				62.31
5	4.04061	0.04	4.04057	32.04471	0.17	0.06	32.41

Atomic Weight of Hydrogen. Third Series.

	Grams O corrected	Grams water	Atomic weight—	
			From H : O	From H : H ₂ O
1	19.39757	21.84042	1.00746	1.00746
2	17.36305	19.55117	1.00784	1.00780
3	21.84345	24.59389	1.00764	1.00768
4		35.75073		1.00803
5	32.07689	36.11762	1.00772	1.00772
		Mean,	1.00767 ± 0.00006	1.00774 ± 0.00006

Mean of all, 1.00771 ± 0.00004

It will be noticed that the amounts of carbon dioxide found are much smaller, owing to the fact that the copper oxide had been freed from it by repeated use. The amount of nitrogen was also smaller, perhaps because of the simpler manipulations involved.

FOURTH SERIES. HYDROGEN AND OXYGEN FROM SULPHURIC ACID COMBINED BY MEANS OF PALLADIUM.

With the best manipulation which it has been possible thus far to secure, the use of copper oxide involves several sources of small constant errors. It is hoped that these errors do not, in the aggregate, exceed 1 part in 10,000, but it seemed very desirable to find some radically different method which should avoid the use of copper oxide. Such a method consists in absorbing hydrogen in

palladium and converting it into water by means of oxygen. This method has already been used by Keiser,¹¹ but his method of manipulation compelled him to use comparatively small amounts of hydrogen, and the degree of concordance which he was able to secure was not such that his value for the atomic weight of hydrogen can be used to decide between Morley's value and that found here.

The problem seemed at first a very simple one, but several months were spent in fruitless experiments before a satisfactory method of manipulation was found.

The 360 grams of palladium foil previously used were placed in a somewhat wider tube and were kept from direct contact with the glass by means of small glass beads strung on platinum wires. The stopcock at one end of the tube was replaced by a tube having a capacity of about 50 cc. At the beginning the palladium tube was heated and charged with hydrogen and partially exhausted several times to remove moisture. It was then heated to 380°–400° and exhausted until the hydrogen showed a residual pressure of 0.50 to 0.70 mm as measured by the McLeod gauge. At this pressure and temperature 360 grams of palladium retain only 2 to 3 mg of hydrogen and the amounts of hydrogen given out for small changes of pressure are as follows:

0.40 to 0.50 mm,	0.11 mg H
0.50 " 0.60 "	0.10 " "
0.60 " 0.70 "	0.094 " "
0.70 " 0.80 "	0.087 " "
0.80 " 0.90 "	0.080 " "
0.90 " 1.00 "	0.073 " "
3.90 " 4.00 "	0.044 " "
16.90 " 17.00 "	0.020 " "

A change of temperature of 10° for pressures less than 1 mm corresponds to a change of 0.03 mg in the amount of hydrogen retained. At the end of an experiment it was easy to bring the apparatus to a condition of temperature and pressure closely approximating that at the beginning, and by means of the table a correction of sufficient accuracy could be applied.

After exhausting and measuring the temperature and pressure as described, the apparatus was allowed to cool, rinsed with distilled

¹¹ Am. Chem. J., 18, p. 354; 1888.

water, wiped, placed on the balance, and on the following morning it was weighed. It was then charged with hydrogen (about 2.3 grams) and weighed again on the morning of the third day. The apparatus was then connected with the oxygen side of the electrolytic generator. The palladium tube was inclined so that the water formed ran down into the receptacle at the end. The oxygen entered rapidly for a few minutes, but the heat generated soon caused the pressure of the hydrogen to increase and check the current. The oxygen then continued to enter slowly, with the stopcock partly closed, for two or three hours. After that the rate at which the oxygen was taken changed quite suddenly, so suddenly that the first time the experiment was tried the electrolyte was drawn over and the electrolytic apparatus was broken. From this time to the end the entrance of the oxygen was regulated by the stopcock and was allowed to enter as rapidly as it was thought desirable to run the generator. More than enough oxygen to combine with the hydrogen present was easily introduced. On the fourth day, after weighing, the apparatus was charged again with hydrogen.

On the fifth day the manipulation varied. In some cases oxygen was admitted in excess as on the third day. In other cases, as the end of the oxidation of the hydrogen approached, the apparatus was weighed roughly at intervals and the admission of the oxygen was stopped while 5 to 10 mg of hydrogen remained unoxidized. By this method of manipulation a day was saved. On the sixth day, when an excess of oxygen was admitted on the fifth, a small excess of hydrogen was admitted. It was necessary to wait till the following day before weighing.

On the sixth or seventh day, according to the method of manipulation chosen, the water and excess of hydrogen were removed and the apparatus brought back to a condition for the beginning of a new experiment. A receptacle similar to that described above was used for the water. On account of the hydrogen which accumulated rapidly in the bulb with the water and stopped the current of water vapor, it was found desirable to give the bulb a capacity of about 100 cc. Any nitrogen present was, of course, carried over with the first portions of the water vapor and hydrogen and this part of the gas was collected and analyzed separately. At the close of the removal of the water the receptacle for the water was replaced by a phosphorus pentoxide tube which permitted of a

free connection between the palladium tube and the pump for the measurement of the residual pressure. It was necessary to heat the palladium tube at least two hours in the electrical air bath to secure constant conditions for the measurement of the residual pressure. This was not understood in the earlier determinations of this series and may be one reason why some of the results are not very satisfactory.

In the results which follow the amounts of nitrogen and carbon dioxide are given as an indication of the magnitude of the errors which may have been occasioned by traces of these gases or by leakages, but no correction for these gases has been applied.

Theoretically, the weight of the apparatus at the end of an experiment should be the same as at the beginning. Practically, it was almost always heavier because of the retention of a small amount of water. When the hydrogen was allowed to accumulate in the receptacle for water the moisture would condense in the connecting tubes and stopcocks, and if allowed to remain for a short time in contact with the lubricant of the stopcock could not be entirely removed. In some cases minute drops of water could be seen within the bore of the stopcock, and these would not evaporate during any reasonable length of time. The gain in weight of the apparatus during the experiment was, accordingly, assumed to be due to water which had been retained and was added to the weight of water found directly. In cases where condensation was avoided the amount of water was very small or even became negative. (See the following series.)

Weights of Hydrogen and Oxygen. Fourth Series.

	Grams H	Mg cor. for changes of t and P.	Mg H recovered	Grams H corrected	Mg N	Mg CO ₂	Grams O
1.	2.31840	-0.05	39.19	2.27916	0.48	0.02	18.08455
2.	4.13630	-0.06	8.90	4.12734	0.50		32.76527
3.	4.18248	+0.24	7.16	4.17556	0.52	0.02	33.13449
4.	4.19599	-0.51	2.02	4.19346	0.15	0.16	33.27384
5.	2.31593	-0.01	8.46	2.30746			18.30863
6.	4.60148	-0.23	4.33	4.59692	0.11	0.08	36.48543
7.	4.63931	+0.14	3.20	4.63625	0.07	0.07	36.79354
8.	4.57292	+0.18	0.36	4.57274	0.02	0.28	36.28696

Weights of Water and Atomic Weight of Hydrogen.

	Grams of water	Mg water retained	Grams water corrected	Atomic weight—	
				From H : O	From H : H ₂ O
1	20.29807	63.21	20.36128	1.00823	1.00830
2	36.87182	18.61	36.89043	1.00774	1.00780
3	37.30667	1.20	37.30787	1.00818	1.00821
4	37.46202	2.51	37.46453	1.00822	1.00831
5	20.61671	—3.14	20.61357	1.00825	1.00839
6	41.07653	5.09	41.08162	1.00795	1.00797
7	41.41452	14.53	41.42905	1.00806	1.00808
8	40.85460	3.74	40.85834	1.00813	1.00817
			Mean,	1.00809	1.00815
				±0.00004	±0.00005
			Mean of all,	1.00812	±0.00003

FIFTH SERIES. HYDROGEN AND OXYGEN FROM BARIUM HYDROXIDE COMBINED BY MEANS OF PALLADIUM.

The manipulation in this series was essentially the same as in the fourth series. The series has much greater value than the former one, partly as shown by the concordance of the results, partly because the processes of manipulation had been more thoroughly mastered, and partly because the electrolyte used gives a greater probability of purity for the hydrogen. In those experiments in which oxygen was introduced last a small quantity was left in the bore of the three-way stopcock and was usually found in the subsequent analysis of the first portions of gas removed. In one experiment of this series only one-half the quantity of hydrogen was used and too much oxygen was admitted by mistake. This so complicated the manipulation that the experiment was unsatisfactory and it has been omitted from the table. It gave the values 1.00814 and 1.00823.

Weights of Hydrogen and Oxygen. Fifth Series.

	Grams H	Mg cor. for change t° and P	Mg H recovered	Grams H corrected	Mg N	Mg CO ₂	Mg O	Grams O
1	4.61443	—0.07	2.56	4.61180	0.11	0.17		36.60909
2	4.62877	0	5.19	4.62358	0.08	0.03	0.13	36.69575
3	4.60386	—0.02	5.31	4.59853	0.22	0	1.31	36.50484
4	4.56300	+0.04	4.72	4.55832	0.06	0	0.29	36.17887
5	4.21188	—0.04	7.85	4.20399	0.11	0	0.11	33.37000

Weights of Water and Atomic Weight of Hydrogen.

	Grams water	Water retained	Grams water corrected	Atomic weight—	
				From H: O	From H: H ₂ O
1	41.21934	1.71	41.22105	1.00779	1.00779
2	41.30740	9.07	41.31647	1.00798	1.00806
3	41.09872	3.40	41.10212	1.00776	1.00780
4	40.74186	—2.82	40.73904	1.00795	1.00790
5	37.56732	6.04	37.57336	1.00782	1.00786
			Mean,	1.00786	1.00788
				±0.00003	±0.00003
			Mean of all,	1.00787	±0.00002

The mean of the four series, giving them equal weight, is 1.00787 from the hydrogen and oxygen and 1.00791 from the hydrogen and water. The second and fifth series have, however, much greater weight than the third and fourth, and when we consider, further, that there was probably a trifling loss of water, especially in the third series, the value 1.00787 seems to be the most probable result which can be calculated from the work.

It has seemed of interest to recalculate Morley's results on the oxygen basis for comparison with the result here given.

Atomic Weight of Hydrogen. Morley's Results.

1	1.00765	1.00778
2	1.00751	1.00766
3	1.00768	1.00774
4	1.00755	
5	1.00773	1.00750
6	1.00773	1.00778
7	1.00777	1.00786
8	1.00764	1.00761
9	1.00759	1.00752
10	1.00752	1.00737
11	1.00746	1.00734
12	1.00744	1.00771
Mean,	1.00761	1.00763
	± 0.00002	± 0.00003
Mean of all,	1.00762	± 0.00002

The difference between this result and that given above is greater than can be accounted for on the basis of accidental errors. While the average of the results of the best determinations by other observers approaches more nearly to the result of this investigation than to Morley's value, it does not seem justifiable to calculate a mean on such a basis. For reasons which will be given in a later paper the most probable value which we can get at the present time seems to be the average between Morley's value and that here given. That average is 1.00775. Morley's value and my own each differ from this by one part in 8,000.

CHANGE IN WEIGHT OF THE HYDROGEN WHEN CONVERTED INTO WATER.

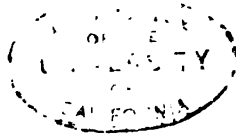
As was stated in the introduction, one purpose of this investigation was to secure, if possible, some evidence of a change of weight which might occur in a reaction in which a large amount of energy is dissipated. In twenty-five experiments the same hydrogen was weighed, first, as absorbed in palladium and, second, after conversion into water by means of the copper oxide. The results were as follows:

Gain of CuO	Loss of Pd	Diff. Mg.	Gain of CuO	Loss of Pd	Diff. Mg.
1.04216	1.04295	+0.79	0.95151	0.95207	+0.56
1.46493	1.46642	+1.49	1.98130	1.98053	-0.77
0.96114	0.96231	+1.17	1.82248	1.82205	-0.43
1.51279	1.51296	+0.17	2.04840	2.04816	-0.24
1.17961	1.17973	+0.12	1.71048	1.71042	-0.06
1.22273	1.22268	-0.05	1.96621	1.96648	+0.22
1.13563	1.13541	-0.22	0.99685	0.99707	+0.22
1.30184	1.30150	-0.34	2.11434	2.11335	-0.79
1.22851	1.22699	-1.52	2.20439	2.20472	+0.33
0.99563	0.99600	+0.37	1.32683	1.32678	-0.05
1.24243	1.24270	+0.27	2.07207	2.07281	+0.74
1.24639	1.24754	+1.15	1.46752	1.46678	-0.74
1.52682	1.52706	+0.24	36.72299 g	36.72562 g	+2.63mg

An examination of these results show that in fourteen experiments the hydrogen as weighed in the palladium appeared heavier, while in eleven experiments the hydrogen after conversion into water appeared heavier. The average difference between the two weights of a given quantity of hydrogen was 0.52 mg or 1 part in 3,000, while the average amount by which the hydrogen as weighed in the palladium appeared heavier was 0.11 mg or 1 part in 14,000. It is clear that these results do not justify any conclusion either way with regard to a change in weight of the hydrogen during the reaction other than that if any change of weight occurs it must be very small. It seems scarcely possible that a change of so much as 1 part in 10,000 of the weight of the hydrogen takes place. It is, possibly, of interest to note that so far as the results furnish any indication whatever they point toward a trifling loss of weight, a result which would agree with Landolt's elaborate study of the question.¹² It is hoped that this problem may be taken up again by a method capable of giving more accurate results.

WASHINGTON, September 11, 1907.

¹² Zs. physik. Chem. 55, p. 589.



ON THE BEST METHOD OF DEMAGNETIZING IRON IN MAGNETIC TESTING.

By Charles W. Burrows.

The magnetic permeability of a specimen of iron is a function of its previous mechanical, thermal, and magnetic history as well as of its chemical composition. While therefore the magnetic permeability of a given piece of metal may be measured quite accurately at a given instant, in general the very process of measurement adds a new chapter to the history of the metal and may leave it with magnetic properties different from those it had at the beginning of the test. Such a test may be quite misleading as regards the real magnetic properties of the specimen. Hence we are most concerned with those properties and conditions that can be associated and reproduced. Some little space will therefore be devoted to the consideration of the after effects of various details in the history of the magnetic substance, and a plan will be outlined whereby a specimen may be freed from all effects of previous magnetic treatment, and so be brought to a standard condition. We shall thus define a set of normal conditions and specifications under which the specimen is to be tested. When this is done, the permeability becomes a definite physical constant, and may be redetermined at will. All numerical quantities and equations are given in the electromagnetic c. g. s. system of units. Field strength and induction per square centimeter are expressed as so many *units*, although many writers prefer the more distinctive term *gauss*.

THE RESULTS OF EARLIER OBSERVERS.

Considerable work on the influence of previous magnetic treatment and certain other related details on the value of the induction has been done and some of the more important conclusions are here

given. Ewing in his classic work "Magnetic Induction in Iron and other Metals," third edition, 1900, has recorded the following phenomena, most of which are founded on magnetometric measurements either by himself or by earlier investigators.

(1) Demagnetization by hand reversals or by rotating commutator is complete if the maximum current is at least as great as the greatest used since the last demagnetization.¹

(2) If a force is repeatedly applied and removed, the values of the induction and the residual magnetism form two increasing sequences of numbers whose last terms approach each other.² This has been called "molecular accommodation."

(3) During repeated reversals of the same magnetizing force, the change of induction on reversal gradually becomes smaller and finally reaches a constant value.³

(4) After the change of induction on slow reversal of current has become constant, a sudden reversal will at first increase the change of induction on reversal, but later reversals make this change smaller than ever.³

(5) After application of an intense magnetizing force the tendency to a diminution of the change of induction on repeated reversal of a given magnetizing force has disappeared.⁴

(6) "Creeping" produces a greater induction than is at first shown on the application of the magnetizing force.⁵

In an article entitled "Studies in Magnetic Testing,"⁶ Professor Searle gives the results of an extended investigation carried out at the Cavendish Laboratory. An alternating current of 90 cycles controlled by a liquid rheostat furnished the demagnetizing force. Inductions were measured ballistically. The specimens used were samples of transformer iron 4 cm wide, 68.8 cm long, and about 0.035 cm thick, and were so arranged that the magnetic circuit was

¹ Magnetic Induction, etc., p. 46.

² Fromme, Pogg. Ann. Ergbd., 7; 1875. Wied. Ann., 4; 1878.

³ Magnetic Induction, etc., p. 340. My experience seems to indicate that this later decrease is nothing more than a part of the original decrease accelerated perhaps by the sudden reversal.

⁴ Magnetic Induction, etc., p. 341. This does not accord with my experience, and is therefore probably found only in certain specimens.

⁵ Ewing, Phil. Trans., p. 569, 1885.

⁶ G. F. C. Searle, Proc. of Inst. of Elect. Engrs., Part 170, 84, pp. 55-118; 1904.

a square about 50 cm on a side. The more important conclusions of this paper are here given.

(1) Complete demagnetization is obtained by an alternating current with a frequency of 90 cycles per second, provided the initial demagnetizing force is greater than a certain "critical value."

(2) An imperfect demagnetization results in a lower apparent induction.

(3) The effect of many reversals of a force H_y on the apparent induction due to a small force $H=1$ is modified by many reversals of a force H_x as follows:

(a) If H_x is as large as the critical value, it wipes out the effect of H_y and leaves its own after effect.

(b) If H_x is less than the critical value its effect in raising the apparent induction is less as H_y is greater. In every case it leaves its own after effect.

(c) If H_x is very small, less than one, the after effect in certain cases is to give a lower apparent induction than if H_x were omitted.⁷

(d) If $H_y=0$ the apparent induction may be raised by a small value of H_x .⁸

(4) Many reversals of a force H_x cause a lowering of the apparent induction due to a smaller force H_o , and the lowering increases with H_x .

(5) A constant force superposed upon the force used in determining the induction reduces the apparent induction. A constant force of $H=0.2$ in one case caused a 20 per cent reduction for low values of B .⁹

(6) Virgin iron gives lower values of the apparent induction than demagnetized iron in the lower values of B . In the upper regions the difference disappears. This iron, although it is called virgin because it had experienced no other magnetizing force than the

⁷I have not attempted to verify this. It seems analogous to the decrease in apparent induction noted in some cases when the upper limit of the demagnetizing force was too low.

⁸This is contrary to my experience.

⁹I have noticed a lowering of the apparent induction when the specimen was turned so that the earth's field was parallel to its length.

earth's field, showed a small initial induction even after careful annealing.¹⁰

All of the above conclusions have been carefully examined during the present investigation and have been verified except in certain cases, particularly that on the efficiency of demagnetization by alternating current, which will be discussed latter. While these general conclusions do not cover exactly the same ground as the present work yet, with an exception or two, there are no inconsistencies between the earlier work and my own.

DEFINITIONS.

In Fig. 1, B and H are the magnetic induction per square centimeter and magnetizing force, respectively. Then if we start with a piece of virgin iron and apply successively increasing magnetizing forces and note the corresponding inductions, we get a series of points having the line OA as locus. If the magnetizing forces had been negative instead of positive, the symmetrical curve OB would have been traced. The positive branch of the curve is commonly called the ascending B - H curve, or simply the ascending curve. Consider the point A on the ascending curve and the corresponding point B on the negative curve. In the course of the ascending curve let the magnetizing force be reversed at the point A . This will cause the point which represents the magnetic state of the iron, or state point, to move along the curve ARB' where B' in general is a different point from B . Another reversal will trace the curve $B'R'A'$ where also A' is different from A . If the magnetizing force is reversed many times, the path of the state point eventually becomes a closed curve such as is pictured in Fig. 2. The upper tip of this curve defines a pair of values of B and H . If other loops are traced in this manner, but so that each succeeding loop is larger than its predecessor, the locus of the tips of the loops forms the normal curve of ascending reversals, or, as it is sometimes called, the commutation curve. The locus of the lower tips of these same loops is a curve symmetric with this.

¹⁰I have not been able to obtain any iron free from initial polarization and consequently have no data for virgin iron. The experiment cited is not conclusive, as the small residual magnetization detected would tend to produce the same effect attributed to the fact that the iron is in the virgin state.

Instead of starting with a piece of virgin iron, or with a piece of iron which had been thoroughly freed from all previous polarization, we may start with a specimen which has a residual induction represented by the point P which obviously could lie anywhere on the line RR' . Proceeding as before, we get a system of curves analogous to the preceding but with the symmetry lost. The point corresponding to the center of symmetry has been displaced towards P .

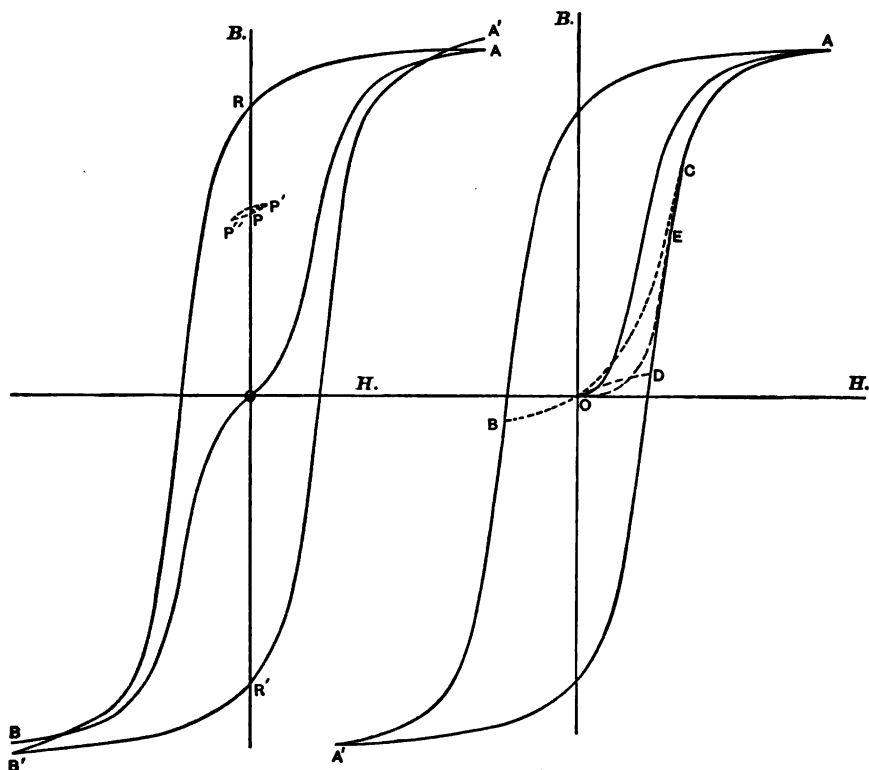


Fig. 1.—Ideal curve to illustrate the definitions of magnetic properties.

Fig. 2.—Ideal curve to illustrate the effect of previous magnetic history on the initial permeability of apparently neutral iron.

This new center may be so far displaced from the origin that on reversals of small values of H the induction does not change sign, but oscillates between two values of the same sign, such as those represented by P' and P'' . In such a case as this the applied magnetizing force does not determine the absolute induction but only

the amplitude of the oscillation. Instead of comparing the absolute values of B with H , we should rather compare with H the value of this oscillation which is expressible as

$$\frac{1}{2}(B_H - B_{-H})$$

In order to avoid circumlocution and ambiguity the following definitions seem desirable.

The *magnetic induction* is one-half the change of magnetic flux per square centimeter observed on the reversal of a given magnetic force, or

$$\frac{1}{2}(B_H - B_{-H})$$

Permeability is the ratio of this induction to the corresponding magnetizing force, or

$$\mu = \frac{B}{H}$$

The *differential permeability* is the ratio of an infinitesimal change in B to the corresponding change in H . Geometrically it is the tangent of the angle which the tangent to the curve makes with the H axis.

The *hysteresis loop* is the path traced by the state point on a double reversal of a given magnetizing force after sufficient repetition has made the path cyclic.

A *normal hysteresis cycle* is one which is symmetrical about the origin. The induction and permeability are *normal* or *apparent* according as the corresponding hysteresis loops are normal or not.¹¹

Iron is in a *neutral state* when it exerts no external field and yields as readily to a positive magnetizing force as to a negative one.

APPARATUS.

In these experiments the magnetic circuit usually consisted of two specimens about 50 centimeters long placed side by side with their adjacent ends joined by suitable wrought-iron yokes. The

¹¹The distinction between "normal" and "apparent" is due to Searle (loc. cit.). Rowland (Physical Papers, p. 44) uses "normal" in the sense of "virgin."

magnetizing coils consisted of two sections, each 40 centimeters long, wound with a single layer of 400 turns of No. 22 double cotton covered magnet wire. The secondary coil was uniformly distributed over the middle portion of the specimen. The shearing correction due to end effects is small and has been allowed for in the results. The reduction to absolute values, as well as other details of manipulation, will be described in a subsequent paper. The nature and dimensions of the specimens used are shown in Table I. For each

TABLE I.

Showing the physical constants of the specimens used in this work.

Specimen	Length	Breadth	Thick- ness	Dia- meter	Cross section	Specific re- sistance in micro-ohms
Transformer iron.....	51.0	1.006	*.0355		.0357	14.0
Common iron.....	51.0	1.00	.065		.065	13.9
Low carbon steel.....	47.1			.578	.262	13.4
High carbon steel.....	45.8			.578	.262	18.9

*Calculated from mass and density.

kind of iron and steel two specimens were cut from the same rod or sheet, and though cut from adjoining parts show a small difference magnetically.

CRITERION OF PERFECT DEMAGNETIZATION.

In order to reduce iron to a magnetically neutral state, it must be freed from the effects of previous magnetization. It must not only be freed from all evidences of polarization which would produce an external field and which would be indicated by a suspended magnet or a movable test coil, but also from all internal polarizations which produce no external field. A test coil shows the integrated value of the induction and therefore does not show whether the specimen is in the neutral state. The specimen may be magnetically homogeneous with no external field and yet not be in the neutral state. This appears from Fig. 2. Let the state point be the origin. If the iron is neutral, a small positive force will cause the state point to move along the solid line OA tracing the ascending curve. If,

however, the iron has been previously carried along the path OABO, the state point is again at the origin, but the iron is so influenced by its recent treatment that a small positive force carries the point along the path OC, which initially is above the normal path OA. Again let it be carried along the path OABA'DO. Once more it is at the origin, but with a tendency to move along the path OE with increase of H . This latter path is below the normal path. And so on, with various preliminary paths for the state points, we could cause it to leave the origin in an infinite number of directions which would mean an infinite number of initial permeabilities. And yet in each of these cases an external magnet or a movable test coil would give no indication of the inherited tendencies of the specimen.

In general, if the origin is made the lower tip of a secondary hysteresis loop, its initial permeability in the ascending curve is less than normal. If, however, the origin is near the middle of the ascending branch of a secondary hysteresis loop, the initial permeability is greater than normal. These facts show that the ordinary criteria for perfect demagnetization are unsatisfactory and indicate that no test is possible which will not modify the neutral state if it should exist.

Before a satisfactory criterion of perfect demagnetization can be determined upon, it is necessary to know something of the nature of the influence a residual induction has on the apparent induction. With this in view, the following experiment was carried out: A specimen was demagnetized as thoroughly as might be and its apparent induction measured. Then the specimen was subjected momentarily to a magnetizing force which would give it a certain residual induction. After the removal of this force the apparent induction for the same force as before was determined. These operations were repeated with successively increasing polarizing forces. The results are shown graphically in Fig. 7. From this curve it is seen that as the polarizing force increases the resultant induction decreases. Applied to Fig. 1, this means that the small cyclic loop $P'P P''$ becomes more nearly parallel to the H axis as P recedes from this axis. In consequence of this fact, which holds in every case examined, *the criterion of perfect demagnetization is that the induction shall be a maximum.*

PREVIOUS METHODS OF DEMAGNETIZATION.

Many methods have been employed to remove residual polarizations and to reduce the specimen to a magnetically neutral state. The method most generally employed is that of *descending reversals* in which the demagnetizing force is repeatedly reversed while it is gradually decreased in value. The reversals may be made by hand with an ordinary reversing switch, by a rotating commutator driven either by hand or by power, or by an alternating current. The reduction of current may be made by any form of rheostat. The liquid resistance has found favor in many quarters, chiefly because it gives a continuous variation of current and not a series of steps, as in most types of rheostats.

Ewing¹⁸ suggests a *potential slide*, made as follows: A tall glass jar is filled with a dilute solution of zinc sulphate. Three blocks of amalgamated zinc are fitted in the jar—one lying on the bottom, one fixed at the top, and the third hung between them by a cord, which passes over a pulley above to a little winch. The battery is connected to the fixed blocks, and the magnetizing coils to one fixed block and to the movable block. By altering the position of the movable zinc block the electromotive force applied to the magnetizing coils may be varied. Reversals were made by a rotating commutator worked by hand.

Searle¹⁹ used an alternating current of 90 cycles supplied from the city mains. His resistance consisted of a narrow glass trough containing dilute copper sulphate solution, and furnished with two copper electrodes, one of which was movable. By tilting the trough so that one end was dry the current could be reduced to a very small value.

CURRENT REGULATION.

In this investigation all the available methods of current control were tried. Several kinds of liquid resistances were used. In some of these the electrodes were moved apart so as to give a longer path through the liquid. In others the liquid was removed from the containing vessel. This was accomplished in several ways—by tilting the vessel, by siphoning off the liquid to another vessel, and again by allowing the liquid to run out through a stopcock in the

¹⁸ Magnetic Induction, etc., p. 44.

¹⁹ Proc. Inst. of Elect. Engrs., 84, p. 61; 1904.

bottom of a tall cylindrical vessel. The latter was the most satisfactory of the liquid type. The ordinary forms of rheostats, with fixed steps, controlled by dial sliding contacts, by plugs, or by links and mercury cups, were tried. Another form used was the Kelvin rheostat, in which the wire is wound from a drum of conducting material to an insulating drum. This gives uniform variation, and is easily controlled, but is laborious. The most satisfactory rheostat used was a cylinder of slate wound with bare wire, having a sliding contact moving parallel to the axis of the cylinder. Several of these resistances of different sizes of wire are connected in series and are highly satisfactory.

Current decrease was also effected by changing the applied electromotive force while keeping the resistance constant. In this case the demagnetizing circuit was connected to two points of an adjustable resistance through which a current was flowing. This method was found to offer no advantage over the preceding, but has some obvious disadvantages.

Early in the investigation it became apparent that the efficiency of demagnetization depended not so much on the total time consumed as on the rate at which the demagnetizing current was reduced. To test this point the demagnetizing current was reduced successively in three different ways. First, so that the rate of increase of resistance was constant; second, so that the rate of decrease of current was approximately constant, and, third, so that the rate of decrease of induction was constant. This last method gave the best results and has been followed in all the later work.

The electromotive force to be used is determined largely by the requirements of the induction measurements to be made after the demagnetization. For these the first requirement is constancy, which is best obtained by a storage battery furnishing no other current. A relatively high voltage is advantageous, because the higher the regulating resistance the easier is the regulation and the lower the time constant of the circuit. These two factors are closely connected with rapidity of the measurements and the accuracy of results. On the other hand, a low voltage means less trouble from arcing at the commutator and less outlay for battery and the extra resistance needed. Twenty volts is an intermediate value, which meets all requirements and has been used in this series of experiments.

POLARIZATION EFFECT.

It is well at the outset to determine something of the nature of the polarization effects which are to be removed by the process of demagnetization. To this end several specimens of iron were examined after having been exposed to different magnetic treatments. The iron was first demagnetized as well as might be and its apparent induction determined for a series of values of the magnetizing force. Then the iron was subjected to a strong magnetizing force of 100 units (gausses) after which the induction was redetermined. This polarizing force was applied and removed several times, but it was not reversed in this first comparison. The results of this experiment are shown numerically in Tables II, III, IV, and V, and

TABLE II.

To illustrate the effect of the residual polarization due to a force of 100 on the apparent induction in annealed transformer iron.

Magnetizing force	Induction after demagnetization	Apparent induction when polarized	Polarization effect	Per cent polarization effect	$\mu = \frac{B}{H}$
.1	68	26	42	62	680
.2	192	67	125	65	960
.3	370	130	240	65	1260
.4	670	230	440	66	1670
.5	1290	480	810	63	2580
.6	1680	720	960	57	2800
.7	2570	1380	1190	46	3670
.8	3370	2510	860	26	4210
.9	4300	3700	600	14	4780
1.0	5130	4750	380	7	5130
1.1	5710	5360	350	6	5170
1.2	6730	6490	240	4	5230
1.3	6880	6750	130	2	5290
2.0	8880	8880	0	0	4440
3.0	10750	10750	0	0	3580
5.0	12750	12750	0	0	2550
10.0	14620	14620	0	0	1460
15.0	15280	15280	0	0	1020

Plotted in Fig. 3.

TABLE III.

To illustrate the effect of the residual polarization due to a force of 100 on the apparent induction in common sheet iron.

Magnetizing force	Induction after demagnetization	Apparent induction when polarized	Polarization effect	Per cent polarization effect	$\mu = \frac{B}{H}$
1	263	97	166	63	263
2	1314	496	818	62	657
3	3770	3130	640	17	1257
5	7890	7730	160	2	1580
7	10270	10130	140	1	1467
10	12170	12160	10	0	1217
15	13610	13610	0	0	907

Plotted in Fig. 5.

TABLE IV.

To illustrate the effect of the residual polarization due to a force of 100 on the apparent induction in low carbon Bessemer steel.

Magnetizing force	Induction after demagnetization	Apparent induction when polarized	Polarization effect	Per cent polarization effect	$\mu = \frac{B}{H}$
1	190	90	100	53	190
2	680	370	310	46	340
3	1930	1460	470	24	640
4	4150	3670	480	12	1040
5	6790	6300	490	7	1360
7	9990	9740	250	3	1440
10	12470	12470	0	0	1250
15	14170	14170	0	0	940

Plotted in Fig. 4.

graphically in Figs. 3, 5, 4, and 6. From these data and curves it is to be noted that the four specimens of iron show several common characteristics.

(1) The induction after demagnetization is greater than the apparent induction after the intense polarization. The difference between

TABLE V.

To illustrate the effect of the residual polarization due to a force of 100 on the apparent induction in high carbon crucible steel.

Magnetizing force	Induction after demagnetization	Apparent induction when polarized	Polarization effect	Per cent polarization effect	$\mu = \frac{B}{H}$
1	108	54	54	50	108
2	249	125	124	50	124
3	405	210	195	48	135
5	780	440	340	44	156
10	2980	2600	380	13	298
15	6680	6610	70	1	445
30	10650	10650	0	0	335
40	11670	11670	0	0	292

Plotted in Fig. 6.

these two inductions is a quantity which comes up again and again and may be called the "polarization effect." The polarization effect is a measure of completeness of the previous demagnetization. If the demagnetization is perfect the polarization effect is zero. The first point noted then is that the polarization effect is always positive.

(2) As the magnetizing force under which the apparent induction is measured increases, the polarization effect at first increases, then passes through a maximum, and finally decreases to zero.

(3) The point at which the polarization effect is a maximum is somewhat lower than the point of maximum permeability and near the point where the differential permeability is a maximum.

(4) The point at which the polarization effect vanishes is somewhat above the point of maximum permeability. This vanishing point is a critical point magnetically. We shall therefore denote the corresponding force and induction as "critical demagnetizing force" ¹⁴ and "critical induction."

(5) While the initial polarization effect initially increases, the normal induction increases so much faster that the percentage polarization effect decreases continuously with increasing magnetizing force.

To test the nature of this polarization effect still further, the

¹⁴ Searle calls this "critical magnetic force."

annealed transformer iron was demagnetized and subjected momentarily to successively increasing polarizing forces. After each application of the polarizing force a magnetizing force of 0.5 was applied and slowly reversed several hundred times. After the iron had reached a cyclic condition the apparent induction was measured.

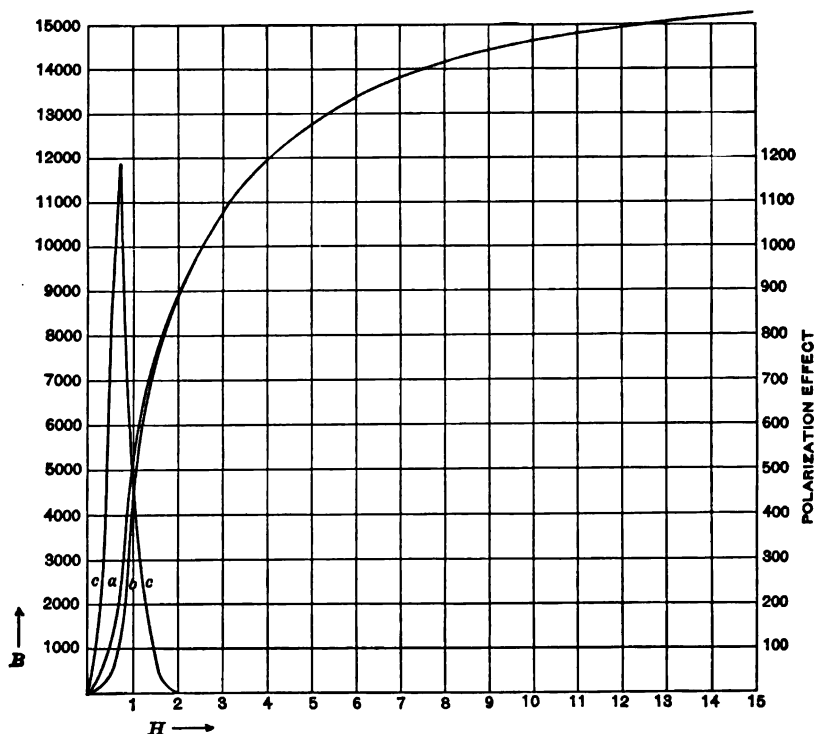


Fig. 3.—Showing characteristics of annealed transformer iron.

(Numerical data in Table II.)

- (a) Normal B - H curve.
- (b) Apparent B - H curve after intense polarization.
- (c) Polarization effect magnified 10-fold relatively to the B - H curves.

The results are shown numerically in Table VI and graphically in Fig. 7. In the curve the apparent induction is plotted against the polarizing force. The same figure may be considered as a curve of polarization effects, for the distance of the curve below the line of normal induction is the polarization effect. It is to be noted that the curve of polarization effect plotted against polarizing force

TABLE VI.

Showing the influence of various polarizing forces on the apparent induction.

Polarizing force	Apparent induction	Polarisation effect	Polarization effect + polarizing force
.5	1290	0	0
.6	1285	5	8
.7	1260	30	43
.8	1210	80	100
.9	1145	145	161
1.0	1095	195	195
1.5	925	365	243
2.0	820	470	235
3.0	680	610	203
4.0	605	685	171
5.0	555	735	147
10.0	490	800	80
20.0	480	810	40

Plotted in Fig. 7.

resembles the ordinary $B-H$ curve with initial slope gradual, then passing through a maximum and finally becoming horizontal. However, for polarizing forces between zero and the force under which the apparent induction is measured, the curve is horizontal. This does not mean that polarizing forces less than the magnetizing force studied are without any effect on the iron, but that any such effect is completely wiped out by the repeated reversals of the chosen magnetizing force.

Certain experiments on the effects of imperfect demagnetization seemed to indicate that the data recorded above do not represent the maximum possible polarization effects for the given polarizing forces. To test this point the polarizing force was applied in different ways. Experiment showed that a given force left a greater residual polarization effect if it were reversed several times than if it were simply applied and removed the same number of times. The data of Table VII substantiate this conclusion. The last column of this table shows the difference in polarization effects under these

two methods of procedure. It is noticeable that these differences are very roughly proportional to the corresponding original polarization effects. This shows that the change due to the reversals of the polarizing force is uniformly distributed, and that the general conclusions as to the nature of the polarization effect hold in this case as well as in the former.

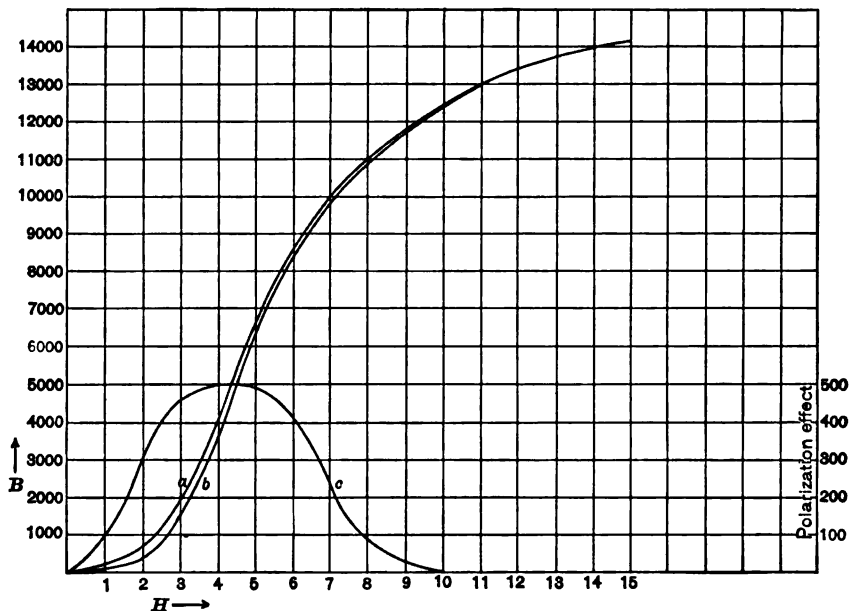


Fig. 4.—Showing characteristics of low carbon steel.

(Numerical data in Table IV.)

- (a) Normal B - H curve.
- (b) Apparent B - H curve after intense polarization.
- (c) Polarization effect magnified 10-fold relatively to the B - H curves.

Since the polarization effect is a continuous function of H and has a maximum at the same value of H , though of different magnitude in the two methods of polarizing used, it was assumed that the polarization effect would be a maximum at the same value of H under all conditions, and when it vanished at this force it would have vanished for all other magnetizing forces. This is a rather bold assumption; but it furnishes a good working basis and economizes time, as it allows the characteristics at a single point to typify

the characteristics of a whole range of points. This assumption is later justified for all the ordinary methods of demagnetization.

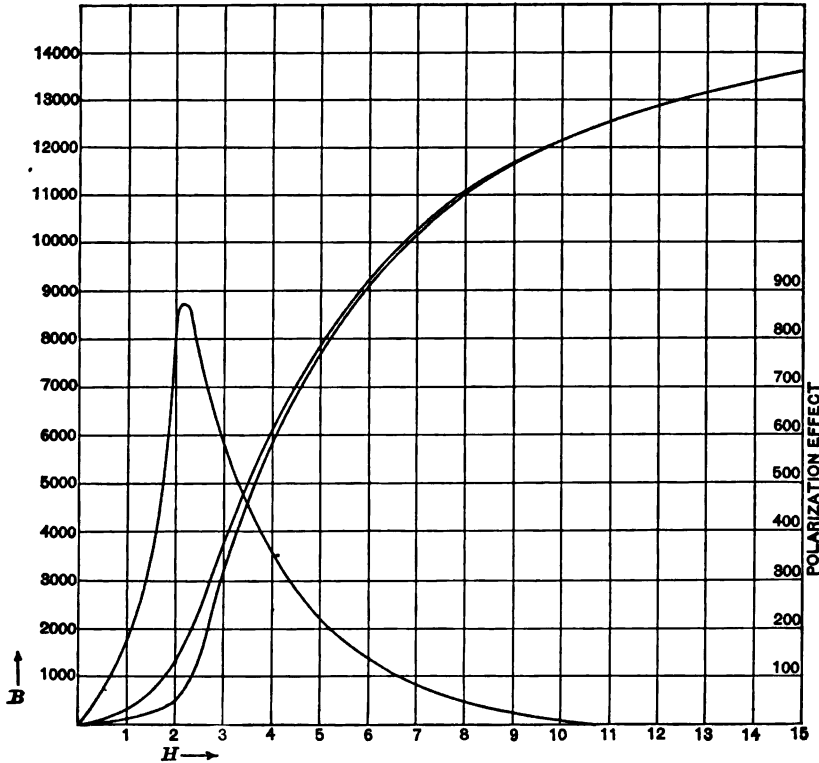


Fig. 5.—Showing characteristics of common sheet iron.
(Numerical data in Table III.)

- (a) Normal B - H curve.
- (b) Apparent B - H curve after intense polarization.
- (c) Polarization effect magnified 10-fold relatively to the B - H curves.

UPPER LIMIT OF THE DEMAGNETIZING FORCE.

It has been assumed in nearly all earlier work¹⁵ that a necessary condition for perfect demagnetization is that the demagnetizing current must carry the iron from an induction greater than any previous one it has experienced down to a vanishingly small one. The question at once arises as to whether a somewhat smaller range

¹⁵ See (1) on p. 206 and (1) on p. 207

might not be equally successful in wiping out the effects of previous polarization.

To determine the maximum value of a necessary yet sufficient demagnetizing force the following experiment was carried out: The specimen was first strongly magnetized by a force of 100 to

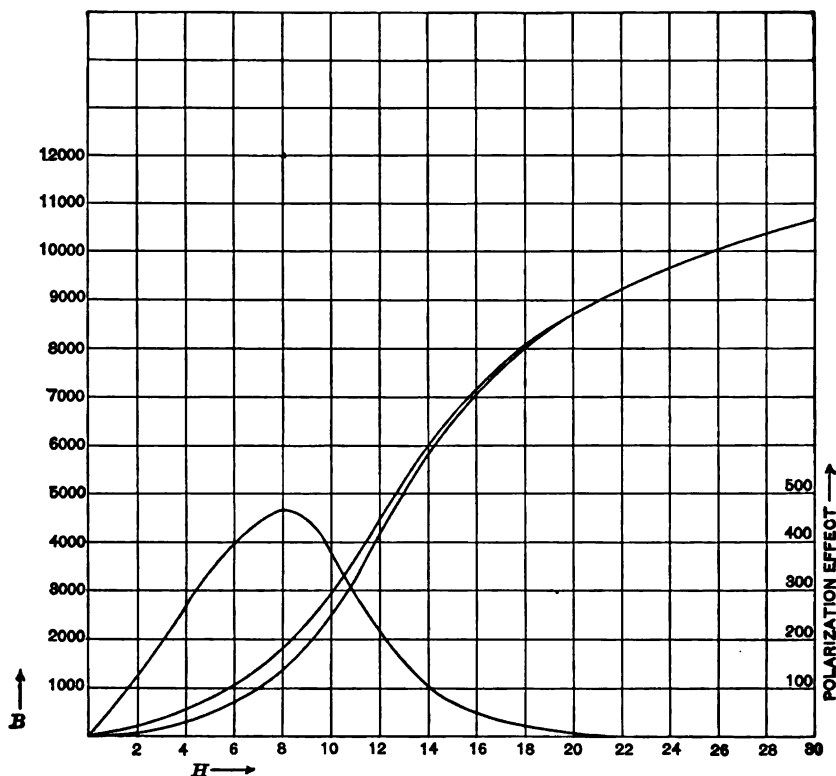


Fig. 6.—Showing characteristics of high carbon steel,

(Numerical data in Table V.)

- (a) Normal B - H curve.
- (b) Apparent B - H curve after intense polarization.
- (c) Polarization effect magnified 10-fold relatively to the B - H curves.

insure a considerable initial polarization. Then it was demagnetized from a certain value of the demagnetizing force down to a vanishingly small force. The cyclic apparent induction was then measured for the value of H , at which the previously observed

polarization effect was a maximum. After this measurement the specimen was again strongly magnetized as before, demagnetized from a second maximum down to the same vanishingly small force, and finally the cyclic apparent induction was measured for the same magnetizing force as before. This process was repeated for a series

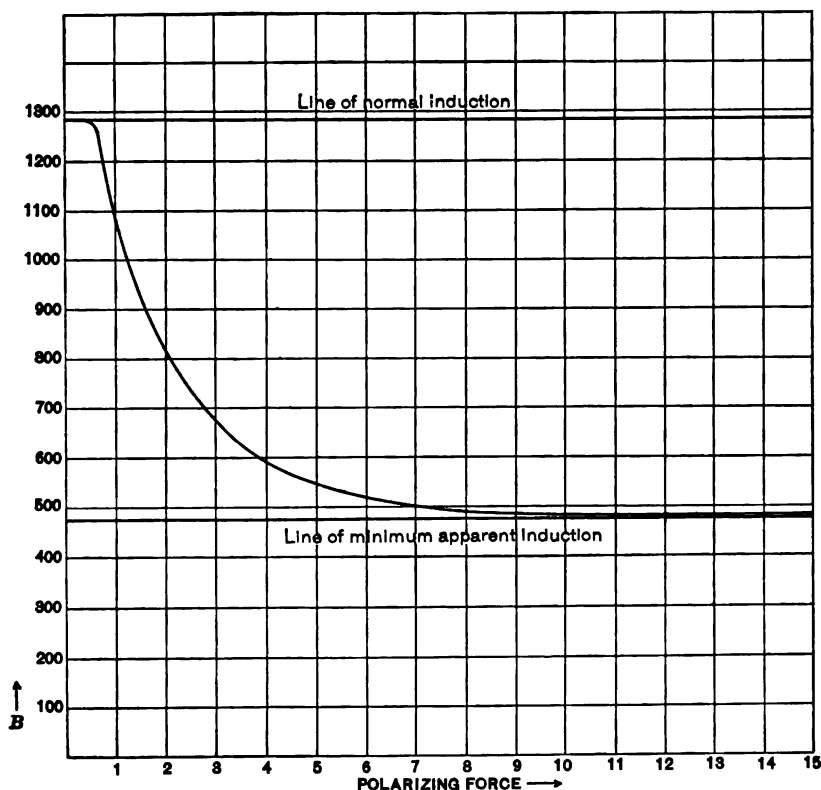


Fig. 7.—Showing the relation between the polarizing force and the apparent induction in annealed transformer iron for $H=0.5$.

(Numerical data in Table VI.)

of values of the maximum demagnetizing force. The results of these experiments are shown graphically in curves marked "curve of upper limits" in Figs. 8, 9, and 10, where the maximum demagnetizing forces are plotted as abscissæ and the apparent inductions as ordinates. From these three curves the following characteristics may be noted:

(1) For maximum demagnetizing forces greater than the critical demagnetizing force the curve is parallel to the horizontal axis. This indicates that the demagnetization is not improved by carrying the demagnetizing force beyond this *critical demagnetizing force*.

(2) For maximum demagnetizing forces less than the magnetic force used to determine the apparent induction the curve is again horizontal. This shows that such demagnetization produces no effect that the simple reversals of the magnetizing force used in determining the induction would not accomplish.

TABLE VII.

Showing that the polarizing effect of a field of 100 is greater on reversal than on simple make and break.

Magnetizing force	Normal induction	Polarization effect of make and break	Polarisation effect of reversals	Difference
.3	370	240	290	50
.4	670	440	520	80
.5	1290	810	910	100
.6	1680	960	1080	120
.7	2570	1190	1310	120
.8	3370	860	1050	190
.9	4300	600	730	130
1.0	5130	380	500	120
1.2	6730	240	330	90
1.4	6950	90	110	20
1.6	7610	30	30	0
2.0	8880	0	0	0
3.0	10750	0	0	0

(3) The two horizontal portions are connected by a smooth curve, indicating that over this range the demagnetization is more or less imperfect but is approaching completeness very rapidly with increase of maximum demagnetizing force.

(4) A comparison of the curves for the soft annealed transformer iron, the moderately hard low carbon steel, and the hard high carbon steel shows that the steepness of this sloping portion of the curve and the sharpness of the bends at its upper and lower extremities decreases as the hardness of the iron increases.

TABLE VIII.

Showing the apparent induction in annealed transformer iron as influenced by the upper limit of the demagnetizing force.

Limits of demagnetizing force	3 (or more) —.2	2 —.2	1 —.2	.5 —.2	No demagnetization
Time interval	92.4	72.4	99	91.8	
Number of cycles	145	107	154	135	
	<i>B</i>	<i>B</i>	<i>B</i>	<i>B</i>	<i>B</i>
<i>H</i> = .3	370	360	320	220	130
.4	670	670	630	360	230
.5	1290	1290	1210	740	480
.6	1680	1680	1570	770	720
.7	2570	2570	2340	1530	1380
.8	3370	3370	2990	2460	2510
.9	4300	4300	3780	3580	3700
1.0	5130	5130	4620	4650	4750
1.2	6730	6730	6370	6370	6490
1.4	6950	6950	6720	6710	6860
1.6	7610	7610	7390	7390	7580
2.0	8880	8880	8730	8730	8880
3.0	10750	10750	10750	10750	10750

TABLE IX.

Showing polarization effects (calculated from Table VIII).

Limits of demagnetizing force	3 —.2	2 —.2	1 —.2	.5 —.2	No demagnetization
.3	0	10	50	150	240
.4	0	0	40	310	440
.5	0	0	80	550	810
.6	0	0	110	910	960
.7	0	0	230	1040	1190
.8	0	0	380	910	860
.9	0	0	520	720	600
1.0	0	0	510	480	380
1.2	0	0	360	360	240
1.4	0	0	230	240	90
1.6	0	0	220	220	30
2.0	0	0	150	150	0
3.0	0	0	0	0	0

TABLE X.

Showing the apparent induction in common sheet iron as influenced by the upper limit of the demagnetizing force.

Limits of demagnetizing force	10-1	7-1	5-1	No demagnetization
Time interval	54	42	32	
Number of cycles	84	64	48	
<i>H</i>	<i>B</i>	<i>B</i>	<i>B</i>	<i>B</i>
1	263	255	255	97
2	1314	1304	1304	496
3	3770	3750	3740	3130
5	7890	7810	7700	7730
7	10270	10200	10170	10130
10	12170	12170	12160	12160
15	13610	13610	13610	13610

TABLE XI.

Showing polarization effects (calculated from Table X).

Limits of demagnetizing force	10-1	7-1	5-1	No demagnetization
<i>H</i> =1	0	8	8	166
2	0	10	10	818
3	0	20	30	640
5	0	80	190	160
7	0	70	100	140
10	0	0	10	10
15	0	0	0	0

TABLE XII.

Showing the apparent induction in low carbon steel as influenced by the upper limit of the demagnetizing force.

Limits of demagnetizing force	15--.2	10--.2	5--.2	No demagnetization
Time interval	70	70	49	
Number of cycles	100	100	70	
	<i>B</i>	<i>B</i>	<i>B</i>	<i>B</i>
<i>H</i> = 1	190	190	183	90
2	680	675	667	370
3	1930	1920	1897	1460
4	4150	4130	3990	3470
5	6790	6745	6590	6300
6	9140	9100	9000	8670
7	9990	9960	9920	9740
8	11070	11060	11040	10950
9	11880	11870	11870	11820
10	12470	12460	12470	12460
15	14170	14170	14170	14170

TABLE XIII.

Showing polarization effects (calculated from Table XII).

Limits of demagnetizing force	15--.2	10--.2	5--.2	No demagnetization
1	0	0	7	100
2	0	5	13	310
3	0	10	33	470
4	0	20	160	480
5	0	45	200	490
6	0	40	140	370
7	0	30	70	250
8	0	10	30	120
9	0	10	10	60
10	0	10	0	10
15	0	0	0	0

The preceding applies to a single value of the magnetizing force. To show that this is typical, and to get further light on the nature of the polarization effect under different degrees of completeness in the demagnetization, the full range of points was taken for a few different sets of magnetizing limits. The data for the apparent inductions after various demagnetizations in which the upper limit

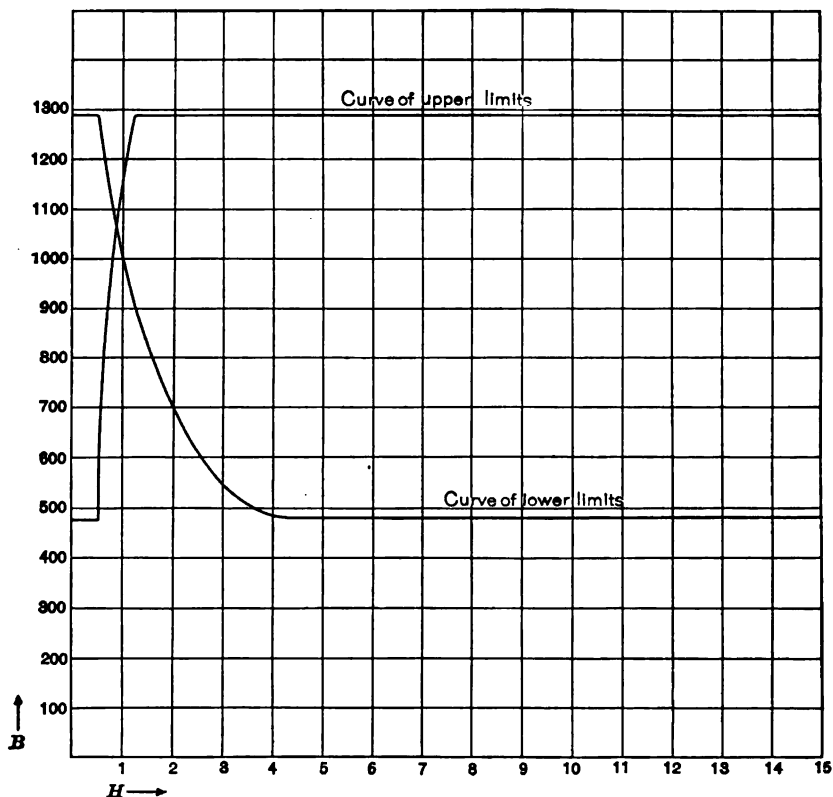


Fig. 8.—Showing the influence of the upper and lower limits of the demagnetizing force on the apparent induction of annealed transformer iron for $H=0.5$.

alone was altered are given in Tables VIII, X, and XII. The corresponding data for the polarization effects are more instructive and are given in Tables IX, XI, and XIII. From these six tables the following conclusions may be drawn:

(1) The demagnetization is complete throughout the whole range if the upper limit is at least as great as the critical demagnetizing force.

(2) If the maximum demagnetizing force is less than the critical demagnetizing force, the demagnetization is in general incomplete, and the incompleteness extends over practically the whole region from the lowest values of the magnetizing force up to the critical value. This region may be called the *domain* of the polarization effect.

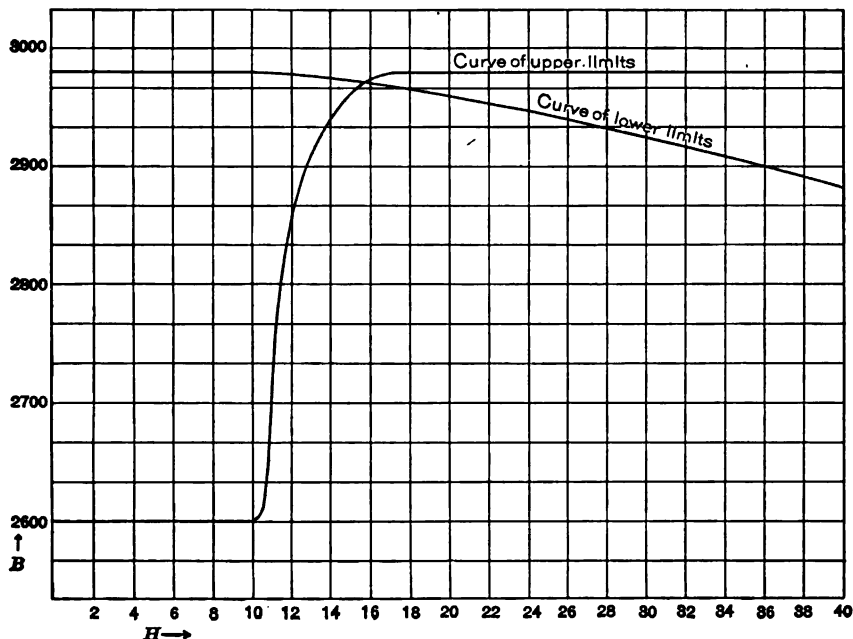


Fig. 9.—Showing the influence of the upper and lower limits of the demagnetizing force on the apparent induction of high carbon steel for $H=10$.

(3) The polarization effect increases as the maximum demagnetizing force decreases, but preserves its general characteristics as described above.

(4) In certain cases the polarization effect is greater after a feeble demagnetization than without any demagnetization.¹⁶ Tables IX and XI for the transformer iron and the common sheet iron, respectively, show this very clearly. This phenomenon occurs at or below the point of maximum permeability. It was this peculiar increase of polarization that suggested the comparison of polarization effects

¹⁶ Cf. Searle's conclusion, 3c on p. 207.

after repeated simple make and break, with those after reversals. A comparison with those results in Table VII will show that the increased polarization effect is still less than the maximum observed on reversal.

Some further observations on the effect of varying the upper limit of the demagnetizing force will be made under the discussion of the influence of frequency of an alternating current demagnetization on the polarization effect.

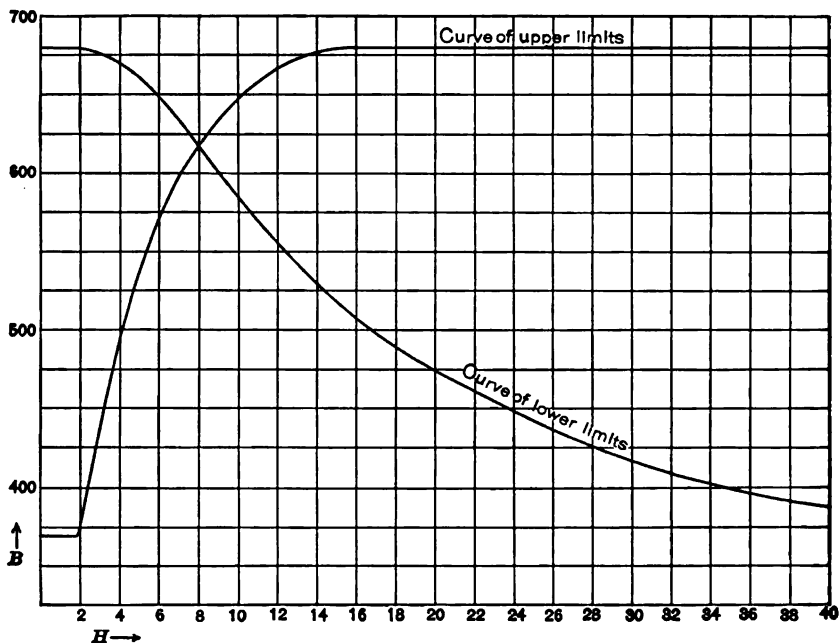


Fig. 10.—Showing the influence of the upper and lower limits of the demagnetizing force on the apparent induction of low carbon steel for $H=2$.

LOWER LIMITS OF THE DEMAGNETIZING FORCE.

Having settled upon a proper value for the upper limit of the demagnetizing force, it now remains to determine the necessary and sufficient final minimum value of this force. For this purpose an experiment similar to the preceding was carried out. The specimen was polarized initially as before. The demagnetization was carried from a point well above the critical demagnetizing force down to a certain minimum value. Finally the cyclic apparent induction was

measured for the same force as in the preceding experiment. These operations were repeated, modifying only the final minimum value of the demagnetizing force. The results of this experiment are shown graphically in the curves marked "curve of lower limits" in Figs. 8, 9, and 10. The minimum value of the demagnetizing force is plotted as abscissa and the corresponding cyclic apparent induction as ordinate. Both coordinates are plotted to the same scale as in the curve of upper limits. From these curves the following conclusions may be drawn:

TABLE XIV.

Showing the apparent induction of annealed transformer iron as influenced by the lower limit of the demagnetizing force.

Limits of demagnetizing force	15-.32	15-.5	15-1.15	15-2.6	No demagnetization
Time interval	95.8	97.8	66	60	
Number of cycles	160	135	104	96	
<i>H</i>	<i>B</i>	<i>B</i>	<i>B</i>	<i>B</i>	<i>B</i>
.3	370	352	250	160	130
.4	670	660	450	340	230
.5	1290	1290	990	645	480
.6	1680	1680	1470	1000	720
.7	2570	2570	2420	1800	1380
.8	3370	3370	3180	2700	2510
.9	4300	4300	4270	3870	3700
1.0	5130	5130	5110	5030	4750
1.2	6730	6730	6730	6690	6490
1.4	6950	6950	6950	6950	6860
1.6	7610	7610	7610	7610	7580
2.0	8880	8880	8880	8880	8880
3.0	10750	10750	10750	10750	10750

(1) For values of lower limit of the demagnetizing force less than the magnetizing force which produces the apparent induction, the curve is a horizontal line. This indicates that the demagnetization is not improved by carrying the demagnetizing force below the lowest magnetizing force to be studied.

(2) For values of the lower limit considerably greater than the critical demagnetizing force the curve is a horizontal straight line and indicates that the demagnetizing force does not modify the previously existing polarization effect; at least it does not modify that portion of it which repeated reversals of the given magnetizing force do not eliminate.

(3) Between the two horizontal portions is a continuous sloping curve, which indicates a partial demagnetization which increases in completeness very rapidly as the lower limit of the demagnetizing force decreases.

TABLE XV.

Showing the apparent induction of common sheet iron as influenced by the lower limit of the demagnetizing force.

Limits of demagnetizing force	15-1	15-2	15-3	15-5	15-7	No demagnetization
Time interval	38	37	31	22	17	
Number of cycles	56	52	48	32	26	
<i>H</i>	<i>B</i>	<i>B</i>	<i>B</i>	<i>B</i>	<i>B</i>	<i>B</i>
1	263	255	197	151	120	97
2	1314	1314	1224	890	820	496
3	3770	3770	3770	3350	3220	3130
5	7890	7890	7890	7890	7860	7730
7	10270	10270	10270	10270	10270	10130
10	12170	12170	12170	12170	12170	12160
15	13610	13610	13610	13610	13610	13610

(4) A comparison of the curves for three grades of iron and steel shows that the steepness of the curved portion and the sharpness of the bends at the upper and lower extremities decrease as we pass from the softer to the harder material. The lower horizontal portion occurs at extremely high values in the hard material. (Figs. 9 and 10.)

To justify the above conclusions drawn from measurements made at a single value of the magnetizing force, and to investigate further the nature of the polarization effect several complete apparent induction curves were obtained under different details of demagnetization.

These complete data are given in Tables XIV, XV, and XVI. From these data the preceding conclusions are justified, and the following generalization appears: *If the demagnetization is carried from the critical demagnetizing force down to a certain point, the demagnetization is complete for all values above the final demagnetizing force. For magnetizing forces below this final value of demagnetization the demagnetization is incomplete, and the incompleteness is greater the greater the interval between the final demagnetizing force and the magnetizing force used to produce the induction desired.*

TABLE XVI.

Showing the apparent induction of low carbon steel as influenced by the lower limit of the demagnetizing force.

Limits of demagnetizing force	15—.2	15—.5	15—.3	15—.5	No demagnetization
Time interval	75	73	167	156	
Number of cycles	83	80	200	180	
<i>H</i>	<i>B</i>	<i>B</i>	<i>B</i>	<i>B</i>	<i>B</i>
1	190	185	175	152	90
2	680	680	668	566	370
3	1930	1930	1930	1795	1460
4	4150	4150	4150	4150	3470
5	6790	6790	6790	6790	6300
6	9140	9140	9140	9140	8670
7	9990	9990	9990	9990	9740
8	11070	11070	11070	11070	10950
9	11880	11880	11880	11880	11820
10	12470	12470	12470	12470	12460
15	14170	14170	14170	14170	14170

EDDY CURRENTS—THEORY.

If we use an alternating current in demagnetizing, we must consider the shielding effect which the eddy currents exert on the interior of the specimen. The magnetizing force at any point is the resultant of the force due to the current in the wire and the force due to the eddy currents in the specimen. The calculation of this

force presents great difficulties unless certain simplifying assumptions are made. Heaviside¹⁷ in a paper entitled "The Induction of Currents in Cores," gives a solution for the special case of a round rod of constant permeability magnetized by a simple harmonic force. In this case the force at any point of the specimen is a function of

μ = the permeability.

ρ = the specific resistance.

N = the frequency.

r = the distance of the point from the axis.

If we let $x = \frac{8\pi^2 \mu N}{\rho}$ the resultant magnetic force at any point is

$$H = AM + BN,$$

where A and B are constants, and

$$\begin{aligned} M &= \frac{1}{2} \left\{ J_0(r\sqrt{-ix}) + J_0(r\sqrt{ix}) \right\} \\ &= 1 - \frac{x^2 r^4}{2^2 \cdot 4^2} + \frac{x^4 r^8}{2^2 \cdot 4^2 \cdot 6^2 \cdot 8^2} - \dots \\ N &= \frac{1}{2i} \left\{ J_0(r\sqrt{-ix}) - J_0(r\sqrt{ix}) \right\} \\ &= \frac{x r^2}{2^2} - \frac{x^3 r^6}{2^2 \cdot 4^2 \cdot 6^2} + \frac{x^5 r^{10}}{2^2 \cdot 4^2 \cdot 6^2 \cdot 8^2 \cdot 10^2} - \dots \end{aligned}$$

The maximum value of H at any point in the core is proportional to

$$\sqrt{M^2 + N^2}$$

By squaring the expressions for M and N , and adding, we have

$$\begin{aligned} M^2 + N^2 &= J_0(r\sqrt{-ix}) J_0(r\sqrt{ix}) \\ &= 1 + \frac{2y}{2^2 \cdot 4^2} \left(1 + \frac{3y}{6^2 \cdot 8^2} \left(1 + \frac{3\frac{1}{2}y}{10^2 \cdot 12^2} \left(1 + \frac{3\frac{1}{2}y}{14^2 \cdot 16^2} \left(1 + \frac{3\frac{1}{2}y}{18^2 \cdot 20^2} \left(1 + \dots \right. \right. \right. \right. \right. \right. \end{aligned}$$

where $y = x^2 r^4$.

Since every term of the series is positive, the amplitude increases as we pass from the axis outward. To get some idea of the relative

¹⁷ Electrical Papers, Vol. I, p. 353 ff.

magnitude of the amplitudes at various distances from the axis, I have calculated values of $\sqrt{M^2+N^2}$ for different values of y . These calculated values are shown graphically in Fig. 25, where $\sqrt{M^2+N^2}$ is plotted as abscissa and y as ordinate. $\sqrt{M^2+N^2}$ is the ratio of the amplitude for a given value of y to the amplitude at the center where y is zero.

Putting

$$\mu = 1000$$

$$\rho = 10000 \text{ c. g. s. units } (= 10 \text{ micro-ohms})$$

$$N = 60$$

we have

$$\begin{aligned} y &= x^2 r^4 \\ &= \left[\frac{8\pi^2 \mu N}{\rho} \right]^{\frac{1}{2}} r^4 \\ y &= 224000 r^4 \end{aligned}$$

The last equation is plotted on the left half of Fig. 25 in connection with the curve of amplitudes; y is plotted upward on the same axis for both curves. The distance from the center r is plotted to the left. Curves are drawn also for $\mu = 500$ and $\mu = 2000$. From this figure the relative amplitudes at any distance from the axis may be obtained, subject of course to the conditions stated above. Apply this to a rod of the dimensions of the low carbon rod already used. If $\mu = 500$, a point on the surface, $r = 0.29$ cm, corresponds to the point P on the (y, r) curve and Q on the $(\sqrt{M^2+N^2}, y)$ curve, so that at the surface of the rod the maximum induction is 4.4 times its value at the axis. For $\mu = 1000$ the maximum induction at the boundary is 1.4 times the value at the axis. The induction has half the boundary value at $r = 0.235$ cm, showing that the resultant force, and consequently the induction, is much greater in the outer layers of the rod. The flatness of the (y, r) curve for small values of r shows that the force and flux are nearly uniform near the axis of the rod. This variation of the resultant magnetic force throughout the cross section of the rod increases with increase

TABLE X.

Showing the apparent induction in common sheet iron as influenced by the upper limit of the demagnetizing force.

Limits of demagnetizing force	10-1	7-1	5-1	No demagnetization
Time interval	54	42	32	
Number of cycles	84	64	48	
<i>H</i>	<i>B</i>	<i>B</i>	<i>B</i>	<i>B</i>
1	263	255	255	97
2	1314	1304	1304	496
3	3770	3750	3740	3130
5	7890	7810	7700	7730
7	10270	10200	10170	10130
10	12170	12170	12160	12160
15	13610	13610	13610	13610

TABLE XI.

Showing polarization effects (calculated from Table X).

Limits of demagnetizing force	10-1	7-1	5-1	No demagnetization
<i>H</i> =1	0	8	8	166
2	0	10	10	818
3	0	20	30	640
5	0	80	190	160
7	0	70	100	140
10	0	0	10	10
15	0	0	0	0

TABLE XII.

Showing the apparent induction in low carbon steel as influenced by the upper limit of the demagnetizing force.

Limits of demagnetizing force	15—.2	10—.2	5—.2	No demagnetization
Time interval	70	70	49	
Number of cycles	100	100	70	
	<i>B</i>	<i>B</i>	<i>B</i>	<i>B</i>
<i>H</i> = 1	190	190	183	90
2	680	675	667	370
3	1930	1920	1897	1460
4	4150	4130	3990	3470
5	6790	6745	6590	6300
6	9140	9100	9000	8670
7	9990	9960	9920	9740
8	11070	11060	11040	10950
9	11880	11870	11870	11820
10	12470	12460	12470	12460
15	14170	14170	14170	14170

TABLE XIII.

Showing polarization effects (calculated from Table XII).

Limits of demagnetizing force	15—.2	10—.2	5—.2	No demagnetization
1	0	0	7	100
2	0	5	13	310
3	0	10	33	470
4	0	20	160	480
5	0	45	200	490
6	0	40	140	370
7	0	30	70	250
8	0	10	30	120
9	0	10	10	60
10	0	10	0	10
15	0	0	0	0

and with the massive yokes, respectively. Graphic representations of the data for the low and the high carbon steels are given in Figs. 12 and 13. Table XVII gives a portion of the numerical data for all three of the specimens in a form which facilitates comparison. From the graphic and numerical data given above a number of conclusions may be drawn.

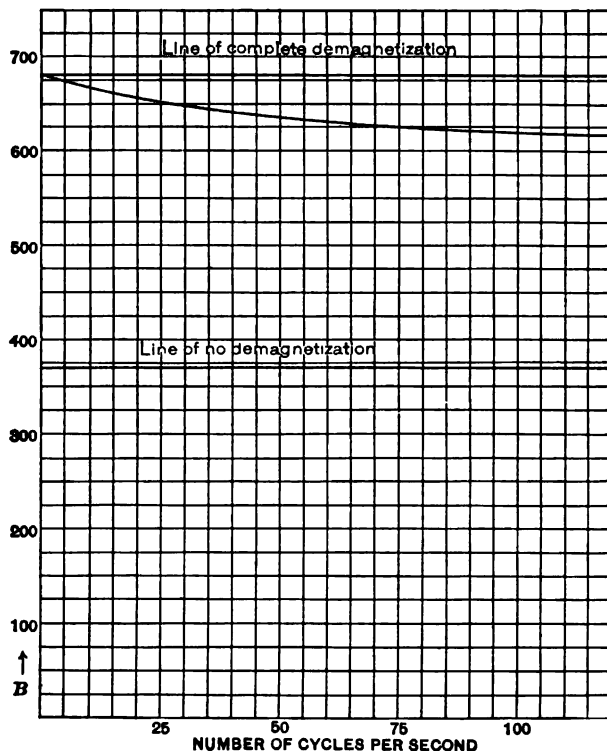


Fig. 12.—Showing the influence of the demagnetizing frequency on the apparent induction of low carbon steel for $H=2$.

(1) In every case an increase in the frequency of the demagnetizing current is accompanied by a decrease in the apparent induction as subsequently measured.

(2) In the case of the transformer iron, the polarization effect which is measured by the distance of the curve below the upper horizontal line is greater as the cross section of the yokes increases. That the polarization effect does not vanish when the yokes are

removed shows that while the yokes modify its magnitude, they are not the cause of it. In comparing the transformer iron with the other specimens the data obtained with the massive yokes should be used, as heavy yokes were used on the other specimens.

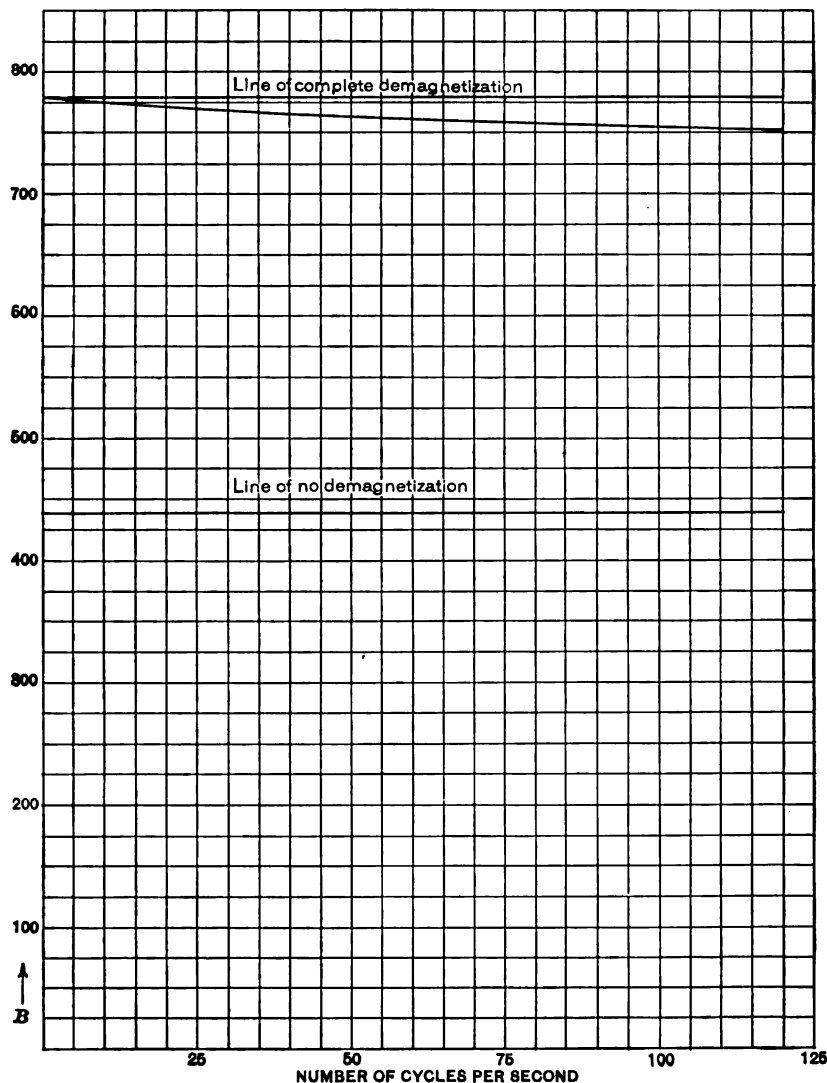


Fig. 13.—Showing the influence of the demagnetizing frequency on the apparent induction of high carbon steel for $H=5$.

(3) The diminution in apparent induction—that is, the polarization effect—is relatively greater in the softer material. This statement holds also for the maximum polarization effect, as measured by the distance between the line of perfect demagnetization and the line of no demagnetization. The rate of increase of the polarization effect with the frequency of the demagnetizing current is greater in the softer materials.

TABLE XVIII.

Showing the nature of the polarization effect remaining in annealed transformer iron after demagnetizing by alternating current of 60 cycles.

Limits of demagnetizing force		*5—.2	3.5—.2	2.6—.2	No demagnetization
Time interval		77	58	34	
Number of cycles		4620	3480	2040	
<i>H</i>	Normal induction	Polarisation effects			
.3	370	20	30	30	240
.4	670	30	30	40	440
.5	1290	50	50	70	810
.6	1680	60	60	90	960
.7	2570	80	80	100	1190
.8	3370	70	70	70	860
.9	4300	60	60	60	600
1.0	5130	50	50	50	380
1.2	6730	40	40	40	240
1.4	6950	20	20	20	90
1.6	7610	10	10	10	30
2.0	8880	0	0	0	0

* This column holds also for the following conditions:

Limits of demagnetizing force	5—.2	20—.2	36—.2	159—.2
Time interval	177	40.8	60	52
Number of cycles	4620	2448	3600	3120

NATURE OF POLARIZATION DUE TO FREQUENCY.

That the above conclusions, based on a single value of the magnetizing force, hold in general is clear from data obtained to show the nature of the polarization effect throughout the whole range of the induction curve. Table XVIII shows such a set of data for the annealed transformer iron where the results are expressed in terms of polarization effects. Several things in this table are worthy of note.

TABLE XIX.

Showing the nature of the polarization effect remaining in low carbon steel after demagnetizing by an alternating current of 60 cycles per second.

Limits of demagnetizing force		90.1—1.2	42.4—1.2	16.2—1.2	No demagnetization
Time interval		122.8	140.6	118	
Number of cycles		7368	6436	7080	
<i>H</i>	Normal induction	Polarisation effects			
1	190	0	12	16	100
2	680	20	50	85	310
3	1930	45	120	200	470
4	4150	100	280	420	480
5	6790	70	280	300	490
6	9140	70	200	210	370
7	9990	60	110	140	250
8	11070	50	60	90	120
9	11880	20	20	20	60
10	12470	0	0	0	10

Plotted in Fig. 14.

(1) The polarization effect exists for all values of the magnetizing force up to the critical demagnetizing force, but is much smaller than the maximum polarization effect observed after intense magnetization without subsequent demagnetization. In this respect it resembles the effect after the imperfect demagnetization due to the upper limit of the demagnetizing force being too low.

(2) The polarization effect is constant for all values of the initial demagnetizing force greater than 5. A reference to Table VIII will

show that for slow frequency the maximum demagnetizing force of 2.0 or over gave constant results. An alternating current then requires a greater initial maximum to produce its full effect.

(3) The polarization effect after a demagnetization by alternating current, even when this has a value great enough to produce its maximum effect, is of considerable magnitude and extends from the smallest values of the magnetizing force up to the critical demagnetizing force.

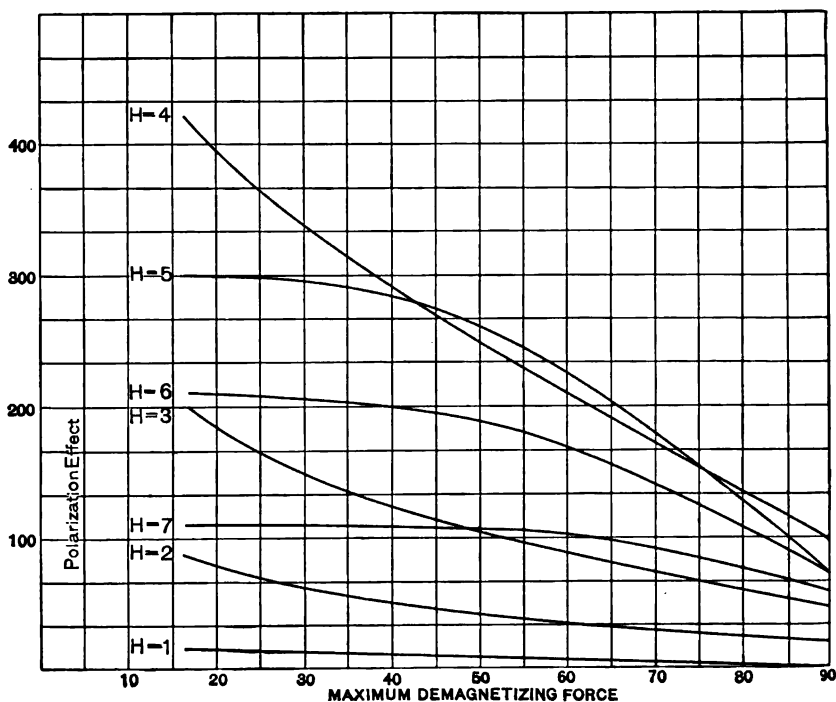


Fig. 14.—Showing how the polarization effect in low carbon steel due to excessive frequency varies with the maximum demagnetizing force.

(Numerical data in Table XIX.)

The low carbon steel was examined in the same way. The data for this have been shown numerically in Table XIX and graphically in Fig. 14. For this specimen also the polarization exists for all values of the magnetizing force up to the critical value. A constant value, however, was not reached at the values of the maximum demagnetizing forces used, even though a value over six times the maximum required at low frequency was used.

THE CAUSE—EDDY CURRENTS.

As was expected, the demagnetizing efficiency of an alternating current is less than that of a slowly reversed direct current. This difference is much greater in the thick round rod of low carbon steel than in the thin strip of transformer iron. (See dimensions in Table I.) Referring to Fig. 25 and assuming a mean permeability of 1000 for the round rod we see that the magnetic force on the axis is only 7 per cent of the magnetizing force exerted by the alternating current of 60 cycles. The alternating current furnished a magnet-

TABLE XX.

Showing the influence of eddy currents during the demagnetization by alternating current on the residual polarization effect.

Limits of demagnetizing force		15—.2	5.3—.2	17.6—.2	28—.2	155—.2	28—.2
Time interval		96	57.2	72	69.8	65.6	53.8
Frequency		1.7	60	60	60	60	120
<i>H</i>	Normal induction	Polarization effects					
.3	370	20	70	30	30	20	40
.4	670	40	130	70	50	50	90
.5	1290	90	250	120	100	100	170
.6	1680	110	400	180	160	120	280
.7	2570	170	620	290	250	230	400
.8	3370	210	720	320	280	250	460
.9	4300	220	740	340	300	260	480
1.0	5130	240	660	350	280	240	500
1.2	6730	200	480	320	250	240	420
1.4	6950	150	350	290	200	190	320
1.6	7610	80	320	250	180	160	300
2.0	8880	30	240	180	100	100	200

Plotted in Fig. 15.

izing force of 30 units, so that in the case of the low carbon steel where the induction does not reach a constant value at the magnetizing forces used, the incompleteness is due to the combined effect of frequency and too low magnetizing current. In the case of the

annealed iron, however, increased current does not reduce the polarization effect to zero, so that here we have an effect due solely to the frequency.

There seems no doubt that this decrease in induction after demagnetizing by alternating current is mainly an eddy current effect;

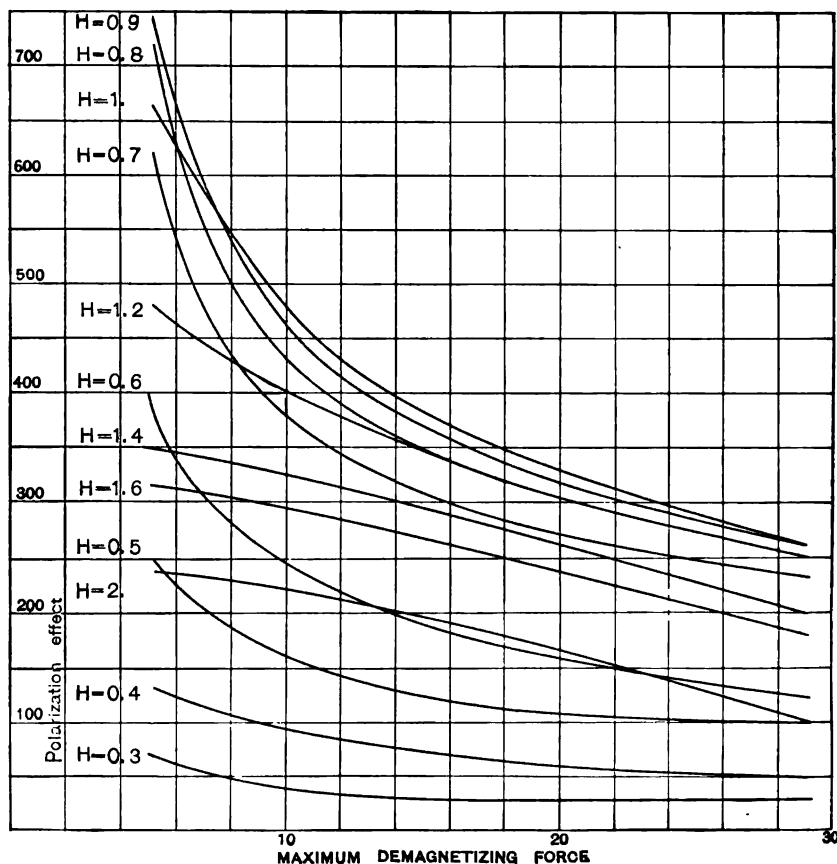


Fig. 15.—Showing how the polarization effect in a specimen of annealed transformer iron surrounded by a copper tube, after demagnetization by an alternating current of 60 cycles, varies with the maximum demagnetizing force.

(Numerical data in Table XX.)

nevertheless, to test this point further, one of the strips of annealed transformer iron was placed in a copper tube and the inductions measured in the regular way after various demagnetizations. The

data have been expressed as polarization effects and are recorded numerically in Table XX and graphically in Fig. 15. From the table we may draw a number of conclusions.

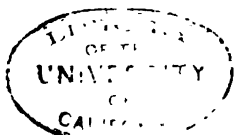
(1) The polarization effect even after slow reversals is not zero as we should expect it to be for perfect demagnetization.

(2) The polarization after a maximum demagnetizing force of 5 is much greater than it was when the copper tube was off. As the maximum force increases the polarization effect decreases throughout the whole range of magnetizing forces in much the same way as it does when the polarization effect is decreased by carrying the upper limit of a slowly reversed demagnetizing force up nearer and nearer to the critical value. The sixth and eighth columns of this table show that doubling the demagnetizing frequency, other conditions remaining unaltered, approximately doubles the polarization effect.

(3) The curves of Figs. 14 and 15 were drawn primarily to show how the polarization effect decreases as the upper limit of the demagnetizing force increases. The form of these curves is interesting. For the lower values of H these curves are concave upward. As the value of H increases the initial curvature increases, reaches a maximum in the neighborhood of the maximum of the permeability, finally decreases and becomes convex upward for the higher values of H . Other data seem to indicate that all of these curves become concave upward for large values of the upper limit of the demagnetizing force.

CONSTANCY OF RESULTS.

Another and very important fact brought out during the course of these experiments is the uniform consistency with which results are reproduced. Thus, while an alternating current of a frequency of 90 cycles per second will give a residual polarization effect it always gives the same residual value, and the induction curve obtained after such demagnetization may be repeated any number of times. No evidence of imperfect demagnetization exists except the fact that higher inductions may be obtained under other methods of demagnetization. This probably accounts for the fact that so many experimenters consider an alternating current demagnetization as entirely satisfactory. Furthermore, the polarization effect remain-



ing after demagnetization by alternating current decreases with decrease in the thickness of the specimen so that in the case of the thin transformer iron, such as that used in this investigation (.036 cm), or in the work of Professor Searle (.034 cm and .037 cm), the error introduced by imperfect demagnetization is negligible except in the steep part of the B - H curve. It is in the case of specimens of large cross section that the objections to alternating current demagnetization have most weight.

TABLE XXI.

Showing the nature of the residual polarization effect in annealed transformer iron caused by the time interval of demagnetization being too brief.

Limits of demagnetizing force		15—.2	15—.2	15—.2	No demagnetization
Time interval		74 or over	58	20	
Number of cycles		110 or over	85	27	
<i>H</i>	Normal induction	Polarisation effects			
.3	370	0	10	30	240
.4	670	0	20	70	440
.5	1290	0	20	80	810
.6	1680	0	30	100	960
.7	2570	0	50	150	1190
.8	3370	0	40	120	860
.9	4300	0	40	110	600
1.0	5130	0	20	60	380
1.2	6730	0	0	40	240
1.4	6950	0	0	0	90
1.6	7610	0	0	0	30
2.0	8880	0	0	0	0

As a general conclusion from the preceding, it is evident that the best demagnetization is obtained with the slowest reversals. As, however, the curves of Figs. 11, 12, and 13 cut the vertical axis at a value of the induction not far different from the value at a demagnetizing frequency of one cycle per second, we may without sensi-

ble difference and with great economy of time use a frequency of one cycle per second in practice. The fact that the constant value of the apparent induction after the best demagnetization which a 60-cycle current can give is less than the normal induction indicates that something else besides eddy currents is reducing the value of the induction. This something is probably closely related to the phenomenon of magnetic viscosity.

TIME INTERVAL OF DEMAGNETIZATION.

Having determined the proper frequency and limits for the demagnetizing force it now remains to determine the effect of variations in the time interval of demagnetization. With this in view the iron

TABLE XXII.

Showing the nature of the residual polarization effect in common sheet iron caused by the time interval of demagnetization being too brief.

Limits of demagnetising force		15—2	15—2	15—2	15—2	No demagnetisation
Time interval		58.2 or over	33.6	18	7	
Number of cycles		82 or over	53	25	11	
<i>H</i>	Normal induction	Polarisation effects				
1	263	0	15	18	28	166
2	1314	0	26	94	124	818
3	3770	0	50	105	130	640
5	7890	0	25	35	60	160
7	10270	0	0	10	20	140
10	12170	0	0	0	0	10
15	13160	0	0	0	0	0

was demagnetized at a slow rate of reversal and between the demagnetizing limits determined upon above. This was done a number of times, varying the time interval of demagnetization; that is, the number of cycles. The numerical results for transformer iron, common sheet iron, and low carbon steel are given in Tables XXI, XXII, and XXIII. From these data it appears that the polarization effect

is zero for all values of the magnetizing force, provided the demagnetization has taken approximately a minute. If the time is too brief the polarization effect extends over nearly the whole range from the lowest values up to that of the critical demagnetizing force. It increases in magnitude as the time decreases. Too brief a time interval has the same effect as too small a current or too high a

TABLE XXIII.

Showing the nature of the residual polarization effect in low carbon steel caused by the time interval of demagnetization being too brief.

Limits of demagnetizing force		15—.2	15—.2	No demagnetization
Time interval		75 or over	67	
Number of cycles		83 or over	100	
<i>H</i>	Normal induction	Polarization effects		
1	190	0	0	100
2	680	0	5	310
3	1930	0	20	470
4	4150	0	25	480
5	6790	0	40	490
6	9140	0	25	370
7	9990	0	20	250
8	11070	0	0	120
9	11880	0	0	60
10	12470	0	0	10
15	14170	0	0	0

frequency. A comparison of the polarization effect after no demagnetization with that after the briefest interval shows that the greater portion of the polarization effect is wiped out by the first few reversals.

Table XXIV shows the effects of variations in the time interval when the demagnetization is carried on at a somewhat higher frequency. The full effect of the demagnetization while less than before is reached in a shorter interval of time and therefore seems to depend on the number of cycles rather than the time. From this it is

TABLE XXIV.

Showing how the time interval of demagnetization influences the apparent induction.

Demagnetizing frequency=14		
Time interval	Number of cycles	Apparent induction
Annealed transformer iron for $H=.5$		
No demagnetization		480
10	140	1225
14	196	1240
25	350	1265
26	364	1265
47	658	1265
130	1352	1265
273	3822	1265
310	4340	1265
Normal		1290
Low carbon steel for $H=2$		
No demagnetization		370
2.4	34	640
3.2	45	649
4.2	59	660
4.6	64	661
7.4	104	662
18.0	252	662
44.0	616	662
Normal		680
High carbon steel for $H=10$		
No demagnetization		2600
26	364	2960
53	742	2960
126	1764	2960
221	3094	2960
Normal		2980

evident that if the time of demagnetization is kept constant while the frequency is varied, there will be a tendency for the induction to increase as the frequency decreases as long as the time interval is great enough to allow the demagnetizing current to accomplish

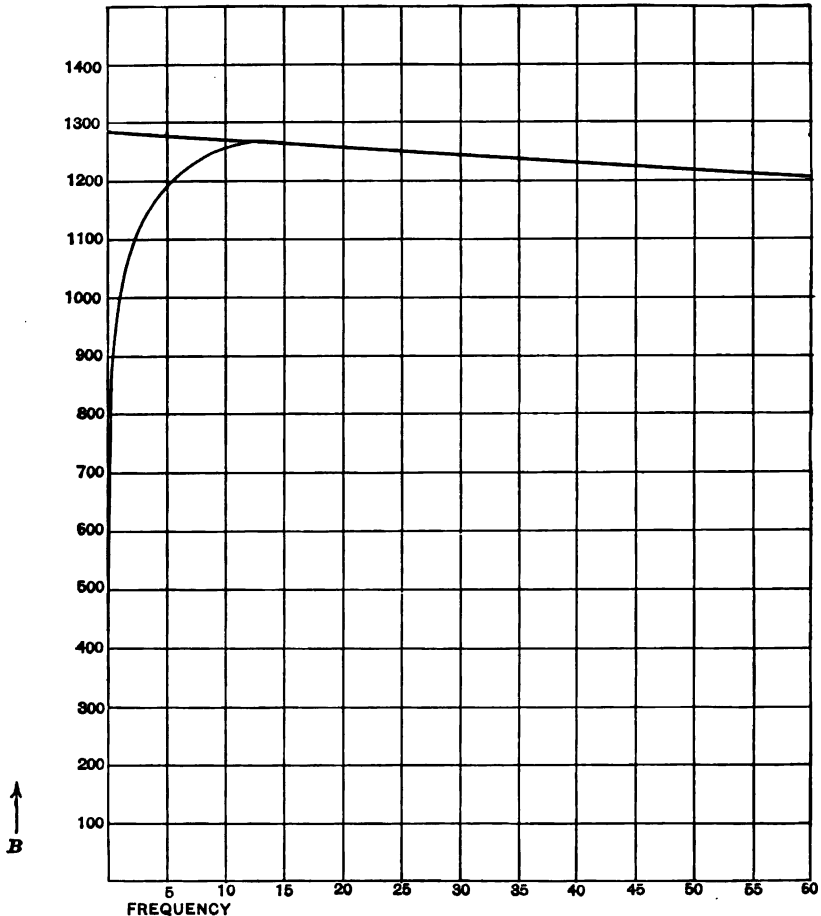


Fig. 16.—Showing the combined effect of frequency and time interval of demagnetization on the apparent induction of transformer iron when $H=0.5$.

its full effect. For the lower frequencies, though a greater efficiency is possible, a longer time is required, and if this is too short the effect of the briefness may exceed the advantage due to the slower frequency and the induction diminishes from a certain point on as

the frequency decreases. This is exactly what happened in one of the earlier experiments of this investigation. A portion of the data obtained is shown graphically in Fig. 16. This curve led to the erroneous conclusion that a frequency of fourteen cycles per second was the best frequency for the demagnetizing current.

THE NORMAL INDUCTION.

After the iron has been thus freed from all traces of previous magnetic polarization it is ready for the ballistic determination.

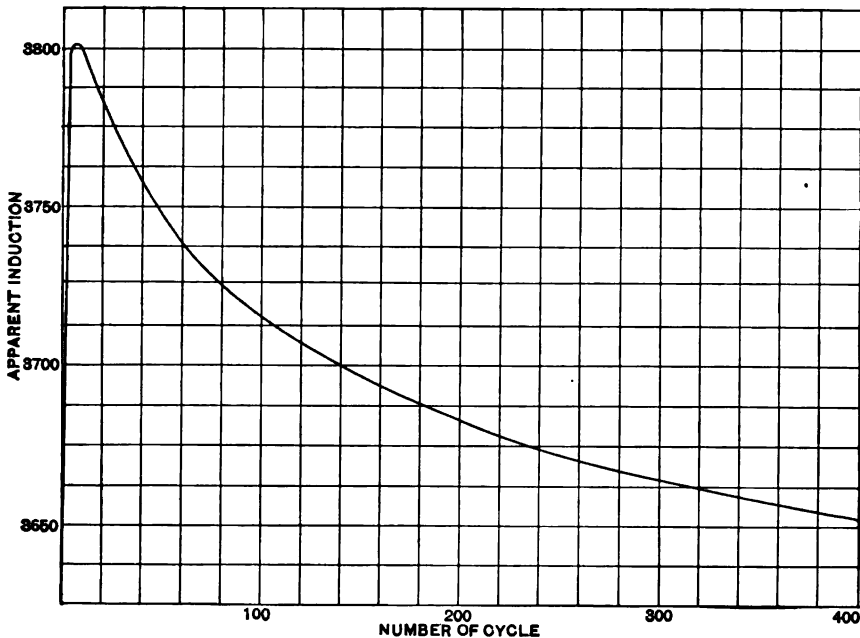


Fig. 17.—Showing the manner in which the apparent induction of an iron ring changes on successive reversals of the magnetizing force.

We have earlier defined true induction as the normal cyclic induction. The reason for this is apparent from the following:

The induction at the point of maximum differential permeability in a bundle of ring stampings of thin annealed transformer iron was determined for successive reversals of the magnetizing force. The results appear in Fig. 17, where the number of the cycle is plotted on the horizontal axis and inductions along the vertical. The ver-

tical scale is greatly enlarged so as to show the characteristics of the curve. This curve shows three characteristic parts. The first few reversals show a rapidly increasing apparent induction, then a maximum value is passed, and finally the apparent induction decreases, at first quite rapidly, then more and more gradually, and finally approaches a lower limit asymptotically. As the test specimen in this case is an endless ring with the magnetizing coil wound

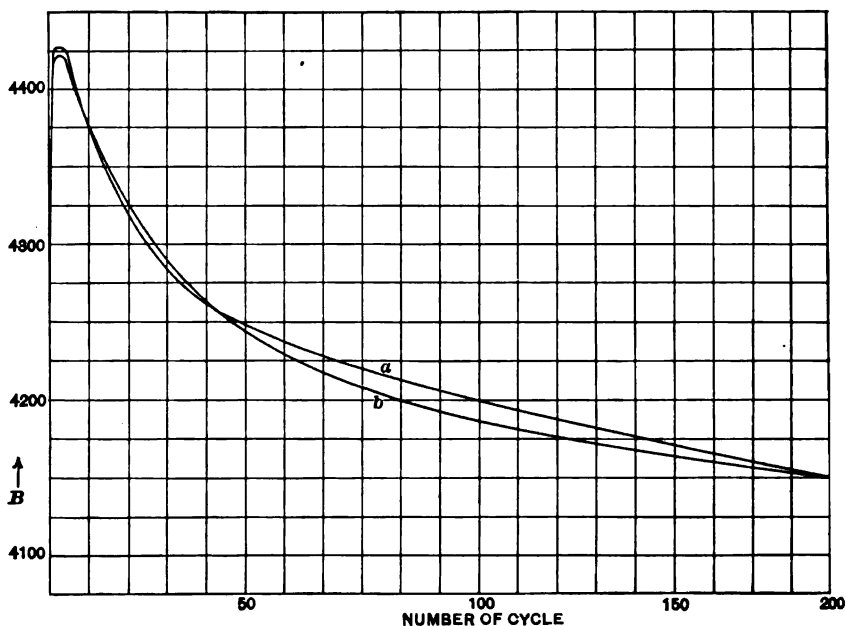


Fig. 18.—Showing the manner in which the apparent induction of low carbon steel changes with successive reversals of the force $H=4$.

(a) with yokes, and (b) without yokes.

uniformly this phenomenon can not be due to any end effect. Similar curves taken with straight rods of low carbon steel are shown in Fig. 18. The curve "a" represents data in which the rods were joined together by yokes, while for curve "b" the rods were not joined but placed as close as the magnetizing coils would allow. These two curves have the same characteristics as the one obtained from the ring. They are so nearly alike that no modifying effect can be attributed to the yokes. To examine more fully the manner in which the cyclic state is reached for various values

of the magnetizing force, complete induction curves were obtained for several values of the magnetizing force after certain numbers of reversals. This was done for annealed transformer iron, low carbon steel, and high carbon steel, and the data are given in Tables XXV, XXVI, and XXVII, and in part are shown graphically in figures 19, 20, and 21. Both from the curves and the summaries at the bottom

TABLE XXV.

Showing the manner in which completely demagnetized transformer iron approaches the cyclic state.

Number of cycle	Apparent induction						
	$H=.5$	$H=1$	$H=2$	$H=3$	$H=5$	$H=10$	$H=15$
1	1300	5350	9100	10850	12800	14620	15280
2	1325	5400	9200	11000	12810	14630	15290
3	1320	5440	9210	11020	12800	14630	15280
4	1320	5470	9200	11020	12780	14630	15280
5	1320	5490	9190	11010	12770	14620	15280
20	1318	5400	9100	10950	12750	14620	15280
50	1310	5300	9030	10880	12750	14620	15280
100	1302	5215	8970	10810	12750	14620	15280
200	1294	5150	8915	10760	12750	14620	15280
400	1292	5140	8890	10760	12750	14620	15280
600	1290	5130	8880	10750	12750	14620	15280
$B_{\max}$	1325	5490	9210	11020	12810	14630	15290
B_{final}	1290	5130	8880	10750	12750	14620	15280
$B_{\max} - B_{\text{final}}$	35	360	330	270	60	10	10
$\frac{B_{\max} - B_{\text{final}}}{B_{\text{final}}}$	.03	.07	.04	.03	.00	.00	.00

Plotted in Fig. 19.

of the tabulated data it is evident that the difference between the maximum apparent induction and its final value increases in all the specimens as the value of H used approaches that of maximum permeability. This difference in apparent inductions divided by the final apparent induction gives a quotient which is a maximum at approximately the point where the differential permeability is a

maximum. No marked difference in the manner in which the various specimens approach the cyclic state can be noticed. Any irregularities that occur in the curve indicating the manner of approach are found during the first few cycles, and are due probably to a

TABLE XXVI.

Showing the manner in which low carbon steel approaches the cyclic state after one initial demagnetization.

Number of cycle	Apparent induction						
	<i>H</i> =1	<i>H</i> =3	<i>H</i> =4.	<i>H</i> =5	<i>H</i> =7	<i>H</i> =10	<i>H</i> =15
1	191	1920	4210	6860	10060	12450	14170
2	192	1930	4240	6890	10070	12510	14180
3	192	1940	4270	6920	10070	12500	14180
4	192	1960	4280	6920	10060	12500	14180
5	192	1970	4290	6925	10060	12490	14180
6	191	1980	4290	6930	10060	12490	14170
7	191	1990	4290	6930	10050	12490	14170
8	190	1990	4290	6930	10050	12480	14170
9	190	1985	4280	6930	10050	12480	14170
10	190	1985	4280	6925	10050	12470	14170
20	190	1980	4260	6905	10040	12470	14170
50	190	1970	4250	6880	10030	12470	14170
100	190	1960	4220	6850	10020	12470	14170
200	190	1950	4190	6810	10010	12470	14170
400	190	1940	4160	6800	9990	12470	14170
600	190	1930	4150	6790	9990	12470	14170
$B_{\max}$	192	1990	4290	6930	10070	12510	14180
B_{final}	190	1930	4150	6790	9990	12470	14170
$B_{\max} - B_{\text{final}}$	2	60	140	140	80	40	10
$\frac{B_{\max} - B_{\text{final}}}{B_{\text{final}}}$	.01	.03	.03	.02	.01	.00	.00

Plotted in Fig. 20.

small residual polarization which is large enough to modify the first reversal but is soon wiped out by continued reversals. Such irregularities might be expected in accordance with the theory of molecular magnets. In view of the preceding experiments it seems quite proper

to fix upon the final minimum value of the apparent induction as the *definition* of the true normal induction. The initial and maximum values are rejected because of their uncertainty and the fact that they can not be verified by repetition without another demag-

TABLE XXVII.

Showing the manner in which demagnetized high carbon steel approaches the cyclic state under various forces.

Number of cycle	Apparent induction					
	$H=1$	$H=2$	$H=5$	$H=10$	$H=15$	$H=40$
1	109	254	790	3130	6780	11660
2	109	254	795	3160	6790	11670
3	109	254	796	3190	6800	11670
4	109	254	795	3190	6805	11680
5	108	254	790	3190	6800	11680
6	108	252	785	3170	6800	11680
7	108	252	785	3150	6800	11670
8	108	250	780	3140	6800	11670
10	108	249	780	3140	6800	11670
20	108	249	780	3130	6790	11670
50	108	249	780	3100	6760	11670
100	108	249	780	3050	6730	11670
200	108	249	780	3020	6700	11670
400	108	249	780	2990	6690	11670
600	108	249	780	2980	6680	11670
800	108	249	780	2980	6680	11670
$B_{\max}$	109	254	796	3190	6805	11680
B_{final}	108	249	780	2980	6680	11670
$B_{\max} - B_{\text{final}}$	1	5	16	210	125	10
$\frac{B_{\max} - B_{\text{final}}}{B_{\text{final}}}$	.01	.02	.02	.07	.02	.00

Plotted on Fig. 21.

netization. These reasons apply whether the induction is to be measured ballistically or magnetometrically, and if results are to be compared by these two methods care must be taken that the inductions have been measured under the same conditions.

ONE DEMAGNETIZATION SUFFICIENT.

It has been suggested by Searle that the specimen should be demagnetized before each determination of an induction. We have already shown that after demagnetization the cyclic induction at

TABLE XXVIII.

Showing the manner in which low carbon steel approaches a cyclic state under various forces, when it has been demagnetized before each new force is applied.

Number of cycle	Apparent induction					
	$H=1$	$H=3$	$H=4$	$H=5$	$H=7$	$H=10$
Make	190	1900	4200	6800	9980	12460
1	191	1920	4250	6890	10000	12500
2	191	1910	4260	6890	10010	12500
3	191	1920	4270	6900	10040	12510
4	192	1950	4290	6900	10080	12510
5	192	1970	4290	6910	10080	12520
6	191	2000	4300	6920	10060	12520
7	190	2010	4300	6920	10060	12510
8	190	2010	4290	6930	10060	12510
9	190	2000	4290	6930	10050	12510
10	190	2000	4280	6930	10050	12500
20	190	1990	4260	6910	10040	12470
50	190	1980	4250	6890	10030	12470
100	190	1960	4230	6850	10020	12470
200	190	1950	4190	6810	10010	12470
400	190	1940	4160	6800	10000	12470
600	190	1930	4150	6790	9990	12470
$B_{\max}$	192	2010	4300	6930	10080	12520
B_{final}	190	1930	4150	6790	9990	12470
$B_{\max} - B_{\text{final}}$	2	80	150	140	90	50
$\frac{B_{\max} - B_{\text{final}}}{B_{\text{final}}}$	.01	.04	.04	.02	.01	.00

any magnetizing force is not affected by any preceding application of smaller forces; but as this is an important detail of manipulation it is desirable to leave no uncertainty. Two sets of data were there-

fore obtained—one for a single demagnetization and another in which the specimen was demagnetized before each measurement of the induction. The two sets of data are given in the Tables XXVI and XXVIII.

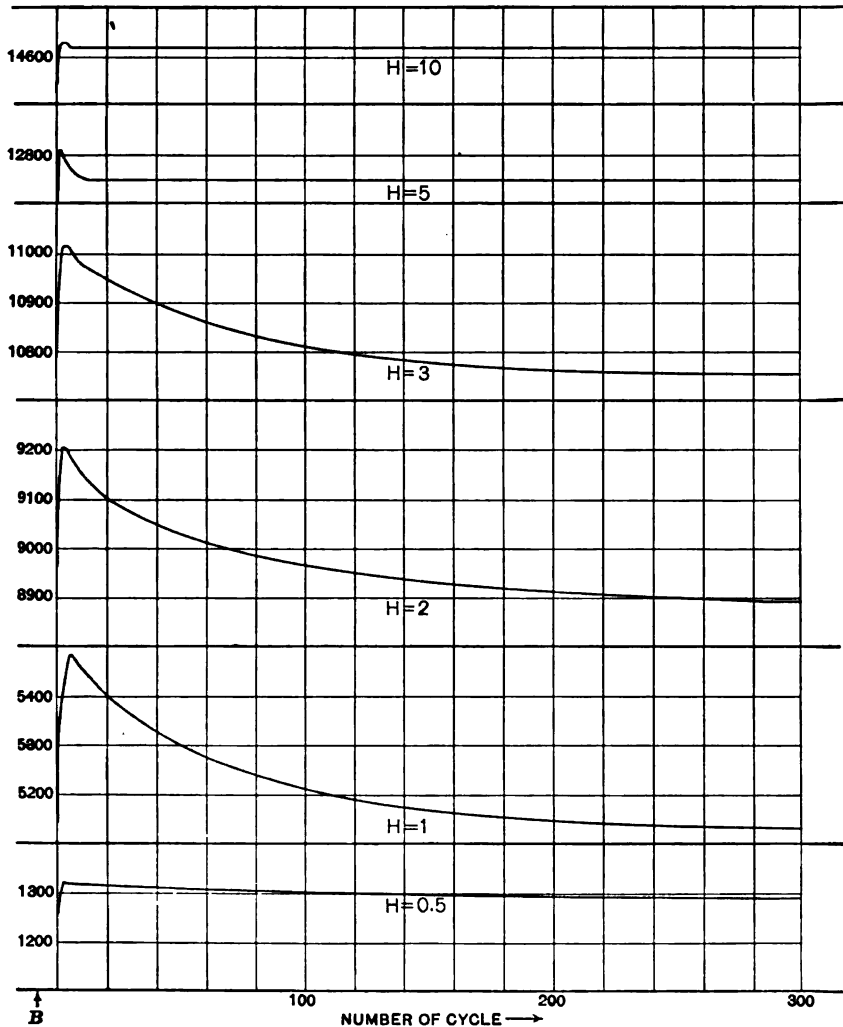


Fig. 19.—Showing the manner in which demagnetized transformer iron approaches the cyclic state for various values of H .

(Numerical data in Table XXV.)

The final values of the induction are the same in each case, and the main characteristics of the manner of approach to the cyclic state

are maintained in each. Whatever differences there are occur in the earlier reversals. This is another argument in favor of the final value as the normal one.

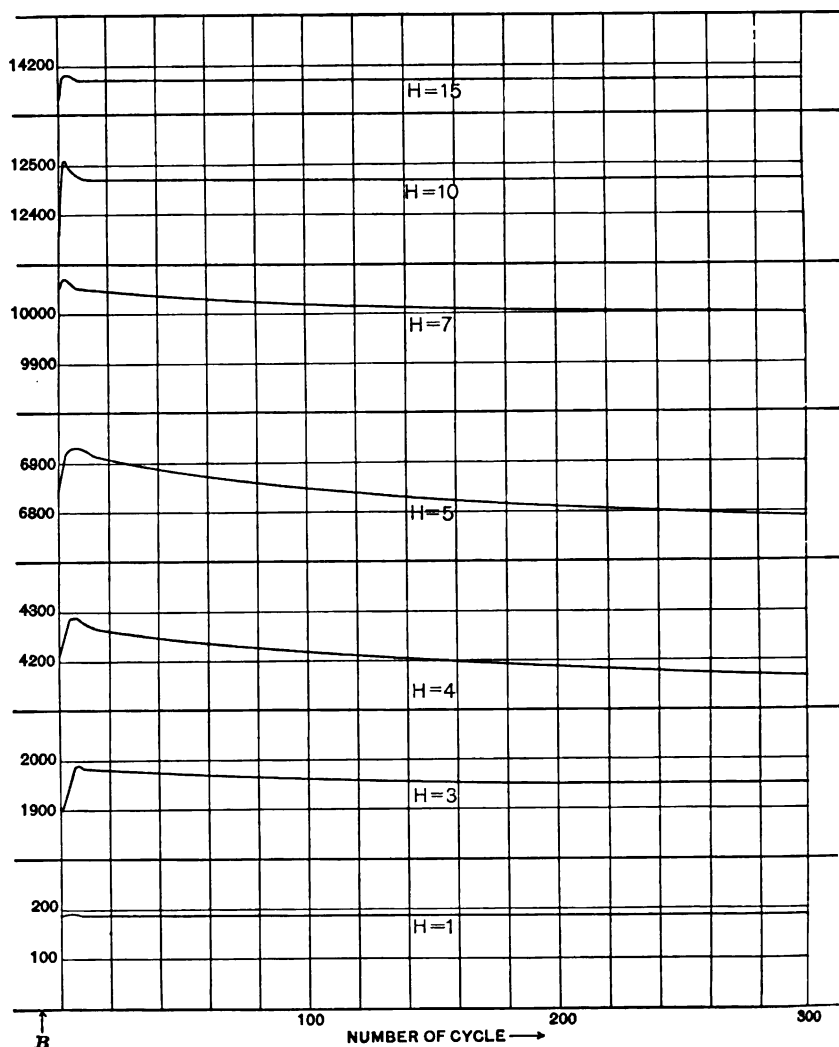


Fig. 20.—Showing the manner in which demagnetized low carbon steel approaches the cyclic state for various values of H .

(Numerical data in Table XXVI.)

EFFECT OF VISCOSITY.

Fig. 22 shows the manner of approach to the cyclic state under two circumstances. The upper curve shows how a specimen of

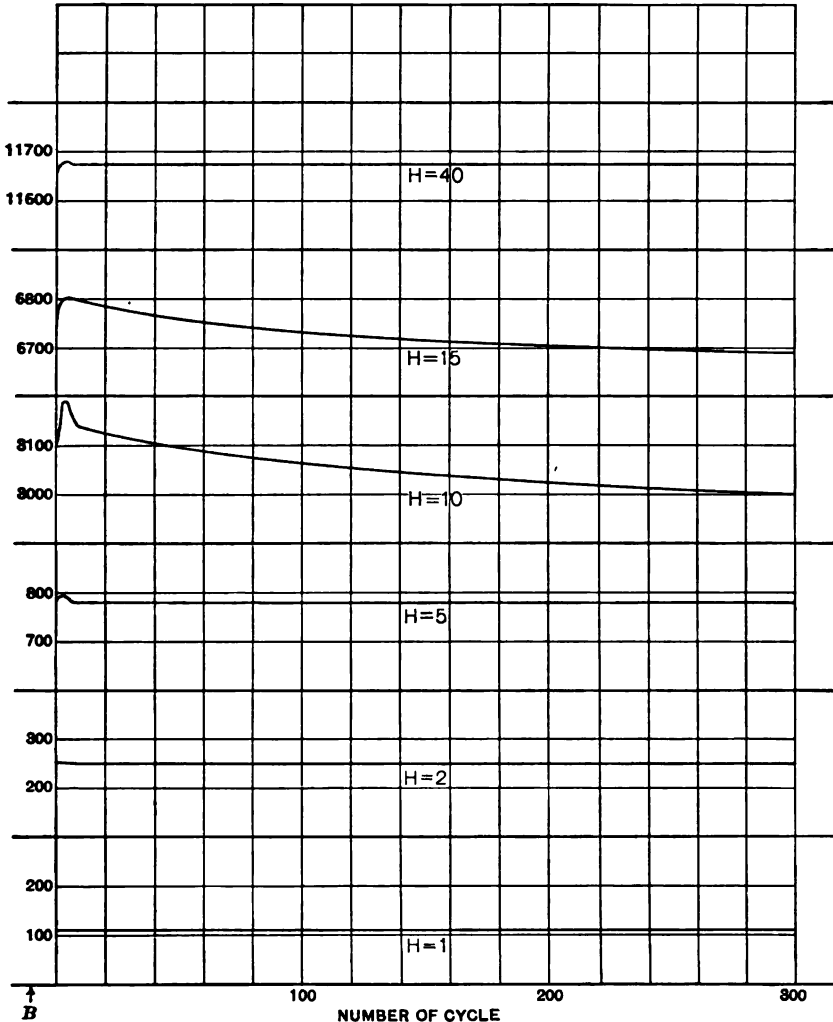


Fig. 21.—Showing the manner in which demagnetized high carbon steel approaches the cyclic state for various values of H .

(Numerical data in Table XXVII.)

high carbon steel comes to a cyclic state on repeated reversals of the magnetizing force by hand switch. The solid portion of the curve

just below this shows apparent inductions as actually observed after imperfect demagnetization. Between the solid portions of this curve reversals were made quite rapidly—several times per second. The dash lines give the hypothetical curve the apparent induction would have followed if the apparent induction had been measured for each reversal and no rapid reversals had been made. There are two causes of irregularities in the curve showing the manner of approach

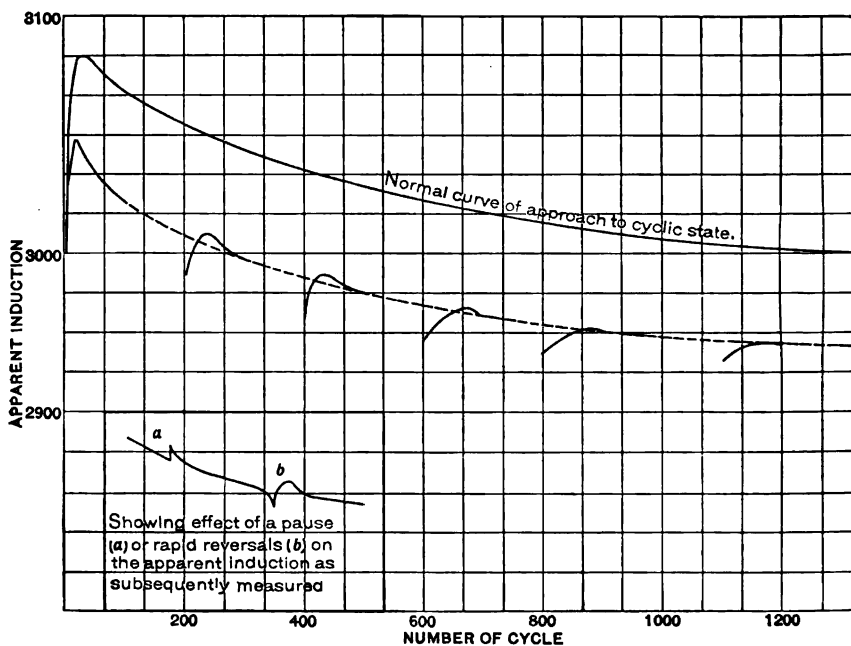


Fig. 22.—Showing how the manner of approach to the cyclic state in high carbon steel is modified by rapid reversals of the magnetizing force and by long pauses.

to the cyclic state. A pause at any point in the curve causes the apparent induction of the succeeding reversal to be a little larger than the one immediately preceding. This is most pronounced in the softer materials and for any specimen is greatest in the steep part of the induction curve and disappears almost completely in the upper parts of this curve. It is noticeable even after the iron has reached a cyclic state under normal conditions. The phenomenon may be attributed to magnetic viscosity. The connection is made clear from the following figure. (Fig. 23). Suppose the iron

has been subjected to forces of $+H$ and $-H$ alternately at the rate of one reversal per second. Then it will trace a hysteresis loop having its vertices at two symmetrical points as A and B. If, however, the reversals are interrupted when the iron is at the point A, the steady application of the force will cause the induction to creep from A to A'. After this creeping has ceased a reversal of the magnetizing force will bring the state point down to B. A further pause will again show a creeping from B down to B', which is symmetric with A'. A magnetometric measurement would take account of this creeping and would measure the induction Oa'. A ballistic measurement however would indicate only the portion Oa, the remainder aa' being lost on account of its slowness. This is on the assumption that a ballistic galvanometer with sufficient torsion in its suspension to bring its movable system back to zero is used. An instrument whose movable system has no directive force, such as the Grassot flux-meter, would measure the full induction as completely as the magnetometer. However, even with an ordinary ballistic galvanometer this full induction

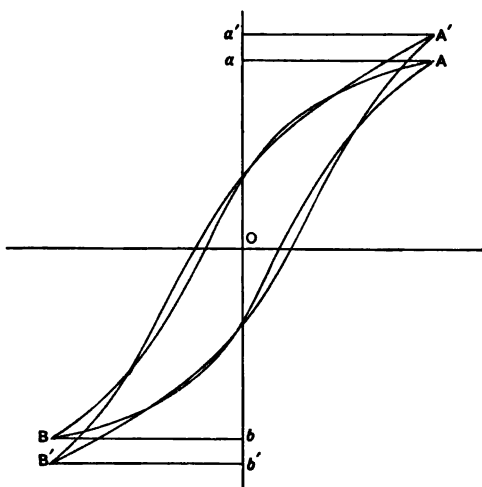


Fig. 23.—Ideal curve to illustrate the effect of magnetic viscosity on the apparent induction.

can be determined. Let a ballistic deflection be taken when the iron is traversing the cycle AB. This will be proportional to ba . After a pause take the deflection on the return reversal. This will be proportional to $a'b$. The second deflection exceeds the first by aa' , which is the amount of creeping at one end of the cycle. The full induction would therefore be proportional to $ba + 2aa'$. As this creeping is greatest in the steep part of the $B-H$ curve a single determination here will tell whether it is of sufficient magnitude to warrant consideration. In some of the irons measured it has amounted to as much as 1 per cent.

Another irregularity in the curve showing the manner of approach to the cyclic state is the peculiar effect of several rapid reversals. After such treatment the first inductions are too low, then the succeeding ones are too high, but after some slow reversals the inductions show no sign of irregularity. The two kinds of irregularity are shown graphically in the lower corner of Fig. 22.

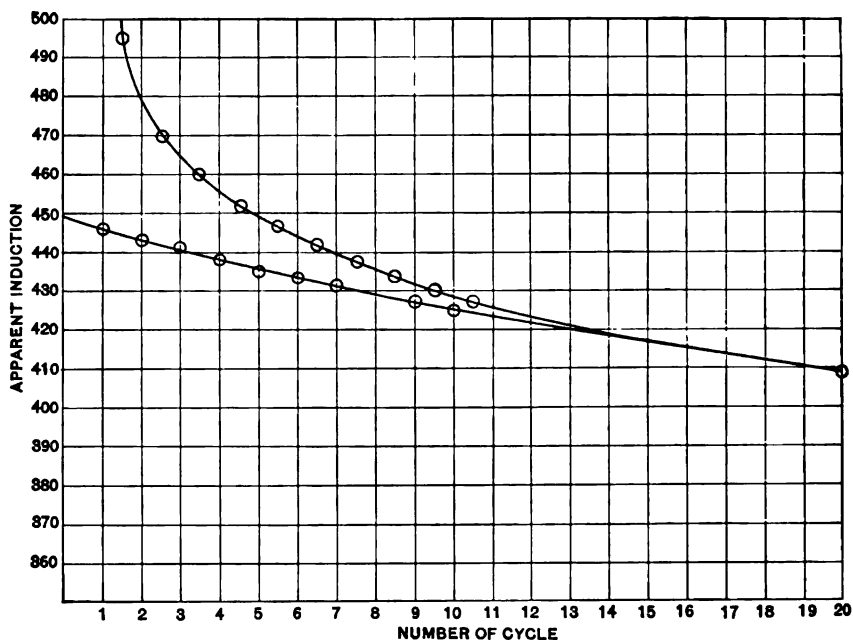


Fig. 24.—Showing the manner in which a highly polarized bar of low carbon steel approaches the cyclic state on repeated reversals of a force $H=2$.

(Numerical data in Table XXIX.)

EFFECT OF REPETITION.

One might suppose that a sufficient number of reversals of the magnetizing force would obviate the necessity of demagnetization. This is not the case, however, as is seen from the manner in which a specimen of low carbon steel approaches the cyclic state after intense polarization by a force of 100. The data for this experiment are shown numerically in Table XXIX and graphically in Fig. 24. The inductions on successive reversals do not form a smooth curve as those obtained after complete demagnetization would, but

those inductions obtained by reversing the magnetizing force from the direction in which the strong polarization was applied are greater than those due to either the preceding or the following reversals in the opposite direction. Thus Fig. 24 shows two lines, converging after about twenty reversals. As the reversals continue the apparent induction becomes gradually smaller, differing

TABLE XXIX.

Showing the manner in which a highly polarized piece of low carbon steel approaches the cyclic state.

Polarizing force, 100; Cyclic force, 2			
Number of cycle	Apparent induction	Number of cycle	Apparent induction
$\frac{1}{2}$ (make)	2500	$7\frac{1}{2}$	438
1 (reverse)	446	8	429
$1\frac{1}{2}$ (reverse)	495	$8\frac{1}{2}$	437
2	443	9	427
$2\frac{1}{2}$	470	$9\frac{1}{2}$	430
3	441	10	425
$3\frac{1}{2}$	460	$10\frac{1}{2}$	427
4	438	20	409
$4\frac{1}{2}$	452	$20\frac{1}{2}$	409
5	435	50	388
$5\frac{1}{2}$	447	100	376
6	433	200	370
$6\frac{1}{2}$	442	1200	370
7	431	Normal induction	680

Plotted on Fig. 24.

more and more from the normal induction. After 200 cycles (400 reversals) the apparent induction has reached a minimum which a thousand double reversals do not alter. In another case a bar of polarized iron was subjected to slow reversals of the magnetizing force for four hours and yet showed no increase in the apparent induction.

STRONG VIBRATIONS.

While the study of magnetism was yet in its infancy Gilbert discovered that a soft iron bar could be quite strongly magnetized by

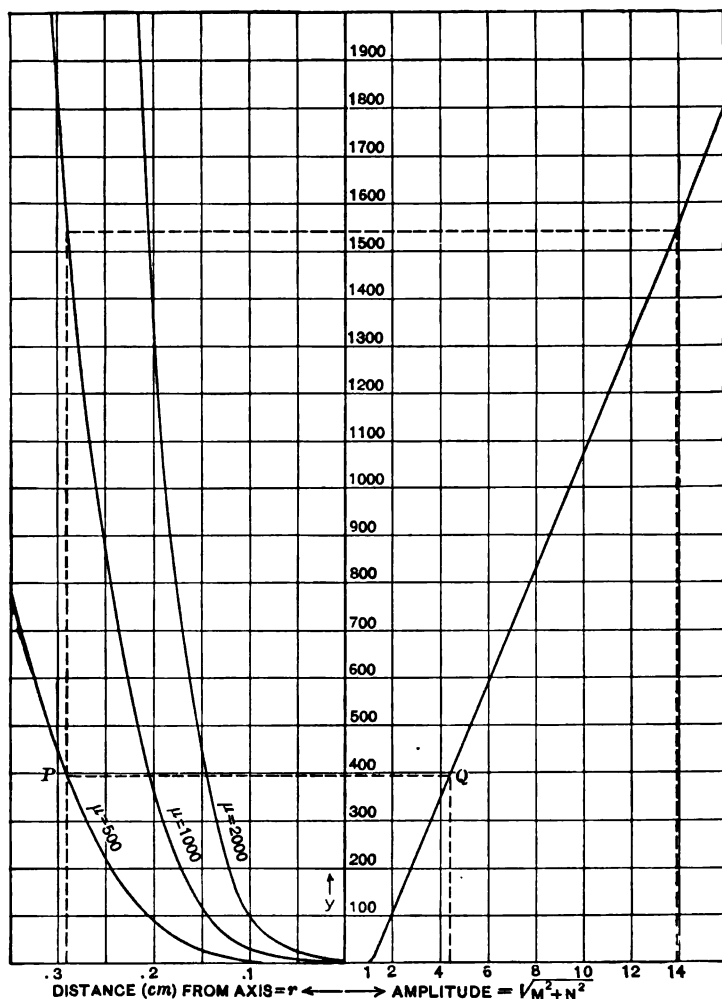


Fig. 25.—Theoretical curves based on Heavisides's formula showing how the amplitude of an alternating magnetic flux varies over the cross section of a round rod for various assumed values of the permeability.

holding it in the earth's field and striking it a sharp blow with a hammer. A soft iron wire placed in the terrestrial field and gently

rubbed takes up a magnetization far greater than its permeability determined in the usual way would indicate. Equally gentle rubbing will remove the major portion of the most intense residual magnetization. Ewing¹⁸ has investigated quite thoroughly the effect of intense mechanical disturbances on the magnetic state of a bar of

TABLE XXX.

Showing the increase of induction in soft iron due to very gentle vibration.

<i>H</i>	<i>B</i> Normal	<i>B</i> With tapping	Increase of <i>B</i> due to tapping	Per cent increase of <i>B</i>
.2	200	211	11	5
.3	370	395	25	7
.4	600	645	45	7
.5	950	1018	68	7
.6	1445	1542	97	7
.7	2400	2525	125	5
.8	3400	3535	135	4
.9	4200	4340	140	3
1.0	4850	4985	135	3
1.1	5400	5525	125	2
1.2	5900	6010	110	2
1.3	6350	6440	90	1
1.4	6700	6770	70	1
1.5	7050	7100	50	1
1.6	7350	7380	30	0
1.7	7600	7620	20	0
1.8	7850	7860	10	0
1.9	8100	8105	5	0
2.0	8300	8300	0	0

Plotted on Fig. 26.

iron. The effect of vibration is to shake in more induction if the magnetic force is acting, and to shake out the residual induction if no magnetic force is acting. The hysteresis loop is contracted and draws in close to the induction curve, which loses its point of inflection and apparently starts out with a maximum permeability which

¹⁸ Magnetic Induction, etc., pp. 77 and 84.

steadily decreases. While the increase in permeability is appreciable in moderately strong fields, it is in weak fields that the effect is most striking. In the case recorded by Ewing and which we have no

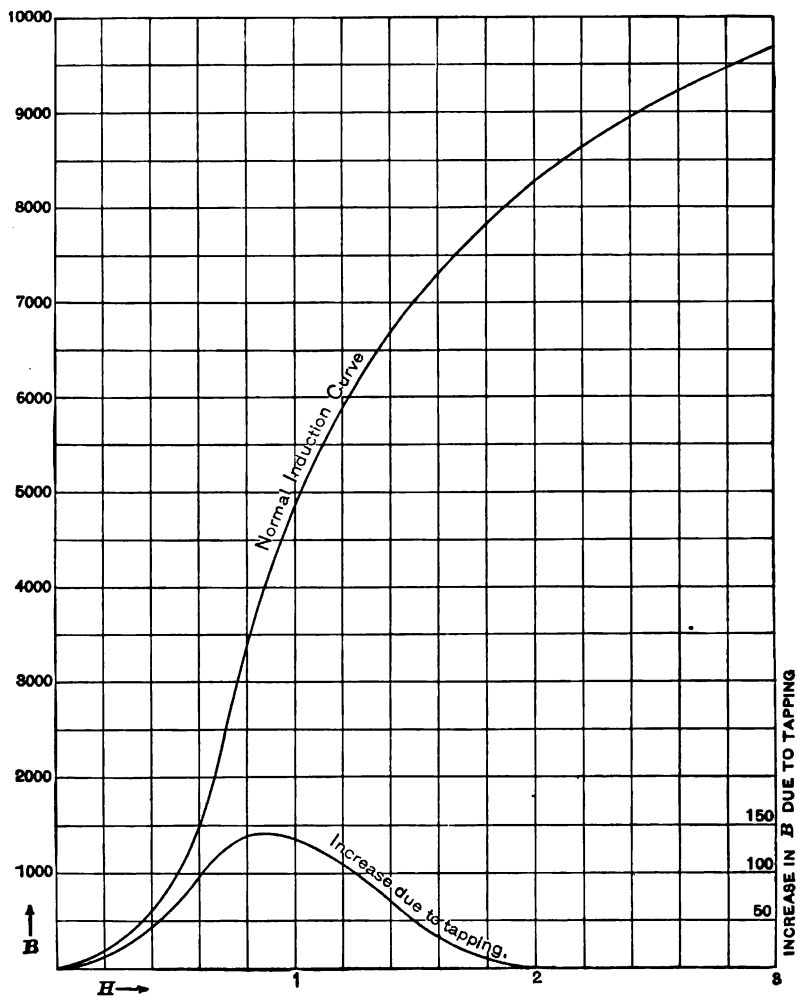


Fig. 26.—Showing the influence of gentle vibrations on the inductions in soft iron.

(Numerical data in Table XXX.)

reason to suppose was unusual the permeability as measured during a mechanical vibration assumed the enormous value of 80,000. These same phenomena are observed in hard iron and steel, but to a much less degree.

GENTLE VIBRATIONS.

The following experiment was undertaken to determine whether the ordinary vibrations of a building containing moving machinery, such as a reciprocating engine, affect the values obtained in a magnetic test. The gong was removed from a small electric bell and the electromagnet mounted so that the hammer might strike against one end of a slender wooden rod 50 centimeters long, the other end of which rested against one of the yokes containing the test specimen. The wooden rod was interposed partly to deaden the effect of the blows delivered by the hammer and partly to permit the removal of the electromagnet to such a distance that its field would not react on the test specimen. To diminish the vibration still further a piece of soft rubber tubing was slipped over the hammer and another piece fastened to the end of the wooden rod. To insure a separation of the natural vibrations of the building from those of the experiment, the magnetic system under test was mounted on several layers of heavy wool felt. As nearly as could be estimated, by placing the hand alternately on the apparatus under test and on the table, the tapping device gave a disturbance of the same order of magnitude as the vibration of the building. With this arrangement the data of Table XXX were obtained. The first column gives the magnetizing force; the second, the value of the induction after the iron had been subjected to many reversals to insure a cyclic state; the third, the induction obtained while the iron was under vibration; the fourth, the increase due to vibration, and the last column gives the proportional increase. These data are shown graphically in Fig. 26. It is to be noticed that the effect of vibration, both in absolute value and in percentage, passes through a maximum at approximately the point where the differential permeability is a maximum. The maximum value of 7 per cent in the disturbing effect shows that for accurate work it is necessary to protect the apparatus from vibration. Even the slamming of a door at one time and the dragging of a heavy box along the floor at another have modified the induction by a measurable amount.

Small changes in the intensity of the mechanical disturbance were accompanied by correspondingly small changes in magnetism, so that the curve may be taken as that characteristic of small disturbances.

To investigate more fully the nature of the effect of these mechanical vibrations the specimen was subjected without vibrations to many reversals of a small force until a cyclic state ensued and the ballistic throw on reversal noted. Next, without altering the current, the vibrator was started. This produced a small deflection. The return reversal was then taken without vibration, then another reversal without vibration, then the tapping, and lastly a reversal while the tapping was in progress. The following table shows the results:

Operations performed		Galvanometer
Magnetic.	Mechanical.	Deflection.
Reversal.	None.	+7.00
None.	Tapping.	+ .23
Reversal.	None.	-7.28
Reversal.	None.	+7.00
None.	Tapping.	+ .29
Reversal.	Tapping.	-7.47

The small deflection produced by tapping after a reversal had been made was the result of the first few taps, for an instantaneous closing of tapping circuit produced the full effect which was not increased by repeated and continued tapping. It is also characteristic that a single reversal wipes out the effect of previous tapping so that the second reversal is normal. What occurs may be shown more clearly by the use of Fig. 28. Here A represents the magnetic state before the first throw. The first reversal brings the state point to B. Tapping runs it up to D. A reversal without tapping brings it back to A. Tapping runs it down to C. Reversal with tapping carries it to D. So that simple reversals cause the state point to traverse cycle AB. Reversals with taps give cycle CD. Reversals followed by taps give cycle CBDA.

A comparison of these effects with those noted under the study of viscosity shows a close connection between the creeping up of the induction due to viscosity and the same thing for tapping. Both these phenomena may be accounted for by assuming a frictional force opposing the movements of the molecular magnets. It was found that such gentle vibrations as these were of little help in demagnetizing either when used alone or with current.

For higher inductions the effect of tapping was appreciable but negligible.

The persistence of previous polarization, as modified by a slight tapping, is shown in the data of Table XXXI. Here the iron has been polarized and then subjected to reversals of a small force till a cyclic condition exists. A few taps raise the apparent induction, which then decreases to a second constant value somewhat higher

TABLE XXXI.

Showing the demagnetizing effect of tapping combined with repeated reversals.

Number of reversal	Galvanometer deflection
1	9.80
2	6.19
3	5.99
4	5.81
6	5.71
8	5.66
10	5.62
20	5.62
100	5.61
200	5.62
202	5.80 after gentle tapping for 15 seconds
204	5.86 " " " " 1 minute
206	5.89 " " " " 2 "
306	5.89 " " " " 100 cycles
406	5.69
500	5.69
1800	5.69

than the first. The gentle vibration has therefore removed a portion of the residual induction, but is not an efficient means of demagnetization.

From the preceding experiment it appears desirable that the magnetic system be protected from small mechanical vibrations. A pad of felt half an inch thick accomplishes this very nicely for all ordinary cases.

INFLUENCE OF TEMPERATURE.

It is well known that for a small rise of temperature above room values the permeability of iron increases for low inductions and

TABLE XXXII.

Showing the induction at various temperatures of an annealed transformer iron ring.

H	B_4 $t=8$	B_{26} $t=26.5$	B_{48} $t=48$	$B_{48}-B_8$	$\frac{B_{48}-B_8}{40}$	$\frac{B_{48}-B_4}{40 B_8}$
0.5	473	495	516	43	+1.1	.0023
1.0	3317	3479	3621	304	+7.6	.0023
2.0	8810	8970	9170	360	+9.0	.0010
3.0	11560	11700	11870	310	+7.7	.0007
5.0	14330	14460	14510	180	+4.5	.0003
10.0	16440	16440	16440	0	0	.0000
15.0	17150	17160	17180	30	0	.0000

See Fig. 27.

TABLE XXXIII.

Showing the inductions at various temperatures of an unannealed transformer iron ring.

H	B_{12} $t=12$	B_{47} $t=47$	$B_{47}-B_{12}$	$\frac{B_{47}-B_{12}}{35}$	$\frac{B_{47}-B_{12}}{35 B_{12}}$
0.5	290	306	16	.5	.0017
1.0	1081	1148	67	1.9	.0018
2.0	4670	4800	130	3.7	.0008
3.0	7225	7310	85	2.4	.0003
5.0	10290	10340	50	1.4	.0001
10.0	13930	12920	-10	-.3	.0000
15.0	15630	15590	-40	-1.1	.0000

See Fig. 27.

decreases for high inductions. At higher temperature this temperature coefficient is always negative and increases rapidly with the temperature, until finally the iron becomes practically nonmagnetic

at some temperature between 690°C. and 870°C. For the ordinary fluctuations of room temperature very little experimental work has been done.

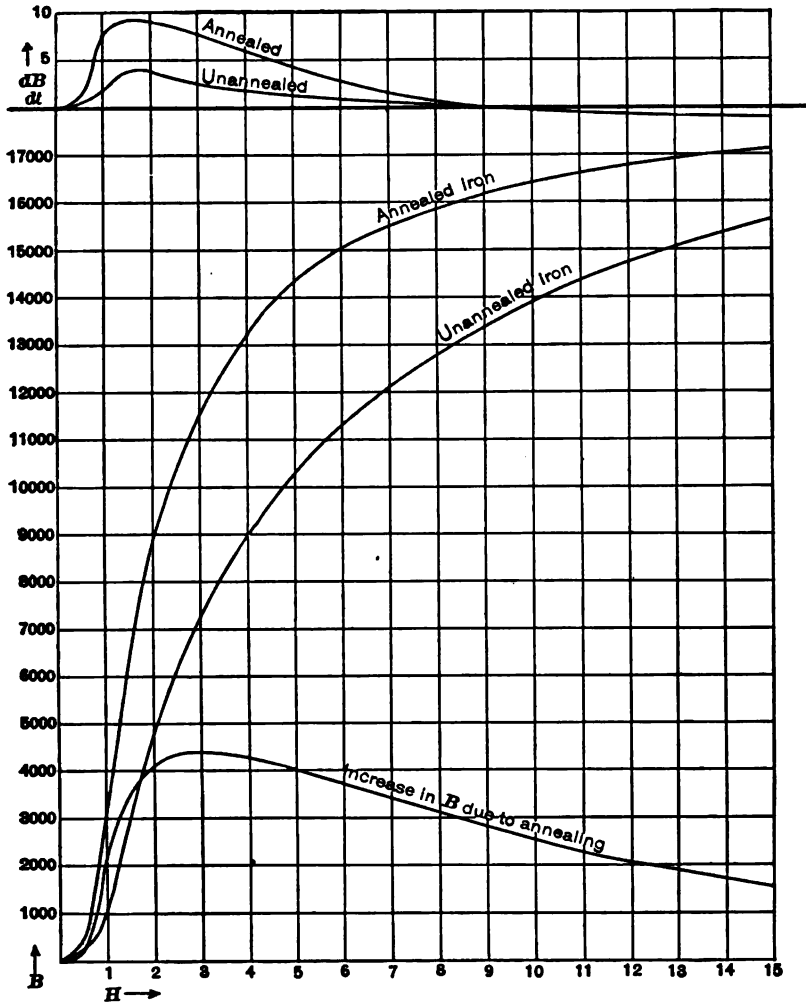


Fig. 27.—Showing the effect of temperature and annealing on the induction in soft iron.

(Numerical data in Tables XXXII, XXXIII, and XXXIV.)

Ewing,¹⁹ from data obtained at 7° and 100°C. , concluded that atmospheric fluctuations need not be considered in magnetic meas-

¹⁹ Magnetic Induction, etc., p. 178.

urements. Permanent magnets such as are used in magnetic surveys and in electrical measuring instruments have a negative temperature coefficient of the order of .05 per cent. Whether or not the temperature coefficient is of sufficient importance to warrant its consideration is a question of the degree of accuracy desired. It seemed worth while therefore to determine the temperature coefficients of a few specimens over the atmospheric range. Four different specimens of transformer iron were examined at different temperatures. A portion of the data is shown numerically in Tables XXXII,

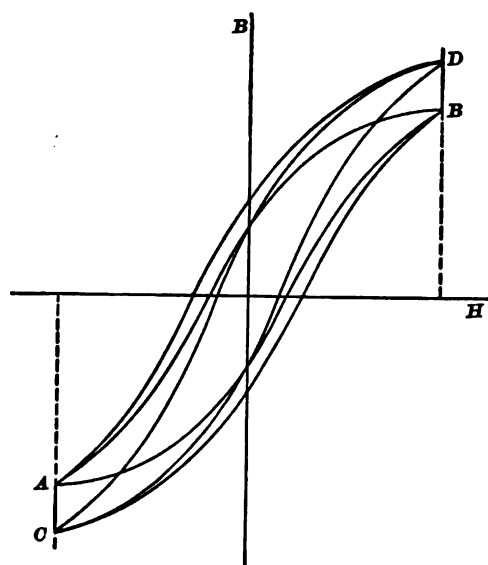


Fig. 28.—Ideal curve to illustrate the effect of vibration on the apparent induction.

XXXIII, and XXXIV, and graphically in Fig. 27. These specimens were two bundles of stamped transformer iron rings about 10 centimeters in diameter, cut from the same sheet. One was annealed and the other not. From the data and curves the following observations may be made:

(1) The change in induction per 1° C. rise in temperature is greatest in the neighborhood of the maximum permeability.

(2) The temperature coefficient is greater for the annealed than for the unannealed iron.

(3) The high value of this maximum temperature coefficient (one-fourth per cent) shows that temperature must be taken account of if an accuracy of 1 per cent is to be attained.

(4) The increase of induction due to annealing is greatest near the point of maximum permeability.

Throughout this work one is impressed with the varying sensitiveness with which the different parts of the B - H curve are influenced by slight changes in the conditions. All those elements, such as polarization, temperature, viscosity, or vibrations, which

tend to modify the apparent induction produce their greatest effects in the steep part of the induction curve. Beyond the critical point the induction is independent of previous magnetization and but lightly influenced by vibrations and temperature changes. Greater sensitiveness is noticed in the soft irons than in the hard ones. The difficulties of permeability measurements, therefore, are found in the earlier part of the induction curve, and it is here that the greatest care must be taken and the largest tolerance allowed.

TABLE XXXIV.

Showing the effect of annealing on the induction in transformer iron (Temperature 10°C). Calculated from Tables XXXII and XXXIII.

H	Annealed = B_a	Unannealed = B_u	$B_a - B_u$	$\frac{B_a - B_u}{B_a}$
0.5	475	289	186	.39
1.0	3332	1078	2254	.67
2.0	8830	4660	4170	.47
3.0	11580	7220	4360	.38
5.0	14340	10290	4050	.28
10.0	16440	13930	2510	.15
15.0	17150	15630	1520	.09

See Fig. 27.

THE BEST METHOD OF PROCEDURE.

In view of evidence offered in the preceding pages the following rule may be outlined as the best (ballistic) method of procedure in magnetic testing:

The magnetic system, consisting of the test pieces and the connecting yokes, should be mounted with its plane perpendicular to the earth's field. If necessary the system should be protected from mechanical vibrations by means of a pad of felt, or something equivalent. If an accuracy of 1 per cent in the steep part of the $B-H$ curve is desired the temperature should be kept at some standard temperature (e. g., 20° C.) constant to 1° or 2° C. It is not feasible to apply a temperature coefficient correction on account of the difficulty in getting its value.

The demagnetization should be accomplished by a current reversed at the rate of approximately one cycle per second, while gradually diminished in such a way that the rate of decrease of the induction is as nearly as may be uniform. An ammeter in circuit and a rough estimate of the shape of the B - H curve will enable one to regulate the rate of decrease of current with sufficient exactness. The initial demagnetizing current should be sufficient to carry the induction beyond the knee of the B - H curve,²⁰ and the final current should be not greater than the smallest magnetizing current to be used.

The full demagnetization may be accomplished in about ninety seconds.

Now apply the lowest magnetizing force desired and reverse at the same speed as in demagnetizing. At intervals get a ballistic deflection. Continue this until two deflections about twenty-five reversals apart show only a negligible difference²¹. This final deflection is the normal induction.

Without demagnetizing again, apply the next larger magnetizing force and reverse as before. Continue in this way till all the required points on the curve have been obtained.

In closing I wish to acknowledge my indebtedness to Professor Rosa, under whose supervision the work has been done, for his continued advice and suggestions.

WASHINGTON, September 3, 1907.

²⁰ Without going to the trouble of a preliminary test an initial demagnetizing force of 15 units may safely be assumed for all specimens of soft iron.

²¹ Several hundred reversals may be required in the steep part of the B - H curve, while half a dozen are sufficient in the upper portions.

A DEFLECTION POTENTIOMETER FOR VOLTMETER TESTING.

By H. B. Brooks.

1. INTRODUCTION.

In a former paper¹ the writer derived an expression for the current through the galvanometer of an unbalanced potentiometer, used in connection with a volt box, and showed how this formula could be applied to the design of a class of instruments, for current and voltage measurements, having properties intermediate between those of the balance type and of the deflection type. A description was given of an instrument constructed on this principle, which was used for measuring voltage in photometric work, and which has now been in service for the past two years, giving satisfactory speed and accuracy.

The object of the present paper is to give some further modifications of circuits which may be used for this purpose, with their advantages and defects; to show what features should be incorporated in an instrument of this kind for such work as the testing of voltmeters, and to describe an instrument of this type recently constructed. This has been designed for voltmeter testing and other precision measurements of voltage in the laboratory and testing room.

2. A STANDARD INSTRUMENT FOR VOLTMETER TESTING.

An instrument to be used as a standard for voltmeter testing should first of all be accurate, permanent, and unaffected by magnetic fields and similar disturbing influences. This requires that the bulk of the unknown voltage be measured by the potentiometer method, as no known type of voltmeter, not even so-called "laboratory standard" instruments, will meet the above requirements. In order

¹This Bulletin, 2, p. 225; 1906.

to save time and avoid the necessity for extreme steadiness of the test voltage, a small portion of the quantity should be measured by the deflection of a galvanometer. This galvanometer should be a pivoted instrument, of good working qualities, and should have a sensibility such that one division of its scale corresponds to say one-tenth to one-fifth of a scale division of a voltmeter of the same maximum range as the potentiometer. This gives great ease of reading, as it is usually sufficient to read the galvanometer to the nearest division, neglecting tenths. These requirements indicate a deflection potentiometer, and as a number of variations are possible, in the arrangement of circuits for such an instrument, these various plans will be examined with a view of determining the one which is the most suitable.

It is desirable that the standard instrument be semiportable, for while such an instrument is commonly used in one place, it is often necessary to make precision measurements in other places on relatively short notice. In the case of the deflection potentiometer, by building the pivoted galvanometer in as a part of the apparatus, the only accessories are the storage battery and the standard cell; the former must be carried separately, and preferably the latter also.

3. ARRANGEMENTS OF CIRCUITS.

In plan 1 (see page 231, former article) the volt box ratio is constant, and the setting is made on the potentiometer wire. The arrangement of circuits may be seen from Fig. 4 of this article by omitting the resistance r_6 ; the symbols used correspond with those of the previous article. The difficulty with this plan lay in the fact that the compensating resistance in the galvanometer circuit, r_4 , was a function of two variables, the resistance of the potentiometer wire up to the setting, r_1 , and the regulating rheostat in the storage cell circuit, r_3 , no satisfactory way of providing this double compensation being available. To avoid this difficulty plan 2 was devised, in which r_1 is a constant; r_4 therefore compensates only for changes in Σr (the total resistance in the galvanometer circuit) due to variations of r_3 . The setting is then made on the volt box, and a compensating resistance r_5 is arranged to take care of variations due to changes of the setting. This plan was adopted for the first instrument of this type, in which the range was from 95 to 125 volts.

A third plan, suggested by Dr. M. G. Lloyd, is like plan 2, except that instead of a constant total resistance R between the emf. terminals of the volt box, with the drop taken from a variable fraction $\frac{1}{p}$ of this, the drop is taken from a fixed resistance and R is variable, as shown in Fig. 1.

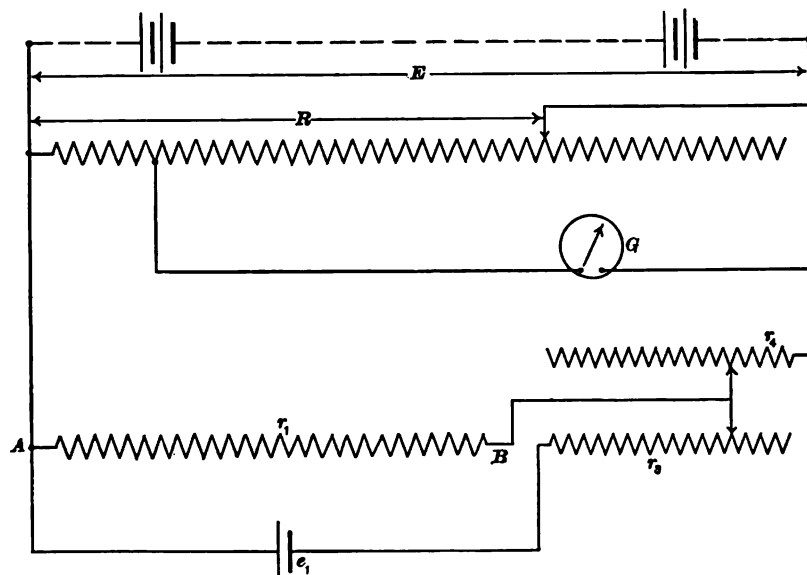


Fig. 1.—Third plan of circuits.

The equations for this case are the same as for plan 2. $R \frac{p-1}{p^2}$ is more nearly constant in plan 3 than in plan 2. A serious disadvantage of plan 3 is the fact that R , the resistance of the volt box, is low on low settings. If the instrument reads down to a small fraction of the maximum, and through mistake the maximum pressure is applied while the dial is set at or near the minimum, a current many times greater than normal will flow through the part of the volt box in circuit, with consequent damage.

Plans 2 and 3 have the disadvantage of variable damping of the galvanometer. For an instrument of limited range (as 95 to 125 volts) this disadvantage practically disappears. By designing the coils so that the value of Σr for the mean setting is equal to the total resistance for critical damping of the galvanometer, the damp-

ing will still be such as to give good working, even at the extremes of the range. If, however, we are to cover a wide range with an instrument using plan 2, it is necessary to modify the circuits so as to preserve critical damping over the whole range. This may be accomplished by the arrangement shown in Fig. 2.

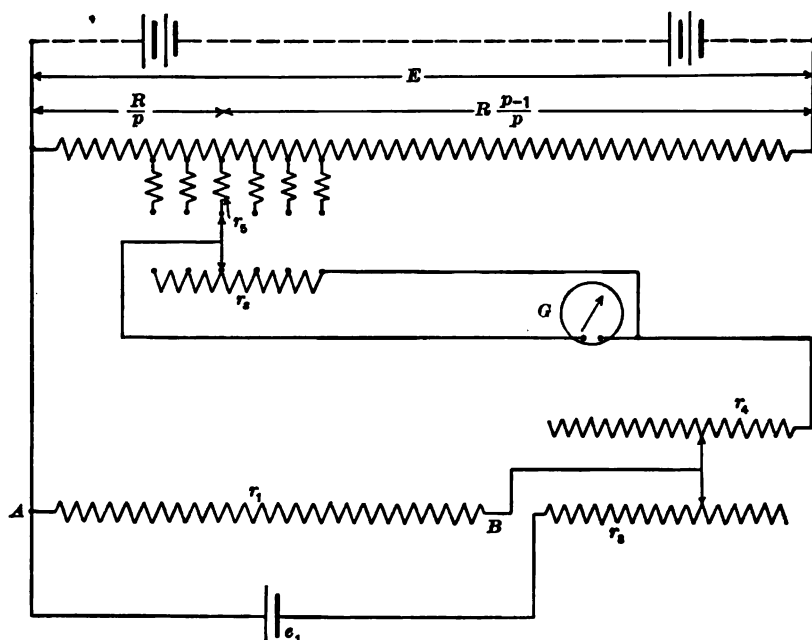


Fig. 2.—Fourth plan of circuits.

This arrangement is the same as plan 2, with the addition of a variable shunt r_s to the galvanometer, which is changed at the same time as the compensating resistance r_c . Since r_s and r_c are independent variables, we may impose two conditions: (1) The current through the galvanometer, for all settings, must be such that one volt ΔE (difference between p times the setting and the applied voltage E) will give a deflection of m scale divisions; (2) the resistance of the damping circuit must be constant and equal to r_a , the total resistance for aperiodic damping. In the former article it was shown that if Σr is the total resistance in the galvanometer circuit, (neglecting all electromotive forces) $\frac{1}{p}$ the fractional part of the volt box from which the drop is taken, m the number of scale

divisions which the galvanometer is to give for one volt unbalanced emf. in the pressure to be measured, the value of Σr is given by the equation

$$\Sigma r = \frac{I}{pm I} \quad (11)$$

If the current I is required by the unshunted galvanometer for one scale division, when the shunt r_s is applied this is increased to

$$\frac{r_s + r_g}{r_s} I$$

By equation (11) we may write for this case

$$\Sigma r = \frac{r_s}{pm I (r_s + r_g)} \quad (12)$$

For convenience we may denote the resistance in the galvanometer circuit, aside from r_s and r_g , by r' ; or

$$r' = r_1 + \frac{r_1 r_2}{r_1 + r_2} + R \frac{p-1}{p^2} + r_3$$

The damping circuit is then as shown in Fig. 3, and for critical damping

$$r_g + \frac{r_s r'}{r_s + r'} = r_a \quad (13)$$

also,

$$\Sigma r = r' + \frac{r_s r_g}{r_s + r_g} \quad (14)$$

By solving these equations we get

$$r_s = \frac{r_a - r_g}{1 - pm I r_a} \quad (15)$$

$$r' = \frac{r_a - r_g}{pm I r_a} \quad (16)$$

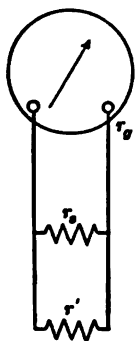


Fig. 3.

Since

$$r' = r_1 + \frac{r_1 r_2}{r_1 + r_2} + R \frac{p-1}{p^2} + r_3$$

$$\frac{r_a - r_g}{pmIr_a} = \text{constant} + R \frac{p-1}{p^2} + r_s$$

$$r_s = \frac{r_a - r_g}{pmIr_a} - R \frac{p-1}{p^2} - \text{constant}.$$

By means of these formulæ we may determine the values of r_s and r_g for a given case.³

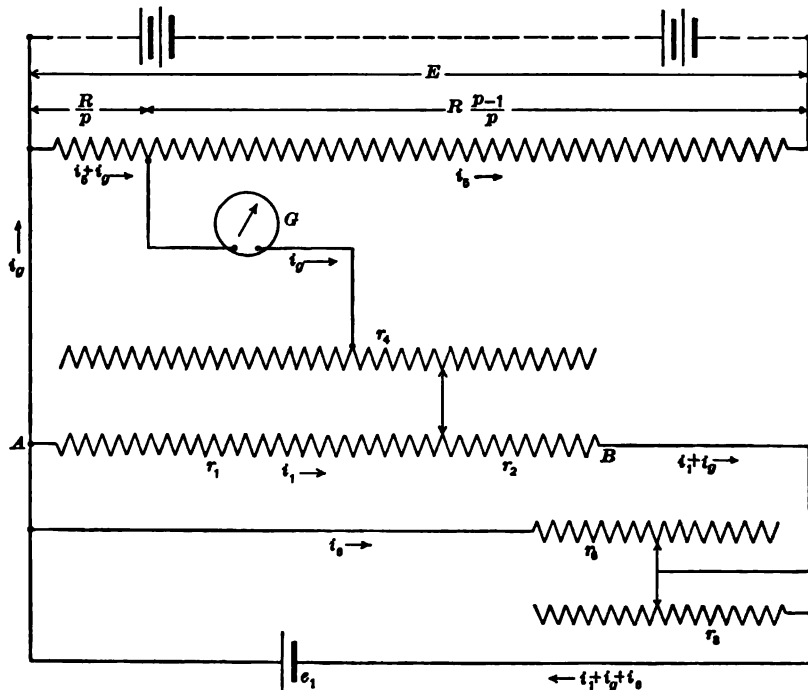


Fig. 4.—Fifth plan of circuits.

The variable shunt to the galvanometer in plan 4 must be made of copper, and should preferably be mounted inside the galvanometer case, to insure equality of temperature. This would require numerous leads from the shunt to the potentiometer, if the galvanometer is not to be built into the potentiometer. Plan 4 also has the disadvantage of requiring a larger number of coils than the

³The method of arranging the compensating coils r_s in Fig. 2 was suggested to the writer by Dr. W. P. White, who has recently described it in the *Physical Review*, 25, 334, 1907; and in the *Zeitschrift für Instrumentenkunde*, 27, 210; 1907.

others for a given range and number of steps. A further practical disadvantage of plans 2 and 4 is that the sections of the voltbox, which require precision adjustment, will nearly all be odd values.

In considering the features which would be desirable in an instrument of this class for voltmeter testing, in which the range from zero to the maximum is to be covered, and several ranges provided, it was found that none of these plans entirely met the requirements, and a plan was sought which would be satisfactory. This was obtained from plan 1 by adding a variable shunt to the potentiometer wire AB, which is varied at the same time as r_3 . This plan is shown in Fig. 4.

In the former article it was shown that for plan 1, in which the resistance r_6 is absent, the current through the galvanometer is given by the equation

$$i_g = \frac{\left(\frac{r_1}{r_1 + r_3 + r_3}\right)e_1 - \frac{E}{p}}{r_4 + r_g + \frac{r_1(r_3 + r_3)}{r_1 + (r_3 + r_3)} + R \frac{p - 1}{p^2}} \quad (6)$$

In plan 5, as r_3 is increased, r_6 is decreased; both of these changes tend to lower the fall of potential on AB. By applying Kirchhoff's laws to this network we get for the value of the galvanometer current

$$i_g = \frac{\left(\frac{r_1}{r_1 + r_3 + \frac{r_1 + r_3 + r_6}{r_6} r_3}\right)e_1 - \frac{E}{p}}{r_4 + r_g + \frac{r_1\left(r_3 + \frac{r_3 r_6}{r_3 + r_6}\right)}{r_1 + \left(r_3 + \frac{r_3 r_6}{r_3 + r_6}\right)} + R \frac{p - 1}{p^2}} \quad (17)$$

By comparing this with equation (6) we see that the effect of r_6 in the first term of the numerator is to make the coefficient of r_3 greater than unity; that is, r_6 assists r_3 in lowering the fall of potential on AB. The third term in the denominator is seen to be r_1 in parallel with the sum of r_3 and the resultant of r_3 and r_6 in parallel. If, therefore, we can choose corresponding values of r_3 and r_6 so as to keep constant the resultant of these two in parallel, while giving the desired regulation of the current through AB,

this third term will be a function of r_1 , since $r_1 + r_2$ is constant; r_4 may then be readily arranged to compensate for this variation, keeping the denominator, Σr , the total resistance in the galvanometer circuit, a constant for all settings and all positions of the regulating rheostat r_3 . It should be noted that if we make $r_6 = \infty$ in equation (17), it becomes identical with equation (6).

The practical limitations of this method should be examined, for while negative values of resistance may satisfy an equation, such resistances cannot be realized. First, r_3 cannot be reduced to zero, for this would make the term $\frac{r_3 r_6}{r_3 + r_6}$ always equal to zero, and no regulation would be possible. It is therefore necessary to have a value of e_1 , when the cells are giving the lowest emf. at which it is desired to use them, which is somewhat greater than the fall of potential to be maintained on AB. Let this fall of potential be represented by a ; then from a consideration of the first term of the numerator in equation (17) we have

$$\begin{aligned} r_1 + r_2 + \frac{r_1 + r_2 + r_6}{r_6} r_3 &= \frac{e_1}{a} (r_1 + r_2) \\ \frac{r_1 + r_2 + r_6}{r_6} r_3 &= \frac{e_1 - a}{a} (r_1 + r_2) \\ \frac{r_w + r_6}{r_6} r_3 &= \frac{e_1 - a}{a} r_w \end{aligned} \quad (18)$$

where r_w is put for $r_1 + r_2$, the resistance of the wire AB. On account of the change of emf. of the storage cell, e_1 will vary between certain limits, and values of r_3 and r_6 must be found for a series of values of e_1 . It is first necessary to find, from equation (11), the constants of the galvanometer and assigned or determined values of p , m , and R , how large a value we may allow for the maximum resistance between the point of contact on the wire AB and the point A. Some resistance should be reserved in a calibrating coil in the galvanometer circuit, which may be altered, if necessary, to correct for change in the galvanometer sensibility with time. This maximum value, r_m , will occur when the slider is at the center of the total resistance in the storage cell circuit; that is, when

$$r_1 = \frac{r_w}{2} + \frac{r_s r_6}{2(r_s + r_6)}$$

and its value is

$$r_m = \frac{r_w}{4} + \frac{r_s r_6}{4(r_s + r_6)} \quad (19)$$

By solving (18) for r_s and substituting this value in (19), solving for r_6 , we get

$$r_6 = \frac{e_1}{a} \cdot \frac{4r_m - r_w}{\frac{e_1}{a} - \frac{4r_m}{r_w}} \quad (20)$$

Substituting this value of r_6 in equation (18) and solving for r_s , we get

$$r_s = \frac{e_1}{a} \cdot \frac{r_w(4r_m - r_w)}{4r_m} \quad (21)$$

In this equation e_1 is the only variable, hence

$$r_s = \text{constant} \times e_1,$$

which is a very convenient formula to use.

It is somewhat more convenient to get r_6 , not by using equation (20), but by putting it into the form

$$\begin{aligned} r_6 &= \frac{(4r_m - r_w)r_s}{r_s - (4r_m - r_w)} \\ &= \frac{kr_s}{r_s - k} \end{aligned} \quad (22)$$

and using this after values of r_s have been determined for the various steps of the rheostat.

Examining equation (20), we see that r_6 will be infinite when

$$\frac{e_1}{a} = \frac{4r_m}{r_w},$$

except when $\frac{e_1}{a}$ is also equal to unity, in which case $r_6 = \frac{0}{0}$. This is

not a practical case. When $\frac{e_1}{a}$ is less than $\frac{4r_m}{r_w}$, r_6 will be negative.

In a practical case this can not occur; r_m must be greater than $\frac{r_w}{4}$, to allow for regulation, and hence the numerators of the expressions for r_s and r_e will always be positive. As r_m is determined by other considerations, we may choose r_w so as to give suitable values for r_e . Since r_w must be less than $4r_m$ and greater than $\frac{a}{e_1} \times 4r_m$, $\frac{a}{e_1}$ being somewhat less than unity, we may give r_w the mean of the two possible values, or

$$\begin{aligned} r_w &= \frac{1}{2} \left(1 + \frac{a}{e_1} \right) 4r_m \\ &= \frac{2(a + e_1)r_m}{e_1} \end{aligned} \quad (23)$$

The value of e_1 used in this formula should be the lowest emf. at which the storage battery is to be used. It is seen that the nearer e_1 approaches a , or the less margin we allow for regulation, the narrower are the limits for r_w . It is desirable to have each step of r_w an even value, as 10, 20, 50, or 100 ohms, so that if the above formula gives a value for each step, which is near an even value, the even value should be used, making the necessary adjustment of values elsewhere.

In the case of plan 5, the fine adjustment rheostat in the storage battery circuit calls for more attention than would usually be necessary. The steps on the series rheostat in the battery circuit being equal in resistance, it would ordinarily be sufficient to give the fine rheostat a resistance equal to or slightly greater than the resistance of one step of the main rheostat. In the case of plan 5, however, this would not be sufficient. When the coarse rheostat is changed one step, both r_s and r_e are changed; when the fine rheostat is changed, r_s only is changed. It is not practicable to use a double fine rheostat, such that r_e as well as r_s will be changed, on account of the large variation in the value of a step in r_e , for uniform steps in r_s . In the instrument about to be described the change in the shunt resistance r_s is more effective in changing the current through the potentiometer wire than the change of r_e , and to produce, by a variation of r_s alone, an effect equal to that due to changing r_s and r_e , requires a relatively large change in r_s .

Let a fine adjustment rheostat r_s be connected so as to form a continuation of r_s (Fig. 4). The current through the wire AB, when no current flows through the galvanometer, will be, by equation (17),

$$i = \frac{e_1}{r_w + \frac{r_w + r_s}{r_s} r_s} \quad (24)$$

If we reduce r_s by one step, Δr_s , increasing r_s by Δr_s , the current will increase. The insertion of the fine rheostat r_s in series with r_s should reduce the current to the original value; hence,

$$r_w + \frac{r_w + r_s}{r_s} r_s = r_w + \frac{r_w + r_s + \Delta r_s}{r_s + \Delta r_s} (r_s - \Delta r_s + r_s)$$

from which

$$r_s = \Delta r_s + \frac{r_s r_w \Delta r_s}{r_s (r_w + r_s + \Delta r_s)} \quad (25)$$

This shows that r_s must always be greater than Δr_s , unless r_s be made infinite, which gives plan 1. The expression also shows that the amount by which r_s is in excess of Δr_s will increase with increasing r_s , r_w , and Δr_s , and decrease with increasing r_s . It will be found that the value of r_s for a given instrument will not be the same at all points of the coarse rheostat. In the present instrument, r_s should be 7.5 ohms when the coarse rheostat is in the position of maximum r_s , and 10.4 ohms for minimum r_s . It is desirable to have a small amount of overlap, hence this rheostat was made of 11 ohms resistance.

It is often desirable to use an external resistance or "multiplier" to increase the range of a voltmeter. The same thing can be done with the deflection potentiometer, though not with the same facility for every plan. With plans 1 and 5, an external multiplier can be used to increase the range, with a small (usually negligible) error which is a constant per cent of the deflection reading, for any setting of the main dial. This may also be done with plan 2, if the range is not great; but the relative error in the deflection then varies with the setting, being inversely proportional to p . It is impracticable to use a multiplier with plan 3, as the resistance of the multiplier would have to change with the setting, so as to be always the

same multiple of the variable R . The general expression for the error involved in the use of an external multiplier is given in a later paragraph.

It will be seen that plan 5 is the most suitable one for a standard instrument for voltmeter testing, having the most practical points of advantage in construction and operation. The volt box resistance is high and constant; the resistance of the galvanometer circuit is constant, and the galvanometer may thus be critically damped at all settings, without the use of shunts. The steps on the main dial are

4. DESCRIPTION OF INSTRUMENT.

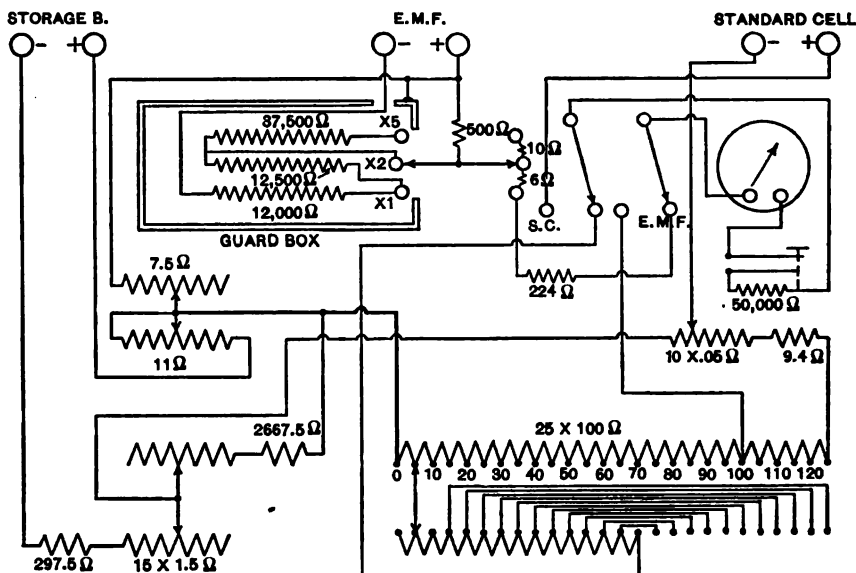


Fig. 5.—Connections of potentiometer (diagrammatic).

equal resistances, and by suitable design may also be made even values, which facilitates checking and adjusting. Multiple ranges are readily provided, and if it is desired to measure pressures in excess of the highest for which the instrument is built, this may be done by the use of an external multiplier, giving a very small but constant percentage correction to the deflection reading.

An instrument has been designed in accordance with plan 5, for use in voltmeter testing in the Bureau of Standards. A diagrammatic plan of connections is given in Fig. 5.

The main dial has 25 steps of 100 ohms each. This is in series with a coil of 9.4 ohms and a dial of 10 steps of .05 ohm each. The Weston standard cell is balanced around 509.4 ohms plus the amount on the dial, and as the standard current is .002 ampere, cells of 1.0188 to 1.0198 volts may be used, and if need be, this range may be varied by changing the 9.4 ohm coil. In the storage cell circuit is a series rheostat whose minimum resistance is 297.5 ohms, increasing from this value in 15 steps of 1.5 ohms each. This is r_s of the preceding discussion. At the same time that r_s is increased, the resistance in shunt to the potentiometer wire, r_0 , decreases from a maximum of 6,814 ohms to a minimum of 2,667.5 ohms. A fine rheostat of 11 ohms in the battery circuit covers any step of the coarse rheostat, and has a compensating resistance of 7.5 ohms maximum in the galvanometer circuit.

The drop is taken from the ends of a 500 ohm coil, which is in series with 12,000 ohms when the range switch is set on $\times 1$. When this switch is at $\times 2$ and $\times 5$ the total resistance between the emf. posts is 25,000 and 62,500 ohms, respectively. All of these coils, except the first 500 ohms, are mounted within and well insulated from a brass box which entirely encloses them. The negative emf. terminal post is inside of, and well insulated from, a brass sleeve which projects into the box and is soldered to it. This box is connected by a wire to the positive emf. post, and acts as a "guard wire" to prevent leakage currents from flowing through the circuits of the potentiometer proper. This is a very important precaution, which will be appreciated when we consider the high pressure (625 volts) available for producing leakage, and the fact that the full deflection of the galvanometer is produced by .00006 ampere, a current which 625 volts would send through a resistance of over ten megohms; while a current sufficient to give a readable deflection would flow, under this pressure, through 3,000 megohms. With the arrangement shown, the maximum pressure which may produce leakage through the galvanometer is 6 volts.

The main dial has a set of compensating coils, r_1 , for keeping constant the resistance between the sliding contact and the 0 end of this dial. By arranging the values of r_s , r_0 , and the standard cell coils so that the resultant resistance beyond the 125 end of the dial is 300 ohms, the total resistance in the storage battery circuit being

2,800 ohms, the point of maximum resultant resistance to the galvanometer current will come at the setting 70, when r_1 is 1,400 ohms; the resistance at 75 will be the same as at 65, and so on. Hence compensating coils are used up to the point 70, and by the use of cross connections no additional coils are needed beyond this point.

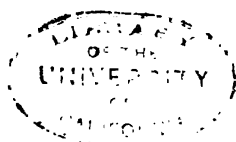
A double-pole double-throw switch is used to change from standard cell to emf., and a push button is provided in the galvanometer circuit. This button operates a double successive contact key having a protective resistance of 50,000 ohms in the first contact. The fine adjustment rheostat consists of two semicircular coils of insulated wire, the insulation being removed on the face to allow the sliding brush to make contact.

While the lowest range provided is nominally 0 to 125 volts, the range 0 to 5 volts may be had by using as the — emf. terminal the lever of the range switch, and adding sufficient resistance to the galvanometer circuit to make the proper total. In other words, the normal range of the potentiometer is from 0 to 5 volts, readable to .0004 volt (one-tenth of a scale division of the galvanometer). This range may be increased as desired by adding resistance in the emf. circuit at the rate of 100 ohms per volt. The reason for not making the lower ranges immediately available by setting the range switch is the danger of accident to these low ranges, due to the inadvertent application of the usual working pressures of 100 volts and over. The lower ranges are not frequently required, and may be arranged for specially as above described. It is proposed to construct an external rheostat which may be quickly connected up to give the ranges 0 to 12.5, 0 to 25, and 0 to 62.5 volts, the proper compensating resistances being thrown into the galvanometer circuit for each range. The presence of this special accessory apparatus would constitute an indication of the need for caution as to the value of the voltage to be applied.

A view of the instrument is shown in Fig. 6, from which it will be seen that the galvanometer is built into the rubber top, making the apparatus self-contained with the exception of the cells. The scale of the galvanometer is shown in Fig. 7. It has two sets of figures; an upper with 0 in the center, and a lower with 5 in the center. Ten divisions of this scale correspond to one volt, the



Fig. 6.—Deflection potentiometer, model 2 (one-third size.)



upper set of figures being used when the dial reading ends in 0, the lower set when it ends in 5. Thus, if the dial reads 110 and the galvanometer needle stands at 22 divisions to the right, the emf. to be measured is $110 + 2.2 = 112.2$ volts. In practice, the dial reading is mentally set at the center of the galvanometer scale, using upper or lower figures as required, when the value of the voltage can be read off from the scale, reading directly to tenths of a volt and estimating hundredths. The above applies to the first or principal range; when the range switch is at $\times 2$ or $\times 5$ the operation of the instrument is exactly the same, but the values are to be multiplied by 2 or 5, giving a maximum range of 625 volts. Should it be necessary to extend this by using a multi-

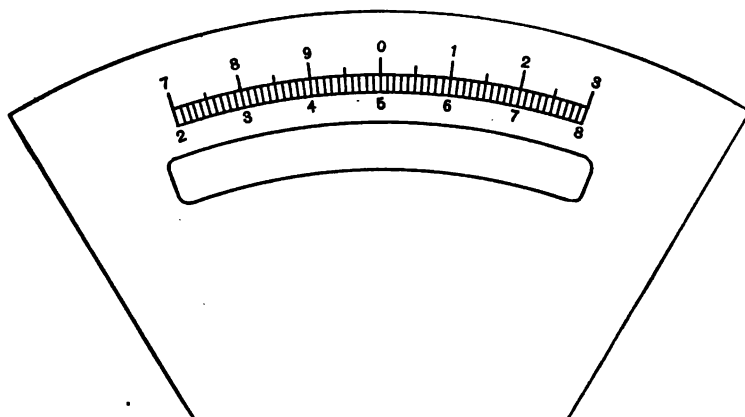


Fig. 7.—Galvanometer scale.

plier, this may be done with an error less than 0.2 per cent of the deflection portion of the reading. At full scale deflection this is about .05 division, or about the limit of setting the needle on a line. Hence, in this design, multipliers introduce a negligible error.

This instrument was built by The Leeds & Northrup Company, of Philadelphia, Pa., the galvanometer being supplied by the Weston Electrical Instrument Company, of Newark, N. J. It is so rapid and dead-beat in operation that the only limit to the rapidity of testing voltmeters with it is the ability of the operator to control the voltage and read the voltmeter under test. The appliances now in use at the Bureau for voltage control will be replaced soon by others which will give the maximum speed of working.

While intended primarily for direct current testing, the new instrument will be of assistance in testing alternating current voltmeters. This is done by the use of a transfer instrument which is equally correct on direct and alternating current; the transfer instrument, the potentiometer, and the voltmeter to be tested being connected in parallel with each other, first on alternating, then on direct emf. The change from alternating to direct voltage is accomplished by a quick-acting snap switch, the circuit being broken for so short a time that the deflection of the transfer instrument is maintained. This results in a saving of time and the avoidance of zero errors, and is a very accurate and convenient method.

Tests of this potentiometer in comparison with a five-dial standard potentiometer showed very small errors; the result (using the 125 volt range) being correct to within .01 to .02 volt when the deflection of the galvanometer is small, and within .04 volt at the points of greatest error. For the higher ranges, and for the 5-volt range, the relative errors are equally small. This result will be improved by making a more accurate scale for the galvanometer, as some of the lines of the present scale are out of place by 0.2 scale division.

5. SOURCES OF ERROR.

The error in a measurement with the deflection potentiometer is the algebraic sum of two distinct errors. The first is due to the error of adjustment of the coils constituting the potentiometer, considered as a null instrument, and affects the result when the galvanometer deflection is zero, as well as for any value of the deflection. This error is present in standard potentiometers of every form; but the accuracy possible in the adjustment of resistance coils is such that it can be made negligibly small in comparison with the second, namely, the error in the value of $\mathcal{A}E$, as given by the deflection of the galvanometer. The coils in the Σr circuit may not be correctly adjusted; the fine rheostat in the battery circuit cannot be perfectly compensated for all settings on the main dial; the scale of the galvanometer may be imperfectly graduated, and changes of temperature may affect the reading of the galvanometer. It is therefore necessary to investigate these points, in order to keep these errors small and be able to allow for them when necessary. Errors in the adjustment of the coils may be determined at any time by careful

resistance measurements. If the error in the relative values of the coils in the main dial and of the volt box is such that corrections for the potentiometer as a null instrument are appreciable, a table of corrections can be made for the settings on the main dial. These corrections will then apply, whether the galvanometer is deflected or not. The accuracy required in the adjustment of the coils in the galvanometer circuit is much less than for the main dial and volt box coils, and errors due to this cause should be inappreciable, with good workmanship.

It is necessary to investigate the error in the deflection reading, due to the fact that the fine rheostat in plan 5 makes the resultant resistance of r_3 and r_4 no longer a constant. This error will be greatest when the main dial setting is a maximum and the coarse rheostat r_3 a minimum. To reduce the error we may let one-half the total resistance in the fine rheostat take the place of an equal amount in the coarse rheostat r_3 ; that is, one-half the fine rheostat may be considered part of the r_3 for which the values of r_4 are determined. Thus the fine rheostat will affect the resultant resistance one-half as much as would otherwise be the case. We have, then, the greatest error, in the present instrument, when $r_3 = 297.5$ and $r_4 = 6,814$. In the middle position of the fine rheostat 5.5 ohms is added to r_3 , and the (normal) resultant resistance is

$$\frac{303 \times 6814}{303 + 6814} = 290.1 \text{ ohms.}$$

Now let the remaining half of the fine rheostat be put in circuit, increasing r_3 to 308.5 ohms. The resultant resistance becomes

$$\frac{308.5 \times 6814}{308.5 + 6814} = 295.14 \text{ ohms.}$$

For maximum setting of the dial, for the above values of r_3 , the resistance in the galvanometer circuit between the contact point on the potentiometer wire and the zero end of the dial will be

$$\frac{(290.1 + 9.9) 2500}{290.1 + 9.9 + 2500} = 267.86 \text{ ohms,}$$

$$\frac{(295.14 + 9.9) 2500}{295.14 + 9.9 + 2500} = 271.87 \text{ ohms,}$$

the difference, 4 ohms, entering the total of 2,000 ohms in the Σr circuit, and requiring a compensation of 4 ohms. For the maximum value of r , the compensation required is 3.45 ohms, to be added or subtracted for the two extreme positions of the fine rheostat. The mean of these was taken, and the compensating rheostat made 2×3.75 ohms. The error for this extreme position of the fine rheostat, if not compensated, will be $\frac{4}{2000}$ of the deflection,

or .06 scale division, which is about the limit of setting on a line. If this were the only source of error to be considered, it might be neglected; but it is desirable to avoid the accumulation of small errors, and by proper arrangement of circuits it is easy to arrange for compensation. It is evident that if we compensate for the error which enters at the maximum setting of the main dial, we introduce the full error at the zero position of the main dial, and a proportionally decreasing error as the dial setting increases to a maximum. This is not objectionable, however, for on low readings of an instrument under test the accuracy is necessarily low, and the greatest accuracy is desired at or near full scale deflection.

The method of combining the fine rheostat and the compensating rheostat in one simple dial may be seen diagrammatically in Fig. 5. It will be seen that the contact resistance of the lower end of the slider does not enter into the resistance of the wire AB, and the contact resistance of the upper end is in the galvanometer circuit. It is possible to put the fine rheostat in other places, but it is believed that this plan is the simplest and most satisfactory.

Errors due to imperfect graduation of the galvanometer scale depend upon the care taken by the maker, and may be determined by testing the galvanometer as an ammeter. The corrections to the deflection reading may be found for the galvanometer with its circuit by the following plan. The storage cell is removed, and the terminals to which it had been connected are joined by a wire of negligible resistance. This reduces to zero the first term in the numerator of equation (17), and the deflection is now proportional to E ; in other words, this makes E and ΔE identical. The value of E may now be adjusted to give suitable values of the deflection, E being measured by a standard potentiometer or calibrated voltmeter of suitable range. This method is necessary in plans 2, 3, and 4,

where a zero setting on the main dial is impossible. For plans 1 and 5 the storage cell may be in position and the main dial set at zero.

Inasmuch as the 2,000 ohms of Σr contains, in the present design, 576 ohms of copper in the galvanometer, the deflection part of the reading will vary with the temperature. The galvanometer was found by trial to have practically zero temperature coefficient when used as an ammeter, the opposite coefficients of magnet and springs being sensibly equal. Hence the temperature coefficient of the galvanometer in use in the potentiometer is numerically that of the resistance of the Σr circuit, or $\frac{576}{2000} \times .0038 = .0011$ per degree C.

Hence, for 10° C above or below that at which the instrument is adjusted, the error will be 1.1 per cent of the deflection, and for maximum deflection of 2.5 volts the error would be 0.03 volt. The simplest way to care for this is to have a table giving values of ΔE corresponding to various readings, at different temperatures. Another plan would be to have a resistance dial marked in degrees, so that Σr might be maintained at the correct value, for any temperature. A third plan would be to use the device known as the Bristol compensator, in which the rise and fall of mercury in a tube varies the length of a platinum wire in circuit. This could be made of such proportions as to just compensate for the effect of the variations in the galvanometer resistance. It is probable, however, that the first plan above is the best as well as the simplest, if all things are considered. For when the most accurate work is done, the temperature is usually kept near the standard, and as the galvanometer scale is not perfect, corrections will be required for the imperfections, if not for temperature. Hence if corrections are to be made at all, they may as well include those for temperature. For most purposes, even in the laboratory, the correction for temperature can be neglected.

It is important to note that in many applications of the deflection potentiometer the error of the result is almost independent of the error in the deflection part of the reading. For example, in testing a voltmeter the points checked are almost always a multiple of 5 or 10, so that the galvanometer reading is quite small, unless the voltmeter is considerably in error. The reading of the galvanometer

is the correction to the voltmeter at the given point, and may be so read off, using the + and - signs marked on the galvanometer scale. Thus the error in the galvanometer deflection is a fraction of a per cent of the small correction to the voltmeter, and is negligible. In using a deflection potentiometer to determine a definite voltage, as 100 or 120 volts, in order to mark the blank scale of an instrument, the method is a null one, and the result is independent of errors in the deflection part of the apparatus.

A point that should be investigated, in a proposed design, is the accuracy with which the current from the storage battery can be adjusted, in terms of the standard cell. While this depends partly upon the available number of steps in the fine rheostat, it also depends upon the galvanometer sensibility and the value of the total resistance in the circuit containing the standard cell and the galvanometer. In the present design the drop for the standard cell is taken from 510 ohms, which is shunted by 2,290 ohms, giving a resultant resistance of 417 ohms. To this must be added 576 ohms galvanometer resistance and 160 ohms internal resistance of the Weston standard cell, giving a total of 1,153 ohms. If the current through the potentiometer wire is to be standardized in terms of the standard cell, to within one part in 10,000, then this fraction of the emf. of the cell, or .0001 volt, must give a noticeable deflection of the galvanometer. This emf. will send $.0001 \div 1,153$, approximately 1×10^{-7} ampere through this resistance, and the resulting deflection will be one-twentieth of a scale division, which is about the limit, but can be observed when the needle is over, or close to, a line. If greater accuracy of the standard current were desired, using this galvanometer, it could be secured by using two or more standard cells in series, and taking the drop from a greater portion of the potentiometer wire. This would double or treble the value of the small unbalanced emf. corresponding to a given error in the current, without increasing the resistance in the same proportion. An accuracy of one in 10,000 is, however, sufficient for all purposes for which this form of instrument would be used.

It has been asked whether an error would arise if there is considerable resistance in the source of the unknown emf. to be measured. This question arises because the denominator of the expression for the galvanometer current—equations (6) and (17)—contains the term

$R \frac{p-1}{p}$, the resultant resistance of $\frac{R}{p}$ paralleled by the remainder of R . To test this point we may assume a resistance r , in the source of E (page 231 of former article), when equations (1), (2), (3), and (4) remain unchanged, and (5) becomes

$$i_s \left(R \frac{p-1}{p} + r \right) + i_g \frac{R}{p} = E + i_g r,$$

which reduces to the form previously given, namely:

$$i_s R \frac{p-1}{p} + i_g \frac{R}{p} = E \quad (5)$$

In this case, as before, E is taken to mean the potential difference at the terminals of the resistance R , which is the value it is desired to measure. The value of i_g is thus unchanged, and the accuracy of the reading is independent of whether there is or is not resistance in the source of the pressure to be measured. The addition of resistance in the source would (theoretically) slightly affect the damping of the galvanometer, but in a practical case this effect would be inappreciable.

It is of interest to determine the magnitude of the error arising from the use of an external multiplier to extend the range. This error is, of course, zero when no current flows through the galvanometer, but is present when such a current flows. In equation (5) let E increase to E' , and let R be increased by the addition of an external multiplier R' . Then the equation becomes

$$i_s R \frac{p-1}{p} + i_g \frac{R}{p} + i_g R' = E' = E + i_g R'$$

which reduces to the original form of equation (5).

Considering the currents at the junction of $\frac{R}{p}$ and $R \frac{p-1}{p}$ we have

$$i_s - i_g - i_g = 0 \quad (2)$$

From equations (2) and (5) we get

$$\begin{aligned} i_s &= \frac{E}{R} - \frac{1}{p} i_g \\ &= \frac{E}{R} - \frac{1}{p} \cdot \frac{\Delta e}{\Sigma r} \end{aligned}$$

Further,

$$\begin{aligned}
 E' &= E + i_g R' \\
 &= E + R' \left(\frac{E}{R} - \frac{1}{p} \cdot \frac{\Delta e}{\Sigma r} \right) \\
 &= E \left(1 + \frac{R'}{R} - \frac{R'}{p \Sigma r} \cdot \frac{\Delta e}{E} \right)
 \end{aligned} \tag{26}$$

When no current flows through the galvanometer this reduces to the ordinary formula for the use of an external multiplier,

$$E' = E \left(1 + \frac{R'}{R} \right) \tag{27}$$

Hence, when the galvanometer current is not equal to zero, and the ordinary factor $1 + \frac{R'}{R}$ is applied to the reading of the instrument to determine the emf. to be measured, the relative error ϵ is given by the formula

$$\begin{aligned}
 \epsilon &= \frac{\frac{R'}{p \Sigma r} \cdot \frac{\Delta e}{E}}{\frac{R + R'}{R}} \\
 &= \frac{RR'}{(\Sigma r)(R + R')} \cdot \frac{\Delta e}{pE}
 \end{aligned} \tag{28}$$

It is desirable to express the small quantity Δe (which is to be distinguished from ΔE) in terms of E , as follows:

$$\begin{aligned}
 \Delta e &= \frac{r_1}{r_1 + r_2 + r_3} e_1 - \frac{E}{p} \\
 &= \frac{ps - E}{p} \\
 &= \frac{\Delta E}{p}
 \end{aligned}$$

Equation (28) thus becomes

$$\epsilon = \frac{RR'}{(\Sigma r)(R + R')p} \frac{\Delta E}{E} \tag{29}$$

The reason for this correction term is simply that when the resistance $\frac{R}{p}$ is shunted by a larger value than the normal amount, $R \frac{p-1}{p}$, namely, by $R \frac{p-1}{p} + R'$, the resultant resistance is increased, and if the galvanometer gave a correct indication before, it will now give too small a deflection. This can be taken care of by applying the correction formula (29), or we may arrive at the same result by calculating the increment to Σr , and increasing ΔE in the same ratio. Thus, if R is increased by an amount R'

$$\Delta(\Sigma r) = \frac{\frac{R}{p} \left(R \frac{p-1}{p} + R' \right)}{R + R'} - R \frac{p-1}{p^2}$$

which reduces to the form

$$\Delta(\Sigma r) = \frac{RR'}{(R + R') p^2}$$

Hence the relative error in Σr is

$$\frac{\Delta(\Sigma r)}{\Sigma r} = \frac{RR'}{(\Sigma r)(R + R') p^2}$$

and the relative error in the observed value of the unknown emf. is

$$\begin{aligned} \epsilon &= \frac{\Delta \Sigma r}{\Sigma r} \cdot \frac{\Delta E}{E} \\ &= \frac{RR'}{(\Sigma r)(R + R') p^2} \frac{\Delta E}{E} \end{aligned} \quad (30)$$

In equation (26), if a positive value of Δe corresponds to an excess of the potentiometer setting over $\frac{E}{p}$, or to values of E lower than that for which the galvanometer current is zero, E' will be lower than $E(1 + \frac{R'}{R})$. When E is higher than the value for which the galvanometer current is zero, the correction term in (26) becomes positive, and E' is greater than $E(1 + \frac{R'}{R})$.

6. OUTLINE OF METHOD OF DESIGN.

In designing a deflection potentiometer, the central and determining feature is the galvanometer. From the current I for one scale division, the number of divisions m it is to give per volt of ΔE , and the total resistance for critical damping, which latter is substituted as Σr , we may determine an approximate value of p from equation (11), namely:

$$\Sigma r = \frac{1}{pmI} \text{ or } p = \frac{1}{(\Sigma r)mI} \quad (11)$$

As the resistance for critical damping is not sharply defined, we may modify the result somewhat, to give p a convenient integral value. From this tentative value of p and the maximum emf. which it is desired to measure, we may find the fall of potential on AB, which is $\frac{E}{p}$. For plan 5 this should be approximately a multiple of

1.7 volts, and will show how many storage cells will be required. Having fixed the value of p , determine Σr by substitution in equation (11). Deduct 5 or 10 per cent from Σr for a calibrating coil, and from the remainder deduct the resistance of the galvanometer.

The rest of Σr is to be divided between $R \frac{p-1}{p^2}$ and the resistance through the potentiometer proper, including the compensating resistance r_4 . This latter makes the resistance through the potentiometer (from the point of contact on the wire AB to the point A) constant and equal to one-fourth of the total equivalent resistance of the circuit supplied by the storage cells. As the emf. of these cells, when freshly charged, will be about 2.1 volts each as a maximum, this total equivalent resistance will be approximately $\frac{2.1}{1.7} r_w$, where r_w is the resistance of the wire AB; hence the constant resistance through the potentiometer will be approximately $\frac{2.1}{6.8} r_w$. As a first approximation, divide the rest of Σr equally between this and $R \frac{p-1}{p^2}$, and determine the values of r_w and R . It is desirable, for adjusting and checking, that each step of r_w have an even and usual value of

resistance, as 20, 50, or 100 ohms; with this in view, r_v may be chosen and R modified to suit. R may then be given an even value, the difference being taken up in the calibrating coil. It is desirable to make R large, to reduce energy loss and consequent heating of the volt box coils; it is also desirable to make r_v large, as the drain on the storage cells is thereby reduced, and slight variations in rheostat contacts do not then exert as much influence on the current strength.

The number of steps of the main dial is determined by two considerations. The smaller each step, the more accurate the instrument, because less of the total quantity is read by deflection. To go too far in this direction increases the size and cost of the instrument for a given range, and reduces the convenience and speed of working. The minimum number of steps is determined by the range of the galvanometer, assuming that emfs. are to be measured continuously from the lowest to the highest.

The mechanical design should conform to good practice in such matters as reliable dial contacts, subdivision of coils in the volt box to reduce the dielectric stress per coil, and sufficient area in these coils to keep the heating small. The guard box around these coils is an essential feature, particularly when the instrument is to be used in damp weather. The grouping of the parts of the apparatus should be made with a view to rapidity and convenience of working, and will depend upon the judgment of the designer.

7. CONCLUSION.

In this and in the preceding article stress has been laid upon the desirability of using a pivoted portable form of galvanometer. This has been done because the aim has been to produce instruments that can be quickly and conveniently operated and moved from one place to another without requiring the special setting up which is necessary with reflecting galvanometers having delicate suspensions and small clearance between coil and pole pieces. Any galvanometer can be used which has a good zero and a constant that is reasonably permanent. It is desirable, if a reflecting galvanometer is to be used, to employ a taut suspension above and below, heavy enough to give quick working and good zero. For a given set of conditions the sensibility required can be determined, and the suspensions can

then be chosen to give this sensibility. The matter of critical damping should be taken into account, as before mentioned.

In the case of a deflection potentiometer for current measurements in connection with suitable shunts, which might have say 150 millivolts as the upper limit of its range, the small amount of power available imposes much greater requirements upon the galvanometer than in the case of an instrument for the measurement of voltage, such as the one above described. The design of such a potentiometer for current measurements has been begun.

While the potentiometer for voltage measurements, as described in this article, is best adapted to laboratory use, where voltmeters of various ranges must be tested, a simpler and less expensive instrument would answer all requirements in central station work. For example, a range of 105 to 125 volts, with provision to double and quintuple the range, would cover the most important requirements of the light and power station. The sensibility in this case would not need to be as great as for the laboratory instrument, so that a less sensitive galvanometer would be satisfactory. Preliminary work on the design of such an instrument has been begun.

WASHINGTON, October 11, 1907.

THE SELF AND MUTUAL INDUCTANCES OF LINEAR CONDUCTORS.

By Edward B. Rosa.

Formulæ for the self and mutual inductances of straight wires and rectangles are to be found in various books and papers, but their demonstrations are usually omitted and often the approximate formulæ are given as though they were exact. I have thought that a discussion of these formulæ, with the derivation of a number of new expressions, would be of interest, and that illustrations of the formulæ, with some examples, would be of service in making such numerical calculations as are often made in scientific and technical work.

I have derived the formulæ in the simplest possible manner, using the law of Biot and Savart in the differential form instead of Neumann's equation, as it gives a better physical view of the various problems considered. This law has not, of course, been experimentally verified for unclosed circuits; but the self-inductance of an unclosed circuit means simply its self-inductance as a part of a closed circuit, the total inductance of which can not be determined until the entire circuit is specified. In this sense the use of the law of Biot and Savart to obtain the self-inductance of an unclosed circuit is perfectly legitimate. I have also shown how, by the use of certain arithmetical mean distances in addition to geometrical mean distances, the accuracy of some of the formulæ can be increased.

In the following demonstrations the magnetic field is assumed to be instantaneous; in other words, the dimensions of the circuit are assumed to be small enough and the frequency of the current slow

enough so that it is not necessary to take account of the finite velocity of propagation of the field. This may be done even when the field is integrated to infinity, as the distant magnetic field is canceled when two or more open circuits are combined into a closed circuit.¹

1. SELF-INDUCTANCE OF A STRAIGHT CYLINDRICAL WIRE.

Let AB be a length l of a cylindrical wire of radius ρ traversed by current i distributed uniformly over the cross section of the wire.

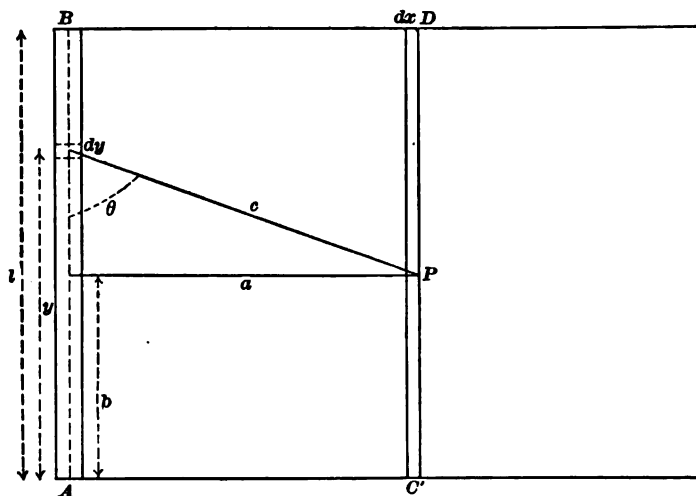


Fig. 1.

The magnetic force at P normal to the paper due to an element of the cylinder of length dy is,

$$i \frac{dy}{c^2} \sin \theta = \frac{i a dy}{[a^2 + (y-b)^2]^{\frac{3}{2}}}$$

It is easy to show² that the force at any point outside a right cylinder is the same as though the current were concentrated at the

¹ For a discussion of the self-inductance of an open circuit closed by displacement currents in which the finite velocity of the field is taken into account see a recent paper by K. Ogura and C. P. Steinmetz, *Phys. Rev.*, **25**, p. 184, Sept., 1907.

² Minchin, *London Electrician*, Sept. 27, 1895.

M. Wien, *Wied. Annal.* **53**, p. 928, 1894, gives a number of formulae for the self and mutual inductance of linear conductors.

axis of the wire. The force at P due to the whole length of the cylinder carrying unit current is then,

$$H = \int_0^l \frac{a \, dy}{[a^2 + (y-b)^2]^{\frac{3}{2}}} = \frac{l-b}{a\sqrt{a^2 + (l-b)^2}} + \frac{b}{a\sqrt{a^2 + b^2}} \quad (1)$$

The number of lines of magnetic force dN , within the strip CD, of breadth dx , is found by integrating the expression for H along the strip.

Thus,

$$\begin{aligned} dN &= \frac{dx}{a} \int_0^l \left[\frac{l-b}{\sqrt{a^2 + (l-b)^2}} + \frac{b}{\sqrt{a^2 + b^2}} \right] db \\ &= \frac{2dx}{a} \left[\sqrt{a^2 + l^2} - a \right] \end{aligned} \quad (2)$$

The whole number of lines of force N outside the wire which will collapse upon the wire when the current ceases is found by integrating dN with respect to x from $x=\rho$ to $x=\infty$. Thus, replacing a by x in (2),

$$N = 2 \int_{\rho}^{\infty} \left[\frac{\sqrt{x^2 + l^2}}{x} - 1 \right] dx = 2 \left[\sqrt{x^2 + l^2} - x - l \log \frac{l + \sqrt{x^2 + l^2}}{x} \right]_{\rho}^{\infty} \quad (2a)$$

$$\text{or, } N = 2 \left[l \log \frac{l + \sqrt{l^2 + \rho^2}}{\rho} - \sqrt{l^2 + \rho^2} + \rho \right] \quad (3)$$

$$L_1 = 2l \left[\log \frac{2l}{\rho} - 1 \right] \text{ approximately} \quad (4)$$

This is the number of lines of force outside the wire due to unit current in the wire, and is therefore that part of its self-inductance L_1 due to the external field. We must now find L_2 due to the field within the wire.

The strength of field at the point P within the wire is $\frac{2ix}{\rho^2}$. The number of lines of force in the length l within the element dx is, therefore,

$$dN = \frac{2ilxdx}{\rho^2}$$

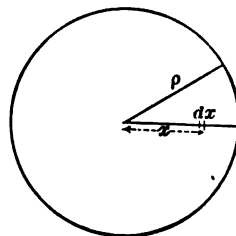


Fig. 2.

If we integrate this expression from 0 to ρ we have the whole number of lines of force within the conductor. Therefore

$$N = \frac{li}{\rho^2} \int_0^\rho 2x dx = li \quad (5)$$

Thus there are i lines or tubes per unit of length within any cylindrical conductor carrying a current i , or *one tube per cm for unit current*.³

The lines within the conductor do not cut the whole cross section of the conductor, as do those without. We must weight them, in estimating their effect on the self-inductance, in proportion to the area of the section of the conductor cut by each elementary tube.

Thus,

$$iL_1 = l \int_0^\rho \frac{2ix}{\rho^2} \cdot \frac{x^2}{\rho^2} dx = l \left[\frac{i}{2} \cdot \frac{x^4}{\rho^4} \right]_0^\rho = \frac{li}{2}$$

or $L_1 = \frac{l}{2}$ (6)

Thus the l lines or tubes within the conductor contribute only half as much toward the self-inductance of the conductor as an equal number of lines outside the conductor would do.

If the permeability of the wire is μ the part of the self-inductance due to the internal field is

$$L_1 = \frac{\mu l}{2} \quad (7)$$

We may derive (6) otherwise thus: The field at P is

$$H = \frac{2ix}{\rho^2}$$

The total energy W inside the wire is

$$W = \frac{1}{8\pi} \int H^2 dv,$$

³For convenience we may, however, speak as though there were many lines or tubes within the conductor due to unit current.

where the integration is taken throughout the entire volume of the cylindrical wire.

Thus, since $dv = 2\pi x dx$

$$W = \frac{1}{8\pi} \int_0^{\rho} \frac{4x^2 x^2}{\rho^4} \cdot 2\pi x dx = \frac{l^2}{\rho^4} \int_0^{\rho} x^3 dx = \frac{l^2}{4} \quad (8)$$

But since $W = \frac{1}{2} L_2 i^2$, we have

$$L_2 = \frac{l}{2},$$

as found above by the first method (6).

The total self-inductance of the length l of straight wire is therefore the sum of L_1 and L_2 , as given by (3) and (6), or

$$L = 2 \left[l \log \frac{l + \sqrt{l^2 + \rho^2}}{\rho} - \sqrt{l^2 + \rho^2} + \frac{l}{4} + \rho \right] \quad (9)$$

$$= 2l \left[\log \frac{2l}{\rho} - \frac{3}{4} \right] \text{ approximately.} \quad (10)$$

$$= 2l \left[\log \frac{2l}{\rho} - 1 + \frac{\mu}{4} \right] \quad (11)$$

where the permeability of the wire is μ , and that of the medium outside is unity. This formula was originally given by Neumann. For a straight cylindrical tube of infinitesimal thickness, or for alternating currents of great frequency, when there is no magnetic field within the wire, we have for the self-inductance instead of (10) or (11)

$$L = 2l \left[\log \frac{2l}{\rho} - 1 \right] \quad (11a)$$

2. THE MUTUAL INDUCTANCE OF TWO PARALLEL WIRES.

The mutual inductance of two parallel wires of length l , radius ρ , and distance apart d will be the number of lines of force due to unit current in one which cut the other when the current disappears. This will be the value of N given by (2a) when the limits of integration are d and ∞ instead of ρ and ∞ as before.

Thus

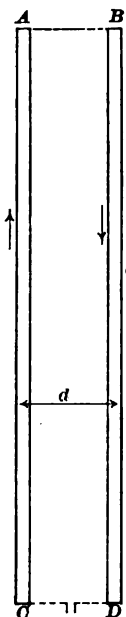


Fig. 3.

$$M = 2 \left[l \log \frac{l + \sqrt{l^2 + d^2}}{d} - \sqrt{l^2 + d^2} + a \right] \quad (12)$$

$$= 2l \left[\log \frac{2l}{d} - 1 + \frac{d}{l} \right] \text{approximately} \quad (13)$$

when the length l is great in comparison with d .

Equation (12), which is an exact expression when the wires have no appreciable cross section, is not an exact expression for the mutual inductance of two parallel cylindrical wires, but is not appreciably in error even when the section is large and d is small if l is great compared with d . The force in that case due to A at all points outside A is exactly the same as though the current were concentrated at the center O_1 of A; and the geometrical mean distance from O_1 to the cross section of B is exactly the distance d between O_1 and O_2 . The *mean distance* from O_1 to all the points in the section of B is not, however, quite the same as d , although the mean of the log of these distances is $\log d$. Hence there is a very slight difference in the last term of (12) depending upon the section of the wires and a still smaller difference in the other terms. (See p. 331.) This is, however, too small to be appreciable in any ordinary case, being a small quantity of the second order when l is large compared with d .

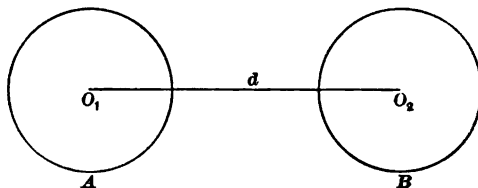


Fig. 4.

3. THE SELF-INDUCTANCE OF A RETURN CIRCUIT.

If we have a return circuit of two parallel wires each of length l (the current flowing in opposite direction in the two wires) the self-inductance of the circuit will be, neglecting the effect of the end connections shown by dotted lines, Fig. 3,

$$L = 2L_1 - 2M$$

where L_1 is the self-inductance of either wire taken by itself, and M is their mutual inductance. Substituting the approximate values of L_1 and M we have

$$L = 4l \left[\log \frac{d}{\rho} + \frac{\mu}{4} \right] \text{ approximately} \quad (14)$$

The same result follows if we integrate the expression for the magnetic force between the wires due to unit current, $H = 2/x$.

Thus,

$$N = l \int_{\rho}^d \frac{2dx}{x} = 2l \log \frac{d}{\rho}$$

Multiplying this by two (for both wires) and adding the term due to the magnetic field within the wires we have the result given by (14). If the end effect is large, as when the wires are relatively far apart, use the expression for the self-inductance of a rectangle below (24); or, better, add to the value of (14) the self-inductance of $AB + CD$ using equation (10) in which $l = 2AB$.

4. MUTUAL INDUCTANCE OF TWO PARALLEL WIRES BY NEUMANN'S FORMULA.

Neumann's formula for the mutual inductance of any two circuits is

$$M = \iint \frac{\cos \epsilon \, ds \, ds'}{r} \quad (15)$$

In this case $\epsilon = 0$ and $\cos \epsilon = 1$, $r = \sqrt{a^2 + (y-b)^2}$, and the integration is along both lines.

$$M = \int dy' \int_0^l \frac{dy}{\sqrt{a^2 + (y-b)^2}} = \int dy' \left[\log \frac{l-b + \sqrt{a^2 + (l-b)^2}}{\sqrt{a^2 + b^2} - b} \right]$$

The quantity in the brackets is the mutual inductance of the line AB and unit length of CD at a point distant b from the lower end, Fig. 4a. Now making b variable and calling it y , and integrating along CD from 0 to l , we have

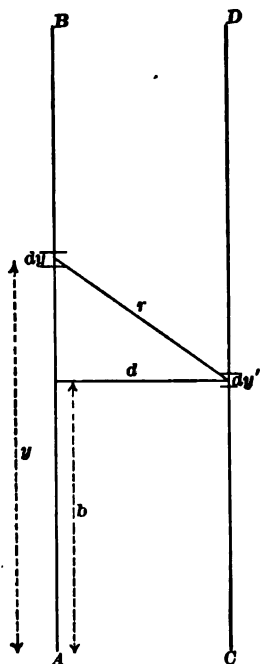


Fig. 4a.

$$M = 2 \left[l \log \frac{l + \sqrt{l^2 + d^2}}{d} - \sqrt{l^2 + d^2} + a \right]$$

which is the same expression (12) found by the other method. That process is more direct and simpler to carry out than to use Neumann's formula.

5. MUTUAL INDUCTANCE OF TWO LINEAR CONDUCTORS IN THE SAME STRAIGHT LINE

We have found the self-inductance of the finite linear conductor AB by integrating the magnetic force due to unit current in AB over the area ABB'A', extending to the right to infinity, equations (3) and (9).

In the same way we may find the mutual inductance of the conductors AB

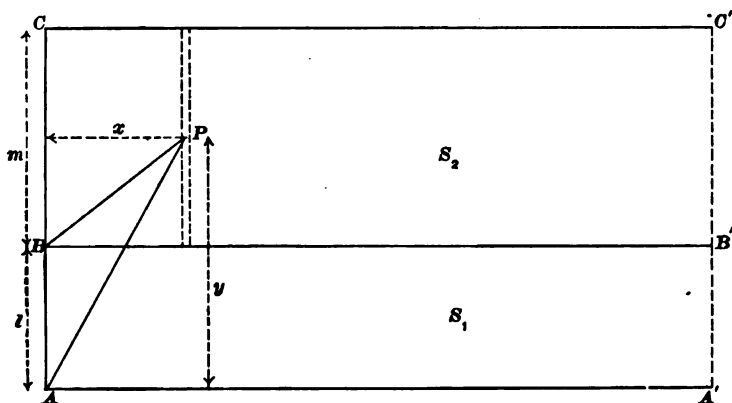


Fig. 5.

and BC, lying in the same straight line, by integrating over the area S_2 (extending to infinity) the force due to unit current in AB.

The magnetic force at the point P (of coordinates x, y , origin at A) due to current i in AB is

$$H_P = \frac{i}{x} \left[\frac{y}{\sqrt{x^2 + y^2}} - \frac{y-l}{\sqrt{x^2 + (y-l)^2}} \right] \quad (16)$$

The whole number of lines of force N_s included in the area S_s is

$$\begin{aligned} N_s &= i \int_0^\infty \frac{dx}{x} \int_l^{l+m} \left[\frac{y dy}{\sqrt{x^2 + y^2}} - \frac{(y-l) dy}{\sqrt{x^2 + (y-l)^2}} \right] \\ &= i \int_0^\infty \left[\sqrt{x^2 + (l+m)^2} - \sqrt{x^2 + l^2} - \sqrt{x^2 + m^2} + x \right] \frac{dx}{x} \\ &= i \left[\sqrt{x^2 + (l+m)^2} - \sqrt{x^2 + l^2} - \sqrt{x^2 + m^2} \right. \\ &\quad \left. + x - l \log \frac{l+m + \sqrt{x^2 + (l+m)^2}}{l + \sqrt{x^2 + l^2}} \right. \\ &\quad \left. - m \log \frac{l+m + \sqrt{x^2 + (l+m)^2}}{m + \sqrt{x^2 + m^2}} \right]_0^\infty \end{aligned}$$

or $M_{lm} = l \log \frac{l+m}{l} + m \log \frac{l+m}{m}$, approximately. (17)

This approximation is very close indeed so long as m does not approach infinity and the radius of the conductor BC (which we have assumed zero) is very small.

If $l = m$,

$$M = 2l \log_e 2 = 2l \times 0.69315 \text{ cm.}$$

If $m = 1000 l$, (17) gives

$$\begin{aligned} M &= l \log_e 1001 + 1000 l \log_e 1.001 \\ &= l \log_e 1001 + l \text{ approximately.} \end{aligned} \quad (18)$$

If $l = 1$ cm, (18) gives

$$\begin{aligned} M &= \log_e 1001 + 1000 \log_e 1.001 \\ &= 6.909 + 0.999 = 7.908. \end{aligned}$$

The self inductance of the short wire AB, suppose 1 cm long and of 1 mm radius, is

$$L_1 = 2 \left(\log \frac{2}{0.1} - .75 \right) = 2(2.9957 - .75) = 4.4915 \text{ cm},$$

which is a little more than one-half of the mutual inductance of AB and BC, BC being 1000 times the length of AB.

In closed circuits, all the magnetic lines due to a circuit are effective in producing self-inductance, and hence the self-inductance is always greater than the mutual inductance of that circuit with any other, assuming one turn in each. But with open circuits, as in this case, we may have a mutual inductance between two conductors *greater* than the self-inductance of one of them.

SECOND DERIVATION OF FORMULA (17).

We may derive formula (17) for the mutual inductance of two linear conductors forming parts of the same straight line by use of formula (10). Let L_l be the self-inductance of AB, L_m of BC, and L that of the whole line ABC. Then we have by (10)

$$\begin{aligned} L_l &= 2l \left(\log \frac{2l}{\rho} - \frac{3}{4} \right) \\ L_m &= 2m \left(\log \frac{2m}{\rho} - \frac{3}{4} \right) \\ L &= 2(l+m) \left[\log \frac{2(l+m)}{\rho} - \frac{3}{4} \right] \end{aligned} \quad (19)$$

The mutual inductance M_{lm} of the two straight lines AB and BC is then given by the expression

$$L = L_l + L_m + 2M_{lm}$$

$$\text{From above } L_l + L_m = 2(l+m) \left[\log \frac{2m}{\rho} - \frac{3}{4} \right] - 2l \log \frac{m}{l}$$

$$\therefore 2M_{lm} = 2(l+m) \left[\log \frac{l+m}{m} \right] + 2l \log \frac{m}{l}$$

$$\text{or } M_{lm} = l \log \frac{l+m}{l} + m \log \frac{l+m}{m}$$

which is equation (17), found independently above by integrating over the area S , the magnetic flux due to unit current in AB.

6. DEFINITION OF SELF-INDUCTANCE OF AN OPEN CIRCUIT.

It is of course impossible to maintain a steady current in a finite straight conductor, or even to start a current in such a conductor without having a return in the form of a displacement current. One can excite an oscillatory current in such a conductor, but the displacement current which closes the circuit will produce magnetic force at a distance, and hence the actual self-inductance of such a circuit is not the value of the self-inductance given by equation (9).

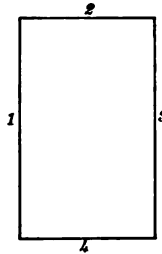


Fig. 6.

The latter is the self-inductance of a part of a closed circuit due to the current in itself. The actual self-inductance of any closed circuit of which it is a part will be the sum of the self-inductances of all the parts, plus the sum of the mutual inductances of each one of the component parts on all the other parts. Thus the self-inductance of a rectangle is the sum of the self-inductances of the four sides (by equation 10) plus the sum of the mutual inductances

of 1 and 3 on each other, and of 2 and 4 on each other (taking account of sign the mutual inductances will be negative). Since the lines of force due to side 1 in collapsing do not cut 2 and 4, the mutual inductance of 1 and 3 on 2 and 4 is zero.

In a recent number of the *Elektrotechnische Zeitschrift*,⁴ Wagner shows that the total magnetic

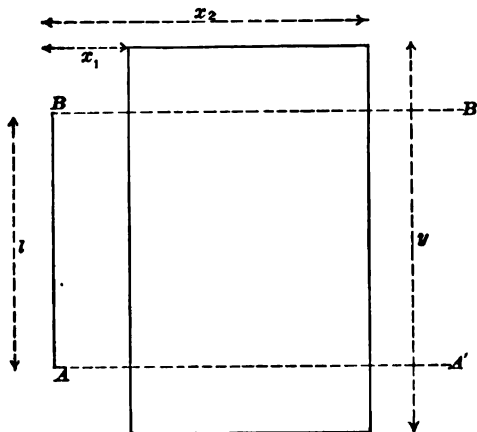


Fig. 7.

flux of a finite straight conductor as derived from the Biot-Savart law has an infinite value, and concludes that the self-inductance is therefore infinite and hence that one can properly

⁴Karl Willy Wagner, *Elek. Zeit.*, July 4, 1907, p. 673.

speak of the self-inductance only for closed circuits. In reaching this conclusion he takes the integral expression given by Sumec³ for the flux through a rectangle of length y and breadth $x_2 - x_1$ due to a finite straight wire of length l , as shown in Fig. 7. He then lets the rectangle expand, x_1 being constant, and the ratio y/x_2 remaining constant until x_2 and y are *both infinite*. This gives an infinite value to the flux, but does not prove the self-inductance of the finite wire AB to be infinite, defining the self-inductance as I have done above. When the current in the wire decreases, the field everywhere decreases in intensity, and we think of the lines as collapsing upon the wire; that is, moving in from all sides upon the wire. But those lines above BB' and below AA' do not cut the wire, and hence contribute nothing to the self-inductance. For no lines of force cut across the lines BB' and AA' (BB' and AA' of course extend to infinity) as the field becomes weaker; the lines above BB' and below AA' collapse upon the *axis extended* of the wire AB.

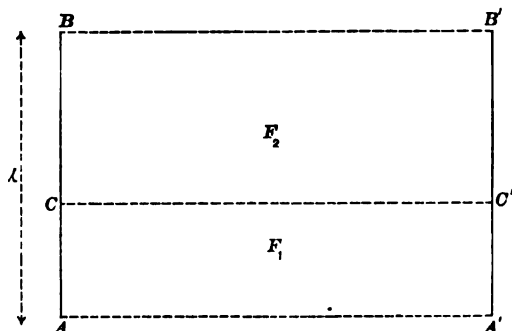


Fig. 8.

Looking at it in another way, suppose the wire is divided into two parts at C, and the field between BB' and AA' is divided into F₁ and F₂. The lines of force in F₁ are due to the whole wire AB and not to AC simply, but in collapsing when the current ceases they cut only AC. So the lines in F₂ are due to the whole wire AB, but they cut only CB. Therefore in getting an expression for the self-inductance of the wire AB we must find the number of lines of

³J. K. Sumec, *Elek. Zeit.*; Dec. 20, 1906, p. 1175.

force included between BB' and AA' integrating to infinity, and this is a finite number, as shown above, (3) or (4).

To repeat what has already been said above, the self-inductance of a finite straight wire means its self-inductance as a part of some closed circuit. The infinite field at a distance due to it is canceled by that due to the other parts of the closed circuit which are not specified. We take account only of those lines which cut the given conductor in calculating its own self-inductance, and of those lines only which cut other parts of the circuit in calculating mutual inductances, ignoring the lines which do neither.

In the case of an oscillating current in a finite straight wire, at the moment when the current i is a maximum and the potential of the wire is sensibly uniform and equal to zero the energy is $\frac{1}{2}Li^2$, where L is the self-inductance and i is the current at the instant.

The value of L is not the value given by (9) nor yet the infinite value found by Wagner, but is a finite value due to the finite conductor taken in connection with the return displacement circuit. It is indeed the self-inductance of a closed circuit, and not simply of the conductor in question. This I take it is what Wagner means, and not that we can not speak of the self-inductance of an unclosed circuit in the sense in which it is done throughout this article.

7. THE SELF-INDUCTANCE OF A STRAIGHT RECTANGULAR BAR.

The self-inductance of a straight bar of rectangular section is, to within the accuracy of the approximate formula (13), the same as the mutual inductance of two parallel straight filaments of the same length separated by a distance equal to the geometrical mean distance of the cross section of the bar. Thus,

$$L = 2l \left[\log \frac{2l}{R} - 1 + \frac{R}{l} \right] \quad (20)$$

where R is the geometric mean distance of the cross section of the rod or bar. If the section is a square, $R = .447 a$, a being the side of the square. If the section is a rectangle, the value of R is given by Maxwell's formula. (E. and M., § 692.) For example, when the rectangular section is 4×1 cm, $R = 1.118$ cm. Thus the self-

inductance of a straight square rod is a little less than that of a round rod of the same diameter, equal indeed to the self-inductance of a round rod of diameter 1.15 times the side of the square.

Sumec has called attention to the fact that the geometrical mean distance for the area of a rectangle is very nearly proportional to the length of the two sides of the rectangle. Putting a and β for the lengths of the two sides of the rectangle, and R for the geometrical mean distance of the rectangle from itself,

$$R = 0.2235 (a + \beta) \text{ nearly for all values of } a \text{ and } \beta.$$

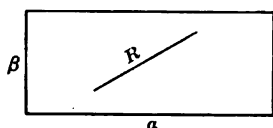


Fig. 9.

The following table shows how nearly constant is the ratio $\frac{R}{a + \beta}$ for rectangles of different proportions:

TABLE I.

a and β are the Length and Breadth of the Rectangles. R is the Geometrical Mean Distance of its Area.

Ratio	R	$\frac{R}{a + \beta}$
1 : 1	0.44705 a	0.22353
1.25 : 1	0.40235 a	0.22353
1.5 : 1	0.37258 a	0.22355
2 : 1	0.33540 a	0.22360
4 : 1	0.27961 a	0.22369
10 : 1	0.24596 a	0.22360
20 : 1	0.23463 a	0.22346
1 : 0	0.22315 a	0.22315

The simple relation between the g. m. d. of a rectangle and the sum of its two sides, $a + \beta$, is rather remarkable and, in view of the complicated formula employed in calculating R for a rectangle, very fortunate. Substituting this value of R in formula (20) we have, since $\log_e \frac{1}{0.2235} = 1.500$ nearly,

$$L = 2l \left[\log \frac{2l}{a+\beta} + \frac{1}{2} + \frac{0.2235(a+\beta)}{l} \right] \quad (21)$$

as the formula for the self-inductance of a straight bar or wire of length l and having a rectangular section of length a and breadth β . Substituting $l=1000$, and $a+\beta=2$ for a square bar 1000 cm long and 1 square cm section we have, neglecting the small last term,

$$\begin{aligned} L &= 2000 \left[\log \frac{2000}{2} + \frac{1}{2} \right] \\ &= 2000 (6.908 + 0.5) = 14816 \text{ cm} \\ &= 14.816 \text{ microhenrys.} \end{aligned}$$

This would also be the self-inductance for any section having $a+\beta=2$ cm.

For a rectangular bar of section 1×4 cm, we have similarly

$$\begin{aligned} L &= 2000 \left[\log \frac{2000}{5} + \frac{1}{2} \right] \\ &= 12.983 \text{ microhenrys.} \end{aligned}$$

For a wire of rectangular section 1×4 mm, and 10 meters long

$$L = 17.588 \text{ microhenrys.}$$

8. TWO PARALLEL BARS.—SELF AND MUTUAL INDUCTANCE.

The mutual inductance of two parallel straight, square, or rectangular bars is equal to the mutual inductance of two parallel wires

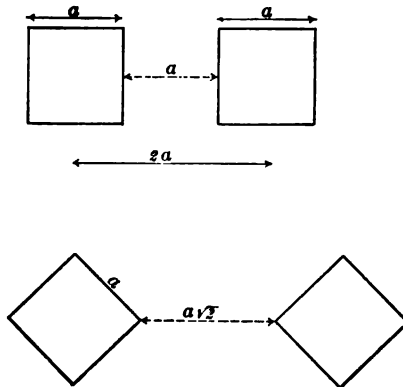


Fig. 10.

or filaments of the same length and at a distance apart equal to the geometrical mean distance of the two areas from one another. This is very nearly equal in the case of square sections to the distance between their centers for all distances, the g. m. d. being a very little greater for parallel squares, and a very little less for diagonal squares⁶ (Fig. 10). We should, therefore, use equation (13) with d equal to the g. m. d. of the sections from one another; that is, substantially, to the distances between the centers. For the two parallel square rods 10 meters long and 1 cm square (Fig. 10) we have therefore for the mutual inductance using (13), and taking $R = 2.0$ cm,⁷

$$\begin{aligned} M &= 2000 \left(\log_e \frac{2000}{2} - 1 \right) \\ &= 2000 (6.9077 - 1) \\ &= 11.815 \text{ microhenrys.} \end{aligned}$$

The self-inductance of a return circuit of two such parallel bars is equal to twice the self-inductance of one minus twice their mutual inductance. That is,

$$\begin{aligned} L &= 2(L_1 - M) \\ &= 2(14.812 - 11.815) \\ &= 5.994 \text{ microhenrys.} \end{aligned}$$

If they were adjacent to one another the self-inductance of the two bars would be (their mutual inductance in that case being 13.190)

$$\begin{aligned} L &= 2(14.812 - 13.190) \\ &= 3.244 \text{ microhenrys.} \end{aligned}$$

These calculations are of course all based on the assumption of a uniform distribution of current through the cross section of the conductors. For alternating currents in which the current density is greater near the surface the self-inductance is less but the mutual inductance is substantially unchanged.

⁶ Rosa, this Bulletin, 8, p. 1.

⁷ Its more exact value is 2.0010, this Bulletin, 8, p. 9.

9. SELF-INDUCTANCE OF A SQUARE.

The self-inductance of a square may be derived from the expression for the self and mutual inductance of finite straight wires from the consideration that the self-inductance of the square is the sum of the self-inductances of the four sides minus the mutual inductances. That is,

$$L = 4L_1 - 4M$$

the mutual inductance of two mutually perpendicular sides being zero. Substituting a for l and d in formulæ (9) and (12) we have

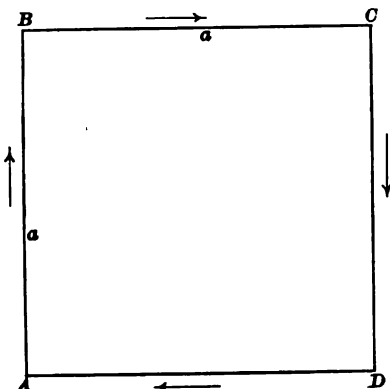


Fig. 11.

$$L_1 = 2a \left[\log \frac{a + \sqrt{a^2 + \rho^2}}{\rho} - \sqrt{1 + \frac{\rho^2}{a^2}} + \frac{1}{4} + \frac{\rho}{a} \right]$$

$$M = 2a \left[\log \frac{a + \sqrt{2}a}{a} - \sqrt{2} + 1 \right]$$

Neglecting ρ^2/a^2 ,

$$L_1 - M = 2a \left[\log \frac{2a}{\rho(1 + \sqrt{2})} - 1.75 + \sqrt{2} + \frac{\rho}{a} \right]$$

$$\therefore L = 4(L_1 - M) = 8a \left[\log \frac{a}{\rho} - \log \frac{1 + \sqrt{2}}{2} + \frac{\rho}{a} - 0.3358 \right] \quad (22)$$

$$\text{or } L = 8a \left(\log \frac{a}{\rho} + \frac{\rho}{a} - 0.524 \right) \quad (22a)$$

where a is the length of one side of the square and ρ is the radius of the wire. If we put $l = 4a =$ whole length of wire in the square,

$$L = 2l \left(\log \frac{l}{\rho} + \frac{4\rho}{l} - 1.910 \right)$$

$$\text{or, } L = 2l \left(\log \frac{l}{\rho} - 1.910 \right), \text{ approximately.} \quad (23)$$

Formulæ (22) and (23) were first given by Kirchhoff⁸ in 1864.

If $a = 100$ cm, $\rho = 0.1$ cm, we have from (22)

$$\begin{aligned} L &= 800 (\log_e 1000 - 0.524) \\ &= 5107 \text{ cm} = 5.107 \text{ microhenrys.} \end{aligned}$$

If $\rho = .05$ cm,

$$L = 5662 \text{ cm} = 5.662 \text{ microhenrys.}$$

That is, the self inductance of such a rectangle of round wire is about 11 per cent greater for a wire 1 mm in diameter than for one 2 mm in diameter.

If l/ρ is constant, L is proportional to l .

That is, if the thickness of the wire is proportional to the length of the wire in the square, the self-inductance of the square is proportional to its linear dimensions.

If in the above case where $\rho = 0.1$, $a = 200$ cm,

$$L = 1600 (\log_e 2000 - 0.524) = 2 \times 5.662 \text{ microhenrys.}$$

That is, for a square 200 cm on a side, L is 11 per cent more than double its value for a square of 100 cm. on a side.

10. SELF-INDUCTANCE OF A RECTANGLE.

(a) *The conductor having a circular section.*

The self-inductance of the rectangle of length a and breadth b is

$$L = 2 (L_a + L_b - M_a - M_b)$$

where L_a and L_b are the self-inductances of the two sides of length a and b taken alone, M_a and M_b are the mutual inductances of the two opposite pairs of length a and b , respectively.

⁸ *Gesammelte Abhandlungen*, p. 176. *Pogg. Annal.* 121, 1864.

From (9) and (12) we therefore have, neglecting ρ^2/a^2 ,

$$\begin{aligned} L = & 4 \left[a \log \frac{2a}{\rho} - \frac{3}{4}a + \rho \right] + 4 \left[b \log \frac{2b}{\rho} - \frac{3}{4}b + \rho \right] \\ & - 4 \left[a \log \frac{a + \sqrt{a^2 + b^2}}{b} - \sqrt{a^2 + b^2} + b \right] \\ & - 4 \left[b \log \frac{b + \sqrt{a^2 + b^2}}{a} - \sqrt{a^2 + b^2} + a \right] \end{aligned}$$

Putting $\sqrt{a^2 + b^2} = d$, the diagonal of the rectangle,

$$\begin{aligned} L = & 4 \left[a \log \frac{2ab}{\rho(a+d)} + d - \frac{3a}{4} - b + \rho \right] \\ & + 4 \left[b \log \frac{2ab}{\rho(b+d)} + d - \frac{3b}{4} - a + \rho \right] \end{aligned}$$

$$\begin{aligned} \text{or} \quad L = & 4 \left[(a+b) \log \frac{2ab}{\rho} - a \log (a+d) - b \log (b+d) \right. \\ & \left. - \frac{7}{4}(a+b) + 2(d+\rho) \right] \end{aligned} \quad (24)$$

For $a = 200$ cm, $b = 100$, $\rho = 0.1$

$$L = 8017.1 \text{ cm} = 8.017 \text{ microhenrys.}$$

(b) *The conductor having a rectangular section.*

For a rectangle made up of a conductor of rectangular section, $a \times \beta$,

$$L_a = 2 \left[a \log \frac{2a}{a+\beta} + \frac{a}{2} + 0.2235(a+\beta) \right]$$

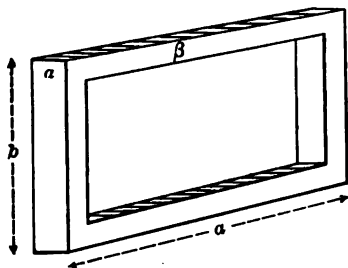


Fig. 12.

$$M_a = 2 \left[a \log \frac{a + \sqrt{a^2 + b^2}}{b} - \sqrt{a^2 + b^2} + b \right]$$

and $L = 2 (L_a + L_b - M_a - M_b)$, as before.

$$\begin{aligned} \therefore L = 4 & \left[a \log \frac{2ab}{(a+\beta)(a+\sqrt{a^2+b^2})} + \frac{a}{2} - b + \sqrt{a^2+b^2} \right. \\ & \left. + 0.2235 (a+\beta)_a \right] \\ & + 4 \left[b \log \frac{2ab}{(a+\beta)(b+\sqrt{a^2+b^2})} + \frac{b}{2} - a + \sqrt{a^2+b^2} \right. \\ & \left. + 0.2235 (a+\beta)_b \right] \end{aligned}$$

Putting as before $d = \sqrt{a^2 + b^2}$ = diagonal of the rectangle, and assuming that the section of the rectangle is uniform; that is, that $(a+\beta)_a = (a+\beta)_b$,

$$\begin{aligned} L = 4 & \left[(a+b) \log \frac{2ab}{a+\beta} - a \log (a+d) - b \log (b+d) \right. \\ & \left. - \frac{a+b}{2} + 2d + 0.447 (a+\beta) \right] \end{aligned} \quad (25)$$

This is equivalent to Sumec's exact formula⁹ (6a), the logarithm of course being natural in (25), as elsewhere in this article.

For $a = b$, a square,

$$\begin{aligned} L &= 8a \left[\log \frac{2a^2}{a+\beta} - \frac{1}{2} + \frac{d}{a} - \log (a+a\sqrt{2}) + 0.2235 \frac{(a+\beta)}{a} \right] \\ &= 8a \left[\log \frac{a}{a+\beta} + 0.2235 \frac{a+\beta}{a} + 0.726 \right] \end{aligned}$$

If $a = \beta$,

$$L = 8a \left[\log \frac{a}{a} + .447 \frac{a}{a} + .033 \right] \quad (25a)$$

If $a = 1000$, $a = 1$,

$$\begin{aligned} L &= 8000 [6.908 + .033] \\ &= 8000 \times 6.941 \text{ cm} = 55.53 \text{ microhenrys.} \end{aligned}$$

⁹ Elek. Zeit., p. 1175, 1906.

For a circular section, of diameter 1 cm, $\rho = 0.5$

$$L = 8000 \left(\log_e 2000 + \frac{1}{2000} - .524 \right) \\ = 8000 \times 7.076 \text{ cm} = 56.61 \text{ microhenrys,}$$

a little *more* than for a square section, as would be expected.

11. MUTUAL INDUCTANCE OF TWO EQUAL PARALLEL RECTANGLES.

For two equal parallel rectangles of sides a and b and distance apart d the mutual inductance is the sum of the several mutual inductances of parallel sides. Writing M_{15} for the mutual inductance of side 1 on 5, etc., we have

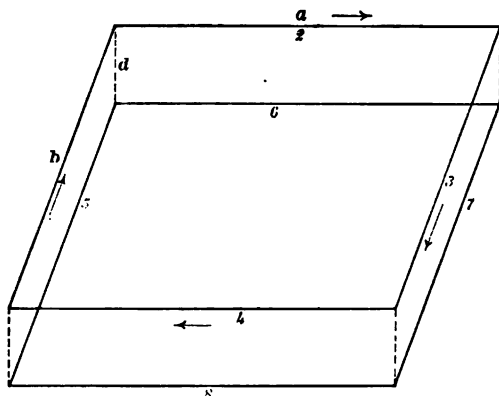


Fig. 13.

$$M = 2 (M_{15} - M_{17}) + 2(M_{26} - M_{28})$$

$$\text{From (12)} \quad M_{15} = 2 \left[b \log \frac{b + \sqrt{b^2 + d^2}}{d} - \sqrt{b^2 + d^2} + d \right]$$

$$M_{17} = 2 \left[b \log \frac{b + \sqrt{a^2 + b^2 + d^2}}{\sqrt{a^2 + d^2}} - \sqrt{a^2 + b^2 + d^2} + \sqrt{a^2 + d^2} \right]$$

$$M_{26} = 2 \left[a \log \frac{a + \sqrt{a^2 + d^2}}{d} - \sqrt{a^2 + d^2} + d \right]$$

$$M_{28} = 2 \left[a \log \frac{a + \sqrt{a^2 + b^2 + d^2}}{\sqrt{b^2 + d^2}} - \sqrt{a^2 + b^2 + d^2} + \sqrt{b^2 + d^2} \right]$$

$$\therefore M = 4 \left[a \log \left(\frac{a + \sqrt{a^2 + d^2}}{a + \sqrt{a^2 + b^2 + d^2}} \cdot \frac{\sqrt{b^2 + d^2}}{d} \right) + b \log \left(\frac{b + \sqrt{b^2 + d^2}}{b + \sqrt{a^2 + b^2 + d^2}} \cdot \frac{\sqrt{a^2 + d^2}}{d} \right) + 8 \left[\sqrt{a^2 + b^2 + d^2} - \sqrt{a^2 + d^2} - \sqrt{b^2 + d^2} + d \right] \right] \quad (26)$$

For a square, where $a = b$, we have

$$M = 8 \left[a \log \left(\frac{a + \sqrt{a^2 + d^2}}{a + \sqrt{2a^2 + d^2}} \cdot \frac{\sqrt{a^2 + d^2}}{d} \right) + 8 \left[\sqrt{2a^2 + d^2} - 2\sqrt{a^2 + d^2} + d \right] \right] \quad (27)$$

These two formulæ (26) and (27) may also be derived by integrating Neumann's formula around the rectangles.¹⁰

Formula (26) was first given by F. E. Neumann¹¹ in 1845.

12. SELF AND MUTUAL INDUCTANCE OF THIN TAPES.

The self-inductance of a straight thin tape of length l and breadth b (and of negligible thickness) is equal to the mutual inductance of two parallel lines of distance apart R_1 equal to the geometrical mean distance of the section, which is $0.22313 \ b$,

$$\begin{array}{ccc} \text{---} \overline{b} \text{---} & (1) & \text{or } \log R = \log b - \frac{3}{2} \\ \\ \text{---} \overline{\frac{+}{b}} \text{---} \overline{\frac{-}{b}} \text{---} & (2) & \text{Thus,} \\ \\ \begin{array}{ccc} \text{---} \overline{\frac{+}{b}} \text{---} & \longleftrightarrow & \text{---} \overline{\frac{-}{b}} \text{---} \\ | & & | \\ | & \xleftarrow{d} \xrightarrow{b} & | \\ | & & | \end{array} & (3) & \\ \begin{array}{ccc} | & \xleftarrow{d} \xrightarrow{b} & | \\ | & & | \end{array} & (4) & \\ \begin{array}{ccc} | & \xleftarrow{b} \xrightarrow{b} & | \\ | & & | \end{array} & (5) & \end{array} \quad \begin{array}{l} L = 2l \left[\log \frac{2l}{R_1} - 1 \right] \text{ approximately} \\ = 2l \left[\log \frac{2l}{b} + \frac{1}{2} \right] \end{array} \quad (28)$$

Fig. 14.

For two such tapes in the same plane, coming together at their edges

¹⁰ Webster, *Electricity and Magnetism*, p. 456. Wallentin, *Theoretische Elektrizitätslehre*, p. 344. Fleming, ———.

¹¹ *Allgemeine Gesetze der Inducirten Ströme*, Abh. Berlin Akad.

without making electrical contact, the mutual inductance is

$$\begin{aligned} M &= 2l \left[\log \frac{2l}{R_1} - 1 \right] \\ &= 2l \left[\log \frac{2l}{b} - 0.8863 \right] \end{aligned} \quad (29)$$

where R_1 is the geometrical mean distance of one tape from the other, which in this case is $0.89252 b$. For a return circuit made up of these two tapes the self-inductance is

$$\begin{aligned} L &= 2L_1 - 2M \\ &= 4l \left(\log \frac{R_2}{R_1} \right) = 4l \log_e 4 \\ &= 5.545 \times \text{length of one tape.} \end{aligned} \quad (30)$$

Thus the self-inductance of such a circuit is independent of the width of the tapes. If the tapes are separated by the distance b , $R_2 = 1.95653 b$ and $L = 8.685 l$.

If the two tapes are not in the same plane but parallel and at a distance apart d (4) Fig. 14, then the geometrical mean distance between the tapes is given by the formula.

$$\log R = \frac{d^2}{b^3} \log d + \frac{1}{2} \left(1 - \frac{d^2}{b^2} \right) \log (b^2 + d^2) + 2 \frac{d}{b} \tan^{-1} \frac{b}{d} - \frac{3}{2} \quad (31)$$

If $d = b$, (5) Fig. 9,

$$\log R_2 = \log b + \frac{\pi}{2} - \frac{3}{2} \quad (32)$$

For a single tape

$$\log R_1 = \log b - \frac{3}{2} \quad (33)$$

Hence $\log \frac{R_2}{R_1} = \frac{\pi}{2}$ and for the case shown at (5) Fig. 14,

$$\begin{aligned} L &= 2L_1 - 2M = 4l \log \frac{R_2}{R_1} \\ &= 4l \frac{\pi}{2} = 2\pi l \end{aligned} \quad (34)$$

In this case also the self-inductance of 2π cm per unit of length of the pair of thin strips is independent of their width so long as the distance apart is equal to their width. The self-inductance is as much greater in this case than in the case shown in (2) Fig. 14, as $\frac{\pi}{2}$ is greater than $\log_e 4$, or 1.13 times.

13. CASE OF TWO PARALLEL PLATES.—NONINDUCTIVE SHUNTS.

If a thin sheet of manganin or other conductor is doubled on itself to form a noninductive shunt we can calculate approximately its self-inductance by the above method.

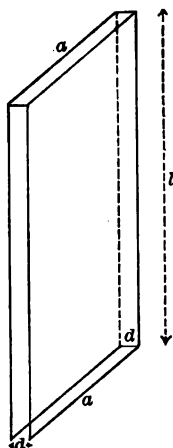


Fig. 15.

$$\text{Let } l = 30 \text{ cm}$$

$$a = 10 \text{ cm}$$

$$d = 1 \text{ cm}$$

$$\text{By (31)} \quad \log R_2 = 1.0787$$

$$\log R_1 = \log_e 10 - \frac{3}{2} = 0.8026$$

$$L = 4l (\log R_2 - \log R_1) = 120 \times 0.2761$$

$$= 33.13 \text{ cm}$$

$$= .0331 \text{ microhenrys.}$$

If the resistance of the shunt is .001 ohm, and the frequency of the current through it is 100 cycles

$$\phi \frac{L}{R} = \tan \phi = .02$$

and ϕ , the angle of lag of the current in the shunt behind the emf. at the terminals is approximately 1° . If $n = 1000$, $\phi = 12^\circ$ nearly. By bringing the two halves of the sheet nearer together ϕ could of course be reduced considerably below 1° for 100 cycles. This would be desirable for high frequencies. If the sheet were used straight in the above example the inductance would be six times as great, unless a return conductor were near.

14. USE OF THE GEOMETRIC MEAN DISTANCE.

In the approximate formula for the mutual inductance of two parallel wires

$$M = 2l \left[\log \frac{2l}{d} - 1 \right]$$

we have only one variable, d . In applying this to determine the self-inductance of a thin, straight strip we make use of the theorem that the self-inductance of a circuit is equal to the sum of all the mutual inductances of the component parts of the circuit; that is, the sum of the inductances of every element upon itself and every other element. If there are n elements each carrying $\frac{1}{n}$ -th of the

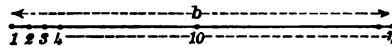


Fig. 16.

current, each of these n^2 component inductances will be multiplied by $\frac{1}{n^2}$, if the total current is unity. Hence we see that the value of the self-inductance is the average value of the n^2 separate mutual inductances. But each mutual inductance is

$$M = 2l (\log 2l - \log d - 1) \quad (35)$$

and as the first and third terms are constant, we have only to find the average value of $\log d$, where d is the distance between every pair of points in the straight line of length b which is the section of the strip. Since

$$\begin{aligned} \frac{1}{n} \left[\log d_1 + \log d_2 + \dots + \log d_n \right] &= \frac{1}{n} \log \left[d_1 d_2 d_3 \dots d_n \right] \\ &= \log \sqrt[n]{d_1 d_2 d_3 \dots d_n} \\ &= \log R \end{aligned}$$

we see that what Maxwell called the geometrical mean distance R of the line is the n^{th} root of the product of the n distances between all the various pairs of points in the line, n being increased to infinity in determining the value of R . This shows why the term *geometrical mean distance* was chosen.

The more exact formula (12) for the mutual inductance of two straight parallel lines may be written

$$M = 2 [l \log (l + \sqrt{l^2 + d^2}) - l \log d - \sqrt{l^2 + d^2} + d] \quad (36)$$

In getting the mean value of this expression for n pairs of points along the line b we must find not only the mean value of $\log d$, but also the mean value of d itself and of d^2 . The latter means will not be the same as the geometrical mean distance R , but are the *arithmetical mean distance* and the *arithmetical mean square distance*. In order, therefore, to obtain an accurate value of the self-inductance L for the strip we should determine these arithmetical mean distances for the section of the strip.

15. DETERMINATION OF THE ARITHMETICAL MEAN DISTANCES OF A LINE.

Let AB be the line of length b , and we first find the arithmetical mean distance S_1 of the point P ($AP = c$) from all the points of the line.

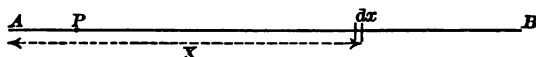


Fig. 17.

This may be done by integration, but the a. m. d. from P to all points in the line to the right of P is, obviously, $\frac{b-c}{2}$, and to the left

$\frac{c}{2}$. Hence for the whole line

$$\begin{aligned} bS_1 &= \frac{b-c}{2}(b-c) + \frac{c}{2}c = \frac{(b-c)^2}{2} + \frac{c^2}{2} \\ \therefore S_1 &= \frac{b}{2} - c + \frac{c^2}{b} \end{aligned} \quad (37)$$

To find S_2 , the a. m. d. of all points of the line from the line, or the a. m. d. of the line from itself we must integrate S_1 over the line. Thus, putting $c = x$,

$$\begin{aligned} bS_2 &= \int_0^b \left(\frac{b}{2} - x + \frac{x^2}{b} \right) dx = \left[\frac{bx}{2} - \frac{x^2}{2} + \frac{x^3}{3b} \right]_0^b \\ &= \frac{b^3}{3} \\ \therefore S_2 &= \frac{b}{3} \end{aligned} \quad (38)$$

Thus while the geometrical mean distance of a line is 0.22313 times its length the arithmetical mean distance is one-third the length.

To find the arithmetical mean square distance S_1^2 from the point P to the line we integrate as follows :

$$\begin{aligned} bS_1^2 &= \int_0^b (x-c)^2 dx = \frac{b^3}{3} - cb^2 + c^2 b \\ \therefore S_1^2 &= \frac{b^2}{3} - cb + c^2 \\ \text{If } c=0, S_1^2 &= \frac{b^2}{3} \end{aligned} \quad (39)$$

That is, the arithmetical mean square distance from one end of a line to the line is $b^2/3$. Also

$$\sqrt{S_1^2} = \frac{b}{\sqrt{3}}$$

To find the a. m. s. d. S_2^2 we integrate again, changing c to x ,

$$\begin{aligned} bS_2^2 &= \int_0^b \left(\frac{b^2}{3} - bx + x^2 \right) dx = \frac{b^3}{6} \\ \therefore S_2^2 &= \frac{b^2}{6}, \sqrt{S_2^2} = \frac{b}{\sqrt{6}} \end{aligned} \quad (40)$$

If now in formula (36) above we make the proper substitutions of geometrical and arithmetical mean distances, we shall obtain a more accurate expression for the self-inductance of a thin strip. Since d is small compared with l , formula (36) is very nearly equal to

$$\begin{aligned} L &= 2 \left[l \log \left(2l \left[1 + \frac{d^2}{4l^2} \right] \right) - l \log d - l - \frac{d^2}{2l} + d \right] \\ &= 2l \left[\log 2l - \log d - 1 - \frac{d^2}{4l^2} + \frac{d}{l} \right] \end{aligned} \quad (41)$$

For $\log d$ put $\log b - \frac{3}{2}$

“ d^2 put $b^2/6$

“ d put $b/3$

Then
$$L = 2l \left[\log \frac{2l}{b} + \frac{1}{2} + \frac{b}{3l} - \frac{b^2}{24l^2} \right] \quad (42)$$

which is the self-inductance of a straight thin strip of length l and breadth b .

This formula neglects only terms in b^4/l^3 , and is therefore quite accurate. The value previously found (equation 28) is the same except for the last two terms. Formula (28) is of course accurate enough for most cases; but it is interesting to see what the more accurate expression is when we make use of the arithmetical mean distances in getting the values of the terms neglected in the first approximation.

16. SELF-INDUCTANCE OF A CIRCLE OF THIN STRIP.

Let us apply the principle of geometrical and arithmetical mean distances to obtain the self-inductance of a circular band of radius a and width b . This is a short cylindrical current sheet, for which we have the formula of Rayleigh:

$$L = 4\pi a \left[\log \frac{8a}{b} - \frac{1}{2} + \frac{b^2}{32a^2} \left(\log \frac{8a}{b} + \frac{1}{4} \right) \right] \quad (43)$$

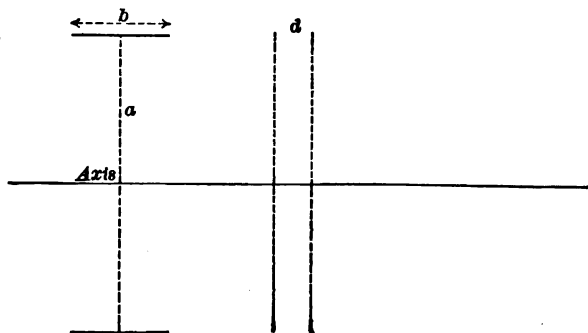


Fig. 18.

Coffin's formula gives some additional terms in b^4/l^3 and higher powers, but when b is not more than one-fourth of a , Rayleigh's formula is correct to within about one part in 100,000, and may therefore serve as a check on the method of geometrical and arithmetical mean distances. The mutual inductance of two parallel

circles of radius a and distance apart d is, neglecting terms in d^3/a^4 and higher powers,

$$M = 4\pi a \left[\left(1 + \frac{3d^2}{16a^2} \right) \log \frac{8a}{d} - \left(2 + \frac{d^2}{16a^2} \right) \right] \quad (44)$$

which may be written

$$M = 4\pi a \left[\left(1 + \frac{3d^2}{16a^2} \right) \log 8a - \log d - \frac{3d^2}{16a^2} \log d - 2 - \frac{d^2}{16a^2} \right] \quad (45)$$

In addition to the g. m. d. to be used in the second term and the arithmetical mean square distance in the first and last terms we have to know the mean value of a product of g. m. d. and a. m. s. d. in the third term; that is, a term of the form

$$S_1^2 \log R_1$$

To get this we must integrate as follows:

$$\begin{aligned} b S_1^2 \log R_1 &= \int_0^b (x-c)^2 \log (x-c) dx = \\ &= \frac{(b-c)^3}{3} \left[\log (b-c) - \frac{1}{3} \right] + \frac{c^3}{3} \left[\log c - \frac{1}{3} \right] \end{aligned} \quad (46)$$

$$\begin{aligned} b^2 S_1^2 \log R_1 &= \frac{1}{3} \int_0^b (b-x)^2 \log (b-x) dx + \frac{1}{3} \int_0^b x^2 \log x dx \\ &= -\frac{1}{9} \int_0^b (b-x)^2 dx - \frac{1}{9} \int_0^b x^2 dx \\ &= \frac{b^3}{6} \left(\log b - \frac{7}{12} \right) \end{aligned} \quad (47)$$

$$\therefore S_1^2 \log R_1 = \frac{b^2}{6} \left(\log b - \frac{7}{12} \right) \quad (48)$$

We may now substitute in (39) as follows:

$$\begin{aligned} \log d &= \log b - \frac{3}{2} \\ 3 d^2 &= \frac{b^2}{2} \\ 3 d^2 \log d &= \frac{b^2}{2} \left(\log b - \frac{7}{12} \right) \end{aligned}$$

This gives

$$\begin{aligned}
 L &= 4\pi a \left[\left(1 + \frac{b^2}{32a^2} \right) \log 8a - \log b + \frac{3}{2} - \frac{b^2}{32a^2} \left(\log b - \frac{7}{12} \right) - 2 - \frac{b^2}{96a^2} \right] \\
 &= 4\pi a \left[\left(1 + \frac{b^2}{32a^2} \right) \log \frac{8a}{b} - \frac{1}{2} + \frac{b^2}{128a^2} \right] \quad (49)
 \end{aligned}$$

which is Rayleigh's equation (43). This confirms the values of the quantities S_1^2 and $S_2^2 \log R_1$ employed in deducing the equation (49) from the formula (44) for M for two parallel circles.

17. ARITHMETICAL MEAN DISTANCES FOR A CIRCLE.

The arithmetical mean distance of any point P on the circumference of a circle from the circle is found by integrating around the circumference. Thus, since $PB = 2a \cos \theta$

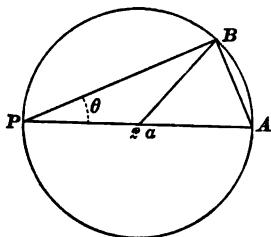


Fig. 19.

$$\begin{aligned}
 \pi a S_1 &= \int_0^{\frac{\pi}{2}} 2a \cos \theta \cdot 2a d\theta = 4a^2 \\
 \therefore S_1 &= \frac{4a}{\pi} \quad (50)
 \end{aligned}$$

Since the a. m. d. is the same for every point of the circle we have also

$$S_2 = \frac{4a}{\pi} \quad (51)$$

For the arithmetical mean square distance we have

$$\begin{aligned}
 \pi a S_1^2 &= \int_0^{\frac{\pi}{2}} 4a^2 \cos^2 \theta \cdot 2a d\theta = 2\pi a^3 \\
 \therefore S_1^2 &= S_2^2 = 2a^2 \\
 \text{and } \sqrt{S_1^2} &= a\sqrt{2} \quad (52)
 \end{aligned}$$

That is, the square root of the arithmetical mean square distance of every point on a circumference of a circle from every other point is equal to the radius of the circle into the square root of 2.

For a point P outside or inside the circle we have, since

$$\overline{PB}^2 = a^2 + d^2 + 2ad \cos \theta$$

$$\pi a S_1^2 = a \int_0^{2\pi} (a^2 + d^2 + 2ad \cos \theta) d\theta = \pi a (d^2 + a^2)$$

$$\therefore S_1^2 = d^2 + a^2 \quad (53)$$

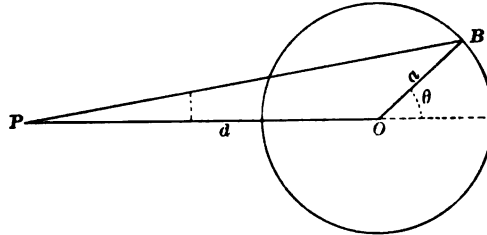


Fig. 20.

For the entire area of the circle with respect to the point P

$$\pi a^2 S_1^2 = \int_0^a (d^2 + r^2) 2\pi r dr = 2\pi \left[\frac{a^2 d^2}{2} + \frac{a^4}{4} \right]$$

$$\therefore S_1^2 = d^2 + \frac{a^2}{2} \quad (54)$$

If $d=0$, $S_1^2 = \frac{a^2}{2}$, the value for the area of the circle with respect to the center of the circle.

For the area of a circle with respect to itself, the a. m. s. d., S_s^2 would be found by integrating $\overline{P_1 P_2}^2 = r_1^2 + r_2^2 - 2 r_1 r_2 \cos (\theta_2 - \theta_1)$ twice over the area of the circle.

This was done in effect by Wien¹² in getting his formula for the self-inductance of a circle, which is a little more accurate than the formula deduced by the use of geometrical mean distances only, when the geometrical mean distances are used for the arithmetical mean distances.

These examples are sufficient to illustrate the differences in the values of the geometrical mean distances and the arithmetical mean

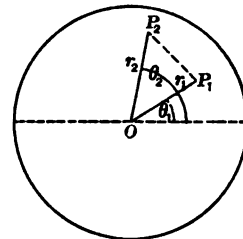


Fig. 21.

¹² M. Wien: Wied. Annal. 58, p. 928, 1894.

distances, and the use of the latter in the calculation of self and mutual inductances.

18. CONCENTRIC CONDUCTORS.

The self-inductance of a thin, straight tube of length l and radius a_2 is, when a_2/l is very small,

$$L_2 = 2l \left[\log \frac{2l}{a_2} - 1 \right] \quad (55)$$

The mutual inductance of such a tube on a conductor within it is equal to its self-inductance, since all the lines of force due to the outer tube cut through the inner when they collapse on the cessation of current. The self-inductance of the inner conductor, suppose a solid cylinder, is

$$L_1 = 2l \left[\log \frac{2l}{a_1} - \frac{3}{4} \right]$$

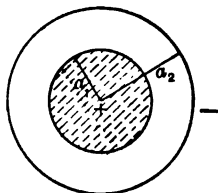


Fig. 22.

If the current goes through the latter and returns through the outer tube the self-inductance of the circuit is,

$$L = L_1 + L_2 - 2M = L_1 - L_2$$

since M equals L_2

$$\therefore L = 2l \left[\log \frac{a_2}{a_1} + \frac{1}{4} \right] \quad (56)$$

This result can also be obtained by integrating the expression for the force outside a_1 between the limits a_1 and a_2 , and adding the term (equation 6) for the field within a_1 , there being no magnetic field outside a_2 . Thus

$$L = l \int_{a_1}^{a_2} \frac{2dr}{r} + \frac{l}{2} = 2l \left[\log \frac{a_2}{a_1} + \frac{1}{4} \right], \text{ as above.}$$

If the outer tube has a thickness $a_3 - a_2$ and the current is distributed uniformly over its cross section the self-inductance will be a

little greater, the geometrical mean distance from a_1 to the tube, which is now more than a_2 and less than a_3 , being given by the expression

$$\log a_g = \frac{a_3^2 \log a_3 - a_2^2 \log a_2}{a_3^2 - a_2^2} - \frac{1}{2} \quad (57)$$

Putting this value of $\log a$ in (56) in place of $\log a$, we should have the self-inductance of the return circuit of Fig. 23.

If the current is alternating and of very high frequency, the current would flow on the outer surface of a_1 and on the inner surface of the tube, and L for the circuit would be

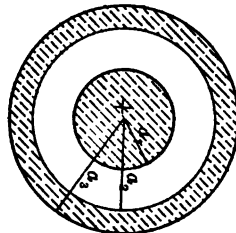


Fig. 23.

$$L = 2l \log \frac{a_3}{a_1} \quad (58)$$

19. MULTIPLE CONDUCTORS.

If a current be divided equally between two wires of length l , radius ρ and distance d apart, the self-inductance of the divided conductor is the sum of their separate self-inductances plus twice their mutual inductance.

Thus, when d/l is small,

$$L = 2 \left\{ \frac{l}{2} \left[\log \frac{2l}{\rho} - \frac{3}{4} \right] + \frac{l}{2} \left[\log \frac{2l}{d} - 1 \right] \right\}$$

$$\text{or } L = 2l \left[\log \frac{2l}{(\rho d)^{\frac{1}{2}}} - \frac{7}{8} \right] = 2l \left[\log \frac{2l}{(r_g d)^{\frac{1}{2}}} - 1 \right] \quad (59)$$

where r_g is the g. m. d. of the section of the wire = 0.7788 ρ .

If there are three straight conductors in parallel and distance d apart, as shown in Fig. 24, the self-inductance is similarly

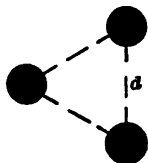
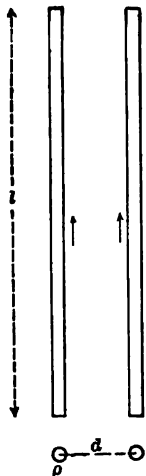


Fig. 24.

$$L = 2l \left[\log \frac{2l}{(r_g d)^{\frac{1}{2}}} - 1 \right] \quad (60)$$

The expression $(r_g d^2)^{\frac{1}{2}}$ is the g. m. d. of the multiple conductor. For example, suppose in the last case $l=1000$ cm, $\rho=2$ mm, $d=1$ cm. Then $(r_g d^2)^{\frac{1}{2}}=0.538$ cm and

$$L=2000 \left[\log_e \frac{2000}{0.5380} - 1 \right] \\ = 2000 \times 7.221 \text{ cm} = 14.442 \text{ microhenrys.}$$

If the whole current flowed through a single one of the three conductors the self-inductance would be

$$L=2000 \left[\log_e \frac{2000}{0.2} - \frac{3}{4} \right] = 17.92 \text{ microhenrys,}$$

or about 25 per cent more than when divided among the three.

Guye has shown¹³ how, by the principle of the geometrical mean distance, one can calculate very readily the self-inductance for any number of similar conductors in multiple when they are distributed in a circle, Fig. 25.

If there are n conductors uniformly spaced on a circle the geometrical mean distance R of the system is given by

$$\log R = \frac{n \log r_1 + n \log (r_{12} \cdot r_{13} \cdot \dots \cdot r_{1n})}{n^2} \\ = \frac{1}{n} (\log r_1 + \log (r_{12} \cdot r_{13} \cdot \dots \cdot r_{1n})) \quad (61)$$

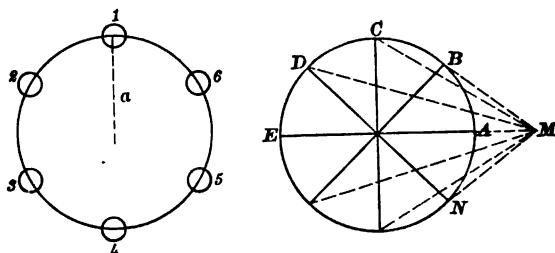


Fig. 25.

where r_1 is the g. m. d. of a single conductor ($=0.7788\rho$, ρ being its radius) and r_{12} is the distance between centers of the conductors 1 and 2, etc. If a is the radius of the circle on which the conductors are distributed

¹³ C. E. Guye, Comptes Rendus, 118, p. 1329; 1894.

$$\log R = \frac{1}{n} \log (r_1 n a^{n-1})$$

$$\text{or } R = (r_1 n a^{n-1})^{\frac{1}{n}} \quad (62)$$

The proof of this, as given by Guye, depends on the following theorem:

If the circumference of a circle is divided into n equal parts by the points A, B, C, . . . and M be any point on the line through OA (inside or outside the circle), then putting $OM = x$

$$X^n - a^n = MA \cdot MB \cdot \dots \cdot MN \quad (\text{Cotes's theorem}).$$

Dividing by $MA = x - a$,

$$x^{n-1} + ax^{n-2} + \dots + a^{n-1} = MB \cdot MC \cdot \dots \cdot MN$$

Making M coincide with A, and hence $x = a$,

$$na^{n-1} = AB \cdot AC \cdot \dots \cdot AN$$

$$= a_{12} \cdot a_{13} \cdot \dots \cdot a_{1n}$$

which substituted in (61) gives (62).

Since the self-inductance of a length l of the multiple system is equal to

$$L = 2l \left[\log \frac{2l}{R} - 1 \right] \quad (63)$$

we see that the calculation for any case is simple when ρ , a , and n are given. Thus, suppose $a = 2$ cm, $\rho = 0.5$ cm, and $n = 6$,

$$r_1 n a^{n-1} = 0.3894 \times 6 \times 32$$

$$\therefore R = (74.765)^{\frac{1}{6}} = 2.0525$$

If the separate conductors have only half the diameter supposed, namely, $\rho = 0.25$, the g. m. d. R will be considerably less. In this case

$$R = (0.1947 \times 192)^{\frac{1}{6}} = 1.8285 \text{ cm.}$$

Thus, in the first case, the self-inductance of the six parallel conductors is equal to that of a thin tube of radius 2.0525 cm, and in the second case to that of a tube of radius 1.8285 cm. As n increases and ρ decreases the value of R approaches 2 cm as a limit, the multiple conductors forming in the limit a tube of infinitesimal thickness, the value of R for which is its radius a , in this case 2 cm.

If a larger conductor at the center carries the going current, and the return is by the multiple conductor, the mutual inductance of the larger upon the others is

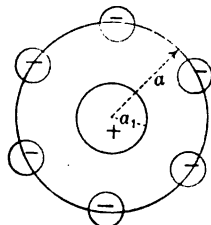


Fig. 26.

$$M = 2l \left[\log \frac{2l}{a} - 1 \right] \quad (64)$$

since the g. m. d. of the central conductor on each of the others is a .

The self-inductance of the return system is

$$\begin{aligned} L &= L_1 + L_2 - 2M \\ \text{where} \quad L_1 &= 2l \left[\log \frac{2l}{a_1} - \frac{3}{4} \right] \\ L_2 &= 2l \left[\log 2l - \log (r_1 n a^{n-1})^{\frac{1}{n}} - 1 \right] \\ M &= 2l \left[\log \frac{2l}{a} - 1 \right] \\ \therefore L &= 2l \left[\log \frac{a^2}{a_1} - \log (r_1 n a^{n-1})^{\frac{1}{n}} + \frac{1}{4} \right] \end{aligned} \quad (65)$$

For $a = 2$ cm, $a_1 = 1$ cm, $\rho = 0.5$, $n = 6$, $l = 1000$ cm

$$\begin{aligned} L &= 2000 \left[\log_e 4 - \log_e (2.0525) + \frac{1}{4} \right] \\ &= 2000 \times 0.9173 = 1.835 \text{ microhenrys.} \end{aligned}$$

If the inner conductor were surrounded by a very thin tube of radius 2 cm for a return, in place of the six wires, the self-inductance of the return circuit would be

$$\begin{aligned} L &= 2000 \left[\log \frac{a}{a_1} + \frac{1}{4} \right] \\ &= 2000 \times 0.9431 \text{ cm} = 1.886 \text{ microhenrys,} \end{aligned} \quad (66)$$

a little greater than in the preceding case.

If the central conductor is also a multiple system, Guye has shown¹³ how to find the mutual inductance of the two when the arrangement is symmetrical, the g. m. d. of the two systems being derived by the aid of Cotes's theorem. In this case

$$\log R_{12} = \frac{1}{n} \log (a_2^n - a_1^n)$$

$$\text{or } R_{12} = (a_2^n - a_1^n)^{\frac{1}{n}} \quad (67)$$

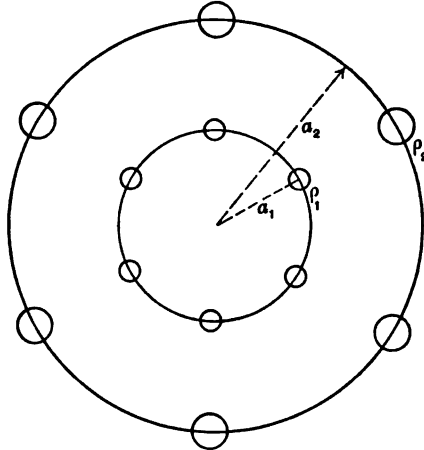


Fig. 27.

Thus, if $a_2 = 2$ cm, $a_1 = 1$ cm, and $n = 6$,

$$R = (64 - 1)^{\frac{1}{6}} = 1.9947$$

and as n increases R_{12} approaches 2 as a limit, as it would be for two concentric tubes of radii 1 and 2. R_1 and R_2 for the two separate systems being given by (62) and R_{12} by (67), the self-inductance of a return circuit with one system for the going current and the other for the return is readily calculated, being

$$L = 2l \log \frac{R_1 R_2}{R_{12}} \quad (68)$$

These examples are sufficient to illustrate the calculation of the self and mutual inductance of multiple circuits by the principle of the geometrical mean distance.

20. SELF-INDUCTANCE OF A "NONINDUCTIVE" WINDING OF ROUND WIRES.

Suppose a to-and-fro winding of insulated wire in a plane, the length of each section being l , the distance apart of the adjacent

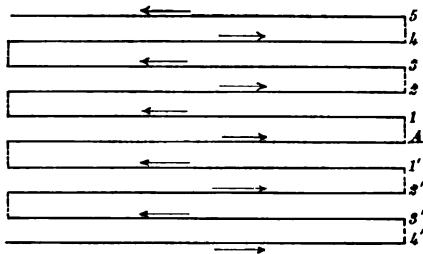


Fig. 28.

wires, center to center, being d , and the wire of radius ρ . The resultant self-inductance of one of the wires is equal to the self-inductance of the wire taken by itself plus the mutual inductance of all the others upon it. The mutual inductance of wires 2, 2', 4, 4', 6, 6', etc., upon wire A at the middle of the group tends to increase the self-inductance of A, while the mutual inductance of 1, 1', 3, 3', 5, 5', etc., tends to decrease the self-inductance of A. Hence, if L_1 is the self-inductance of A by itself and M_1 is the mutual inductance of wire 1 on A, etc., we have for L_A the total resultant self-inductance of wire A of length l the following expression:

$$\begin{aligned}
 L_A &= L_1 + 2M_2 + 2M_4 + 2M_6 + \dots + 2M_{n-1} - 2M_1 - 2M_3 \\
 &\quad - 2M_5 - 2M_7 - \dots - M_n \\
 &= (L_1 - M_1) - (M_1 - M_2) - (M_3 - M_4) - (M_5 - M_6) - \dots \\
 &\quad + (M_2 - M_3) + (M_4 - M_5) + (M_6 - M_7) + \dots \quad (69) \\
 &= 2l \left[\log \frac{d}{\rho} + \frac{1}{4} \right] - 2l \log \frac{2}{1} - 2l \log \frac{4}{3} - 2l \log \frac{6}{5} - \dots \\
 &\quad + 2l \log \frac{3}{2} + 2l \log \frac{5}{4} + 2l \log \frac{7}{6} + \dots \\
 &= 2l \left[\log \frac{d}{\rho} + \frac{1}{4} \right] - 2l \log \left(\frac{2 \cdot 4 \cdot 6 \cdot 8 \dots}{1 \cdot 3 \cdot 5 \cdot 7 \dots} \right) + 2l \log \left(\frac{3 \cdot 5 \cdot 7 \cdot 9 \dots}{2 \cdot 4 \cdot 6 \cdot 8 \dots} \right)
 \end{aligned}$$

$$= 2l \left[\log \frac{d}{\rho} + \frac{1}{4} \right] - 2l \log \left(\frac{2^3 \cdot 4^3 \cdot 6^3 \cdot 8^3 \dots}{1^3 \cdot 3^3 \cdot 5^3 \cdot 7^3 \cdot 9^3 \dots} \right)$$

$$\text{or } L = 2l \left[\log \frac{d}{\rho} + \frac{1}{4} - 2 \log \left(\frac{2 \cdot 4 \cdot 6 \cdot 8 \dots (n-1)}{1 \cdot 3 \cdot 5 \cdot 7 \dots (n-2) \sqrt{n}} \right) \right] \quad (70)$$

(where $2n$ is the whole number of wires)

$$= 2l \left[\log \frac{d}{\rho} + \frac{1}{4} - A \right] \quad (71)$$

where the constant A depends on the whole number of wires. Since

$$\frac{1 \cdot 3 \cdot 5 \cdot 7 \dots (2n-1)}{2^3 \cdot 4^3 \cdot 6^3 \dots (2n)^3} = \frac{\pi}{2}$$

when n is infinite,¹⁴ we see that equation (70) becomes for an infinite number of wires

$$L = 2l \left[\log \frac{d}{\rho} + \frac{1}{4} - \log \frac{\pi}{2} \right]$$

$$= 2l \left[\log \frac{2d}{\pi \rho} + \frac{1}{4} \right] \quad (72)$$

This formula (72) was first given by Mr. G. A. Campbell.¹⁵ He has recently communicated to me by letter his (unpublished) demonstration, which is different from that given above and gives the value of L for an infinite number of wires and not for any finite number, which is the more important case.

For $2n=2$, $A=0$

$$2n=4, A = \log_2 2 = .6931$$

$$2n=6, A = \log_3 \frac{4}{3} = .2876$$

$$2n=10, A = \log_5 \frac{64}{45} = .3522$$

$$2n=14, A = \log_7 \frac{256}{175} = .3804$$

$$2n=18, A = 2 \log_{10} \frac{128}{105} = .3961$$

¹⁴ Loney's Trigonometry II, p. 155.

¹⁵ Elect. World, 44, p. 728; 1904. There is an error in this formula as originally printed; it should have $l/2$ for coefficient instead of l .

$$2n = 38, A = \dots = .4253$$

$$2n = 70, A = \dots = .4373$$

$$2n = \infty, A = \log_e \frac{\pi}{2} = .4516$$

Thus we see that the resultant self-inductance of the middle wire of such a "noninductive" system is always less than that of a single pair, by the quantity $2Al$. If the winding is such that $d = 3\rho$, $\log_e \frac{d}{\rho} + \frac{1}{4} = 1.35$, approximately, and the self-inductance of the middle wire is about two-thirds as much when there is a great number of such wires side by side as when there is a single pair, and about three-fourths as much if there are 10 such wires (5 pairs).

The self-inductance of the end wire will be more, being

$$\begin{aligned} L &= L_1 - M_1 + M_2 - M_3 + M_4 - \dots \\ &= 2l \left[\log \frac{d}{\rho} + \frac{1}{4} \right] + (M_2 - M_1) + (M_4 - M_3) + \dots \\ &= 2l \left[\log \frac{d}{\rho} + \frac{1}{4} + \log \left(\frac{3 \cdot 5 \cdot 7 \dots}{2 \cdot 4 \cdot 6 \dots} \right) \right] \\ &= 2l \left[\log \frac{d}{\rho} + \frac{1}{4} - A_1 \right] \end{aligned}$$

where $A_1 = -\log_e \frac{3}{2}$ for $2n = 4$

$$= -\log_e \frac{15}{8} \text{ " } 2n = 6 \text{ etc.}$$

For the next to the end wire

$$L = 2l \left[\log \frac{d}{\rho} + \frac{1}{4} - A_2 \right]$$

where $A_2 = \log_e \frac{8}{3}$ for $2n = 4$

$$= \log_e \frac{16}{5} \text{ " } 2n = 6, \text{ etc.}$$

For the second from end wire

$$A_3 = \log_e \frac{16}{15} \text{ for } 2n = 8$$

These examples show how the self-inductance of any particular wire of such a winding may be computed and the average or total value found. For a large number the average value of A would evidently not be far from 0.40.

21. CASE OF NONINDUCTIVE WINDING ON A CIRCULAR CYLINDER.

The self-inductance of a single circular turn would be approximately

$$L_1 = 4\pi a \left[\log \frac{8a}{\rho} - 1.75 \right]$$

where a is the radius of the circle and ρ is the radius of section of the wire. The mutual inductance M_1 of two adjacent turns is

$$M_1 = 4\pi a \left[\log \frac{8a}{d} - 2 \right], \text{ approximately,}$$

where d is the distance between centers of the two turns. This approximation is close only so long as d is small compared with a .

Similarly $M_2 = 4\pi a \left[\log \frac{8a}{2d} - 2 \right]$ and hence

$$M_1 - M_2 = 4\pi a (\log 2)$$

$$M_2 - M_3 = 4\pi a \left(\log \frac{4}{3} \right), \text{ etc.}$$

$$\begin{aligned} \therefore L_n &= L_1 - 2M_1 + 2M_2 - 2M_3 + M_4 \\ &= L_1 - M_1 - (M_1 - M_2) + (M_2 - M_3) - (M_3 - M_4) \\ &= 4\pi a \left[\log \frac{d}{\rho} + \frac{1}{4} \right] - 4\pi a \left[\log 2 - \log \frac{3}{2} + \log \frac{4}{3} \right] \text{ for } 2n = 8 \text{ turns} \\ &= 4\pi a \left[\log \frac{d}{\rho} + \frac{1}{4} - 2 \log \left(\frac{2 \cdot 4 \cdot 6 \dots (n-1)}{1 \cdot 3 \cdot 5 \dots (n-2) \sqrt{n}} \right) \right] \text{ for } 2n \text{ turns} \\ &= 4\pi a \left[\log \frac{d}{\rho} + \frac{1}{4} - A \right] \end{aligned} \quad (73)$$

where A has the same values as in the case of parallel straight wires laid noninductively in a plane, provided the length of the coil is small compared with the radius, so that the approximate formula

for M is sufficiently exact. The values of A are positive except for the end wire and depend on the number of wires and the position of the particular wire in the winding. If the radius is not large in proportion to the length of the coil, the constant A is a little less than for the wires in a plane. It is to be noted that the self-inductance of such a winding does not depend on the size of the wire, but on the ratio $\frac{d}{\rho}$, so that if a fine wire has a proportionally close spacing its self-inductance is the same. For a given pitch the self-inductance is of course greater as the wire is finer.

Taking A for the entire spool as approximately equal to 0.40 formula (73) becomes

$$L = 2l \left[\log \frac{d}{\rho} - 0.15 \right] \text{ approximately} \quad (74)$$

where L is the self-inductance of a "noninductive" winding in a plane or on a cylinder of any radius, l being the total length of the wire, ρ is radius and d the mean distance between adjacent turns, center to center. In practice d/ρ can not be obtained with great precision, so that an accurate value of the constant A is not necessary. Moreover, the precise value of L for such a winding is seldom or never required.

A spool of 200 turns of wire with $a = 1$ cm, $d/\rho = 4$, (taking $A = 0.4$) would have a self-inductance of

$$\begin{aligned} L &= 4\pi a (\log_e 4 - 0.15) \times 200 \\ &= 800\pi (1.386 - 0.15) \\ &= 3100 \text{ cm} = 3.1 \text{ microhenrys.} \end{aligned}$$

Since deriving the above expressions Mr. Campbell has sent me an expression for the mean value of the constant A for a noninductive winding in a plane for any number of wires. His demonstration is as follows:

Take $2n$ straight conductors, each of radius ρ , lying in a plane with adjacent wires at distance d center to center as before, the current traversing adjacent wires in opposite directions. We may regard this as a system of n circuits, each consisting of two adjacent wires. The total inductance will be equal to n times the self-inductance of one of the circuits plus $2(n-1)$ times the mutual

inductance of two adjacent circuits plus plus
two times the mutual inductance of extreme circuits. That is,
per unit of length of the system:

$$\begin{aligned}
 L &= n \left(4 \log \frac{d}{\rho} + 1 \right) + 2(n-1) 2 \log \frac{1.3}{2.2} + 2(n-2) 2 \log \frac{3.5}{4.4} + \dots \\
 &\quad + 2.2 \log \frac{(2n-3)(2n-1)}{(2n-2)(2n-2)} \\
 &= 4n \left(\log \frac{d}{\rho} + \frac{1}{4} \right) + 4 \log \left[\left(\frac{1.3}{2.2} \right)^{n-1} \left(\frac{3.5}{4.4} \right)^{n-2} \dots \frac{(2n-3)(2n-1)}{(2n-2)(2n-2)} \right]
 \end{aligned}$$

or for a total length of wire l

$$L = 2l \left[\log \frac{d}{\rho} + \frac{1}{4} - \frac{1}{n} \sum_{k=1}^{n-1} (n-k) \log \frac{4k^2}{4k^2-1} \right] \quad (75)$$

$$\begin{aligned}
 &= 2l \left[\log \frac{d}{\rho} + \frac{1}{4} - \log \frac{\pi}{2} + \frac{1}{4(n-1)} \right. \\
 &\quad \left. + \frac{0.5772 + \log(n-1)}{4n} \right] \text{approximately} \quad (76)
 \end{aligned}$$

The value of the constant A of formula (71) as calculated by Mr. Campbell for various numbers of pairs of wires are given in the following table:

TABLE I.

The Constant A for Formula (71)

n	Value of A by Formula (75)	n	Value of A by Formula (75)
1	.000	10	.350
2	.144	15	.377
3	.213	20	.392
4	.255	25	.402
5	.283	30	.409
6	.304	100	.436
7	.319	200	.443
8	.332	300	.446
9	.342	Infinity	.452

As stated above, the constant A of formula (73) is very nearly the same as in (71), and hence the same values may be used for approximate calculations.

The above results by no means exhaust the subject of the self and mutual inductance of linear conductors; enough has been given, however, to serve in some measure as a guide in solving other cases arising in practice.

WASHINGTON, September 15, 1907.

O



Fig. 1.

THE ATOMIC WEIGHT OF CHLORINE.

By William A. Noyes and H. C. P. Weber.

The ratio of the atomic weights of oxygen and chlorine is of extreme importance on account of the number of atomic weights based either directly or indirectly upon the atomic weight of chlorine.

During the last few years alone, since the determination of the ratio of silver to chlorine by Richards and Wells,¹ the atomic weights of a considerable number of the common elements have been determined, basing them on the value of chlorine. These values have been calculated on the oxygen basis, assuming that the ratio silver : oxygen : 107.93 : 16 is correct. Guye and Ter-Gazarian² have called attention to a possible source of error in the chlorate ratio of Stas, correction for which would bring the value of silver down to 107.89. The newly accepted value 14.01 for nitrogen also points to the lower value of 107.89. Very nearly at the close of this work conclusive evidence has been presented by Richards and Forbes³ and by Richards and Jones⁴ that the value 107.93 for silver is too high. The only direct comparison between hydrogen and chlorine which we have is that of Dixon and Edgar.⁵ In this determination hydrogen was made to burn in an excess of chlorine. The hydrogen was weighed absorbed in palladium and the chlorine in the liquid form in a glass bulb. The hydrogen and chlorine were caused to unite in a large globe of glass containing a small quantity of water to absorb the hydro-

¹ J. Amer. Chem. Soc., **27**, p. 459; 1905.

² Compt. Rend., **143**, p. 411.

³ J. Amer. Chem. Soc., **29**, p. 808; 1907.

⁴ Ibid., **29**, p. 826; 1907.

⁵ Phil. Trans., Series A, **205**, p. 169. Chem. News, **91**, p. 263. The determinations of Deutsch (Dissertation, 1905), from the laboratory of Professor Guye, are scarcely of sufficient accuracy to be considered as atomic weight determinations.

chloric acid formed. Corrections were applied for the quantity of chlorine remaining uncombined, by titrating the amount of iodine liberated by it from a solution of potassium iodide. A further correction was applied for the amount of chlorine used up in liberating oxygen from water, by determining the amount of oxygen set free.

The fact that there was only one such direct determination of the ratio between chlorine and hydrogen, together with the opportunity afforded of carrying out the determination with hydrogen prepared in the same apparatus used for generating the hydrogen in the recent determination of the ratio of hydrogen to oxygen, made a new determination seem to be worth while.

The method we have used, besides being a direct comparison between hydrogen and chlorine, involves the principle of complete synthesis with the determination of the weights of all the substances reacting and of the reaction products formed. Briefly stated, the method consists in weighing the hydrogen absorbed in palladium and the chlorine in the form of potassium chloroplatinate. The hydrogen is passed over the heated potassium chloroplatinate, from which it removes chlorine to form hydrochloric acid. The hydrochloric acid formed is condensed in a third section of the apparatus and weighed. We have thus the weight of hydrogen used, the weight of chlorine removed, and the weight of hydrochloric acid formed. In this manner two series of ratios, each independent of the other, are obtained.

Working in this manner and with hydrogen prepared under the same conditions and at the same time as that used in the determination of the ratio of hydrogen to oxygen by one of us, we believe that we have very favorable conditions for bridging the gap.



PURIFICATION OF MATERIALS AND WEIGHING.

Hydrogen.—The hydrogen used in these experiments was prepared and purified in the same manner as described in a previous paper on the atomic weight of hydrogen.⁶

⁶ Noyes, this Bulletin, 4, p. 179.
J. Am. Chem. Soc., Dec., 1907.

The gas was taken from the generating apparatus at intervals covered by the period of the work on hydrogen. Consequently all remarks concerning its character and purity as used in the hydrogen-oxygen ratio apply to these determinations. As in that work so in this, two methods of generating the hydrogen were employed. In the last series of determinations the hydrogen was obtained by the electrolysis of a solution of barium hydroxide.

Platinum.—The platinum used in the preparation of potassium chloroplatinate was originally obtained in the form of platinum sponge. The preliminary purification consisted in dissolving this in aqua regia and evaporating to remove nitric acid. The separation from other platinum metals was carried on according to the method of Schneider and Seubert, as described in Graham Otto's Lehrbuch.⁷

The solution of chloroplatinic acid was boiled for half an hour with excess of caustic soda, acidified, and the platinum precipitated as potassium chloroplatinate. The chloroplatinate so obtained was reduced with sodium formate. The platinum black was then heated with dilute hydrochloric acid to remove iron and washed until it commenced to go through as colloidal platinum. It was then redissolved and the process repeatedly gone through until the mother liquors from the chloroplatinate precipitation were practically colorless and free from other platinum metals. As the same platinum was continually used, and underwent a large number of successive solutions and reductions during the preliminary work, it seems safe to assume that it was sufficiently pure.

Potassium Chloroplatinate.—During the preliminary work it was soon discovered that the preparation of chloroplatinic acid by the use of aqua regia was unsatisfactory. The removal of nitric acid by the process of repeated evaporation was tedious and, at best, uncertain. To overcome this difficulty and eliminate nitric acid entirely, a process of dissolving the platinum electrolytically in purified hydrochloric acid was devised. This proved quite satisfactory and will be described in the next paper. After solution of the platinum in hydrochloric acid had been effected the solution, which contained approximately 120 g of platinum and measured

⁷ Graham Otto's Lehrbuch, 5th edition, 4, p. 1153.

500 cc, was evaporated to about one-half its volume in a glass-stoppered wash bottle. At the same time a current of chlorine was passed through the boiling solution. The chlorine used for this purpose was prepared by the action of pure potassium permanganate upon chemically pure hydrochloric acid which had been previously boiled with a small quantity of permanganate to insure its freedom from bromine compounds. The solution of chloroplatinic acid thus obtained had a beautiful bright color, matching almost exactly that of a 0.1 per cent solution of methyl orange. It contained about 100 g of hydrochloric acid in excess and after filtration and dilution to 1 liter was used directly for the precipitation of potassium chloroplatinate. For this purpose a solution of potassium chloride was prepared, using an excess of one-third above the theoretical quantity dissolved in 1 liter of water. The excess of potassium chloride as well as the excess of hydrochloric acid in the chloroplatinic acid were deemed necessary to check hydrolytic decomposition. For the same reason the precipitation of potassium chloroplatinate was carried out with as concentrated solutions as practicable. The precipitation itself was carried out by pouring the platinum solution into the potassium chloride in a fine stream, the precipitate meanwhile being agitated thoroughly by a current of air. In some cases as much as four hours were spent in precipitating 300 g of potassium chloroplatinate. The potassium chloroplatinate so obtained was of a pale-yellow color, resembling precipitated sulphur and was microcrystalline. It was filtered from the mother liquor by means of the suction pump, washed with water, and finally with alcohol and ether. After having been removed from the hardened filter it was heated for some time in a large platinum dish on an electric air bath until the greater part of the moisture retained had been driven off. The quantity necessary for one determination was then transferred to a hard glass tube. This tube was placed in a cylindrical air bath which could be raised to a temperature of 400°. The potassium chloroplatinate was gradually raised to this temperature, a current of air, dried successively by sulphuric acid and phosphorus pentoxide passing over it continuously. The behavior of the salt under these conditions served as a criterion of its purity. At the exit of the current of air a wash bottle was placed with a drop of methyl orange. At first a small

quantity of moisture and hydrochloric acid passed off. Finally, however, a point was reached where no further hydrochloric acid could be detected in the issuing air stream. At this point the heating was stopped, usually after the chloroplatinate had been heated to 400° for about 7 hours or more. It seems that the chloroplatinate, when pure, will stand the temperature of 400° indefinitely, in a nonreducing atmosphere, without suffering any change chemically. No appreciable quantity of hydrochloric acid could be detected after the hydrochloric acid and water held mechanically had once been driven off. This all, provided the salt was pure to start with. In the first trials, in which the chloroplatinate had been prepared by the use of aqua regia, its behavior was entirely different. In these cases decomposition set in in the neighborhood of 250° and seemed to go on progressively through the whole mass of the salt.

After having been dried in this manner the chloroplatinate was transferred to the final apparatus with little or no exposure to the air through an opening which was sealed off after filling. In this it was subjected to final drying at 350° and evacuation, as described under the manipulations.

Potassium Chloride.—As a starting point for the preparation of pure potassium chloride the purest commercial article obtainable was taken. This was first recrystallized from a solution made slightly alkaline with potassium hydroxide to remove traces of ammonia which were present. The salt was then redissolved in water and chlorine passed through the hot solution for several hours, after which it was concentrated and the potassium chloride allowed to crystallize out. Following this the salt was recrystallized five times from water, precipitated three times by alcohol, and finally precipitated from aqueous solution by purified hydrochloric acid gas. The crystals in these various processes were separated from the mother liquors by centrifugal drainage.

This potassium chloride certainly contained less bromine than 1 in 50,000. To test for bromine the following process, a modification of that described by Andrews,⁸ was adopted. The flask, A (Fig. 2), is fitted with a ground-glass joint at B, which ends in an extremely fine capillary at C. The side tube, D,

⁸ Jour. Am. Chem. Soc., **29**, p. 275; 1907.

has a trap to prevent spray from being carried over. An ordinary distilling flask is affixed to the end of this side tube with a rubber stopper in such a manner as to bring the side tube well into the bulb of the flask E. The flask A is charged with 200 cc of distilled water, 20 cc of $\frac{N}{5}$ potassium iodate and 20 cc of 2 N nitric acid. The flask E contains about 5 cc of a 4 per cent solution of potassium iodide, which must not liberate iodine upon being acidified. The flasks are then connected by means of the stopper at F and vacuum applied. The capillary tube which is ground into the flask at B is connected with a carbon dioxide generator, and after vacuum has been applied should yield a fairly steady stream of bubbles through the liquid, so as to insure regular boiling and at

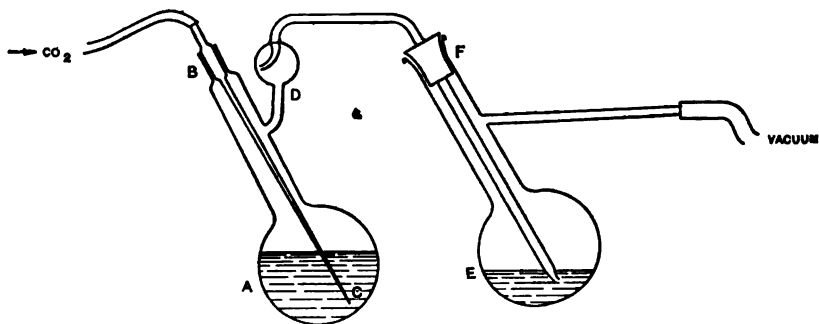
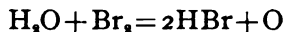


Fig. 2.

the same time not appreciably to diminish the vacuum. It is desirable to have a pure neutral gas, such as carbon dioxide or nitrogen, passing through the capillary, since traces of reducing substances in the air used would vitiate the results. The gas passing through the capillary, besides insuring regular boiling, acts as a diluent for the steam formed and as such checks the reaction:



The flask containing the potassium iodate and nitric acid mixture is then heated on the water bath until 100 cc have distilled into the flask containing the potassium iodide. If the resulting distillate is colored by the presence of free iodine, the process is repeated after adding water to make up for the quantity distilled off, until

a satisfactory blank is obtained. Then 3-5 g of the potassium chloride to be tested are dissolved in a little water and added to the flask containing the iodate and the volume of the solution made up to 250 cc again. The distillate containing the free iodine corresponding to the amount of bromine distilled over is then titrated with a solution of thiosulphate corresponding to 1 mg of bromine per cc. To 5 g of potassium chloride, which had been treated until blank distillates were obtained, the following quantities of bromine were added and found:

Added	1.0	0.5	0.3	0.1	mg
Found	1.0	0.52	0.36	0.12	mg

The potassium chloride used for precipitating potassium chloroplatinate contained less than 0.1 mg bromine in 5 g or 1:50,000. This amount it is safe to assume was further reduced in the preparation of potassium chloroplatinate.

Hydrochloric Acid.—The hydrochloric acid used in the preparation of pure hydrochloric acid was free from sulphuric and nitric acids. It was treated by allowing chlorine to bubble through it for one day. Following this, air was bubbled through the acid saturated with chlorine until the chlorine was expelled and the acid was again colorless. Usually the air was left passing through the acid overnight, a reduction of about one-fifth in the volume of the acid, due to evaporation, taking place with the removal of the excess of chlorine. During the further manipulations in preparing chloroplatinic acid it was further subjected to action of chlorine twice, namely, during electrolysis of the platinum and evaporation of the platinum solution.

Chlorine.—All the chlorine used was generated by the action of pure potassium permanganate on hydrochloric acid which had been previously boiled with a small quantity of potassium permanganate.

Water.—The water used was obtained by redistilling distilled water with alkaline permanganate, rejecting the first part of the distillate until it no longer contained ammonia.

Balance and Weights.—The balance and weights were identical with those described under the atomic weight of hydrogen.⁹ The

⁹ Noyes, this Bulletin, 4, p. 179.
J. Amer. Chem. Soc., Dec., 1907.

air of the balance case was dried by means of a current of air as described in the previous paper. Each piece of apparatus was weighed with a corresponding counterpoise approaching it within 1 cc in volume and 30 g in weight. In the case of the hydrogen the counterpoise was within 0.3 cc of the volume and 2 g of the weight of the palladium tube.

All pieces were rinsed with distilled water after having been used in a determination. After having been wiped dry, they were suspended for some time in a desiccator through which a current of dry air was passing before being transferred to the dry air of the balance. For the hydrogen apparatus 12 to 24 hours were allowed in order to attain constant surface conditions. The current of air through the balance was stopped 15 to 20 minutes before a weighing was made. Duplicate weights were taken at intervals of an hour or more and rarely differed by more than 0.05 mg.

Weights.—The weights were calibrated to vacuum standard in the Bureau of Standards. The small correction necessary on account of the difference in volume between the brass and platinum weights was included in a table with the corrections for the errors of the weights.

METHOD OF CARRYING OUT THE DETERMINATION.

FIRST SERIES.

There are three parts to the apparatus, which was prepared for a determination in the following way:

The Palladium Tube.—This consisted of a cylindrical tube with a stopcock sealed on at each end. The tube at one end was bent downwards at right angles and drawn out to a tip at B. Connection with the hydrogen generator was made by sealing the tip into a corresponding socket with Khotinsky cement. The three-way stopcock at B was then opened in the position so that the small amount of air in the limb of B was swept out. Communication was then established between the palladium and the hydrogen generator and hydrogen was led in until no more was absorbed. The stopcock at A was then opened and the hydrogen allowed to sweep through the apparatus for a short while. The stopcocks were then closed, and after cleaning the

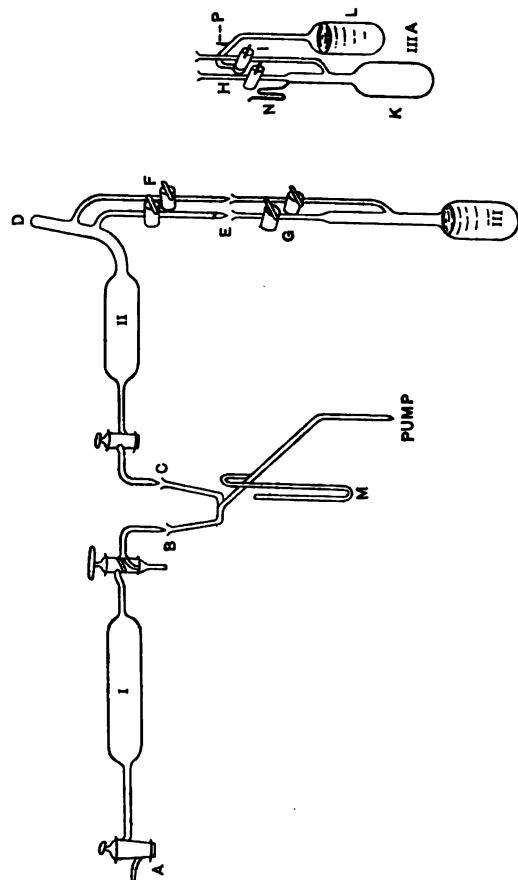
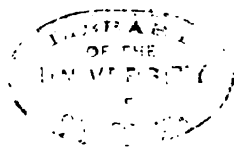


Fig. 3.



apparatus it was ready for weighing. Before the apparatus was used the first time it was repeatedly charged with hydrogen and the hydrogen driven out at 400° . After that the tube was ready for charging anew at the end of each experiment.

Chloroplatinate Tube.—This tube had the general form as shown in the accompanying photograph. The potassium chloroplatinate was introduced through the curved neck which was open at D. For this purpose the hard glass tube in which the chloroplatinate had been heated to 400° was connected to it by a short piece of stout rubber tubing, and the chloroplatinate was shaken into the apparatus. The end D was then sealed off without allowing vapors from the flame to enter the tube. After this was done the apparatus was attached to the Sprengel pump and evacuated to a few thousandths of a millimeter at a temperature of 350° . This temperature was maintained for about 4 hours. After closing the stopcock, disconnecting and rinsing after it was cool, the apparatus was transferred to the desiccator before weighing.

Absorption Tube for the Hydrochloric Acid.—This consisted of a bulb having a volume of about 100 cc. It terminated in two tubes with stopcocks and sockets to fit the tips of the two tubes from the platinum apparatus at E. Fifty cc of boiled distilled water were filled into this bulb. It was then evacuated by means of the Sprengel pump, which was protected by an extra phosphorus pentoxide tube in this case. The evacuation was continued as long as air could be removed from the apparatus or liquid. Usually it was found necessary to allow the partly evacuated apparatus to stand for a day before the last traces of air could be removed from the water. After evacuation the apparatus was rinsed and set in the desiccator for weighing. After the different portions of the apparatus had been weighed they were set up as shown in the photograph. The potassium chloroplatinate and the palladium charged with hydrogen were each enclosed in an electric heater. These are lowered in the photograph to show the apparatus.

The tip, B, of the hydrogen tube and C of the chloroplatinate tube were connected with each other and with the Sprengel pump by means of a T-tube. The two tips from the chloroplatinate tube

were connected to the two corresponding sockets of the absorption tube. These joints were made with Khotinsky cement.¹⁰

The joints were perfectly tight and no leakage was ever discovered. After the three parts of the apparatus had thus been connected with each other and with the vacuum pump the connections were evacuated. The gas in the two connecting limbs at E was removed by opening the stopcocks at F and pumping the gases out over the chloroplatinate, which had meanwhile been brought to a temperature of about 300°. When every part of the apparatus had been evacuated to a few thousandths of a millimeter, as shown by the McLeod gauge attached to the pump, connection with the pump was cut off. The stopcock of the hydrogen apparatus at B was then opened and gradual heating of the hydrogen tube commenced. Then the stopcock at C was opened very gradually and the hydrogen admitted to the chloroplatinate. Next the four stopcocks communicating with the absorption tube were opened. This was done very gradually, to prevent a sudden rush of gas into the absorption tube and consequent carrying over of the finely divided potassium chloroplatinate. In doing this the stopcocks were opened in such order that communication was first established through the tube entering the chloroplatinate apparatus at the lower level. Following this, communication was established by means of the adjoining tube. The purpose of the double connection between the chloroplatinate and the absorption tube was to prevent stagnation of the gas current between the two parts of the apparatus. A little hydrogen was always carried along with the hydrochloric acid toward the absorption apparatus. This would tend to accumulate and block the current. By making the level of the second tube somewhat higher than that of the first, the tendency of the hydrogen to rise would cause a direct and continuous flow. Actually, this arrangement proved entirely satisfactory.

The apparatus remained in this condition for several hours. The temperature of the palladium tube generating the hydrogen was allowed to rise slowly at such a rate that the pressure in the apparatus was kept at or below one atmosphere. This pressure

¹⁰ P. G. Nutting, of this Bureau, has supplied us with a cement of this same character, which seems to answer the purpose as well as the Khotinsky. It is made by fusing together equal parts by volume of rosin and pure rubber.

was measured by means of a small mercury manometer, M, which formed a part of the connecting T-tube, BC. The closed end of the manometer contained a little air. By this means it was possible to measure any pressure from zero to more than one atmosphere with a light manometer 8–10 cm in length.

When it was deemed that a sufficient quantity of hydrogen had been evolved from the palladium tube, the stopcock, B, of this part of the apparatus was closed and the palladium allowed to cool off. The temperature of the tube containing the chloroplatinate was allowed to rise to 350° . At the same time the absorption tube, which had been cooled by ice during the earlier part of the experiment, was now cooled to -20° by a mixture of ice and dilute sulphuric acid. The hydrogen remaining in the apparatus upon cutting off the palladium tube was continuously converted into hydrochloric acid and removed as such by the water. This could be followed by means of the manometer, M, previously mentioned. After two or three hours the manometer had reached the lowest point which it would indicate. At this point the stopcocks, G, of the absorption tube were closed. The conditions prevailing at this moment were as follows:

The chloroplatinate tube still contained an excess of potassium chloroplatinate and was at a temperature of 350° .

The absorption tube was at a temperature of -20° . The residual pressure throughout the apparatus was very low.

Under these conditions but a very minute quantity of hydrogen could have escaped conversion into hydrochloric acid. To determine this, connection with the Sprengel pump was again established and that part of the apparatus between the stopcock, B, of the palladium tube and the stopcocks, G, of the absorption tube was evacuated to the best vacuum obtainable by the pump—usually a few thousandths of a mm. The gas pumped out was collected in a eudiometer and analyzed. In analyzing the gases their volumes were read at a pressure of about one-sixth to one-seventh of an atmosphere, making accurate determination possible. Thus the volume of 1 cc under these conditions represented less than 0.02 mg of hydrogen.

The residual gas having been pumped out of the apparatus, all stopcocks were closed and the apparatus taken apart. All traces of the cement adhering to the glass where the various pieces had been joined were carefully removed. The pieces were then rinsed with distilled water, when sufficiently cool, wiped dry, and transferred to the desiccator. The following day they were weighed. The determination itself required about 8 hours, of which about 2 hours were used for the evolution of hydrogen and about 3 hours for the pressure to drop from atmospheric to practical zero.

SECOND SERIES.

Before giving the results obtained by the first series it may be just as well to turn to the manipulations giving the second set of values. They are essentially the same as in the first series with this difference: the hydrochloric acid generated was condensed as a solid by cooling with liquid air and connection with all other parts of the apparatus was cut off before it was absorbed by water. The point in view was this. During the period of absorption of the hydrochloric acid it might be possible that a transfer of water would take place from the absorption apparatus to the chloroplatinate apparatus and remain there in spite of the fact that there was a difference of 370° in temperature between the two parts and a very high vacuum. This would cause the apparent amount of chlorine transferred to appear lower than the real. Or there was the possibility of the water vapor reacting with the material in the chloroplatinate tube. To eliminate these possibilities the reaction was so carried on that the chloroplatinate was at no part of the determination in contact with water vapor.

Further, this series differs from the first in that the hydrogen used was generated from a solution of barium hydroxide, the hydrogen in the first series being obtained from a sulphuric acid electrolyte. The absorption apparatus in the second series (which simply took the place of the water absorption tube) was constructed as follows:

The bulb, K, was designed for the condensation of the hydrochloric acid and had a volume of 100 cc. It terminated in a neck from which led two exit tubes provided with the stopcocks, H and I. The stopcock at I was provided with two perforations so that

communication could be established either between the chloroplatinate tube and K, or between the two bulbs, K and L. The bulb, L, had a volume of 70 cc and contained the water for the absorption of the hydrochloric acid. At N a miniature mercury manometer was affixed. This was found absolutely necessary in order to follow the conditions of pressure during the transfer of the solid hydrochloric acid in K to the water in L. Indeed, previous to the attachment of this device to the apparatus every determination was lost either by the bursting of the apparatus or the blowing out of the stopcocks. By making the air space above the mercury of capillary tubing, while the remainder was about 1.5 mm in bore, satisfactory readings could be obtained both at normal and reduced pressures and there was no danger of the mercury being sucked out of the manometer.

To prepare the apparatus for weighing, somewhat more than 50 g of water were filled into L, through a small opening at P. The water was then boiled, and when all air had been expelled the opening at P was sealed off. The whole apparatus was then evacuated with the Sprengel pump, the last traces of air being removed from L by opening the stopcock I momentarily until the limit of the pump was reached. Rinsing, drying, and desiccation then completed the preparations for weighing as usual. The apparatus was then put together and prepared for the determination in precisely the same manner as in the first series with the one difference that the absorption apparatus, IIIA, was now connected at E instead of the absorption tube, III. Communication with the water bulb, L, was entirely cut off during the first part of the determination. The condensation bulb, K, was plunged in liquid air at the beginning and kept at the temperature of boiling air throughout the determination.

In opening the stopcocks to obtain communication between the various parts of the apparatus very great care had to be exercised, as the first rush of gases into the chilled condensation bulb often carried some finely divided chloroplatinate with it. In a few cases it was not possible to avoid this. Since this chloroplatinate remained in the condensation bulb, K, its amount could be found at the end of the determination and a suitable correction applied.

After communication had been established between the hydrogen tube, the chloroplatinate tube, and the condensation tube, the hydrogen tube was cautiously heated so as to obtain a constant and steady rise in temperature. Somewhat more care was necessary in this series, as there seemed to be a greater tendency for the pressure in the hydrogen apparatus to fall below that of the remaining parts. This would result in the finding of hydrogen chloride in the palladium tube at the end of the experiment. Suitable corrections were made for this and will be spoken of with the other corrections. When the hydrogen tube had reached a temperature of from 150° to 160° , it was judged that the requisite amount of hydrogen had been evolved. The stopcock from the hydrogen tube was then closed. Any temperature above 150° sufficed for the reduction of the chloroplatinate by the hydrogen, but at the end the temperature of the chloroplatinate tube was run up to 400° . The condensation bulb was kept fully submerged in liquid air. After the supply of hydrogen had been turned off the pressure in the apparatus gradually sank until all hydrogen had been used up and all hydrochloric acid condensed. The conditions then were: Excess of the chloroplatinate at 350° to 400° ; the hydrochloric acid at about -180° , and a very low residual pressure. The stopcocks of the condensation tube were then closed and the residual gases in the remaining parts of the apparatus pumped out.

The condensation tube was then separated from the remaining parts of the apparatus and the transfer of the hydrochloric acid to the water commenced. The water bulb, L, was plunged into ice water and the condensation bulb, K, removed from direct contact with the liquid air but not entirely beyond the cold vapors. As soon as the pressure of the hydrochloric acid had risen to a small part of an atmosphere communication was established with the cold water in L. After some hydrochloric acid had been absorbed by the water the bulb, L, was cooled by ice and dilute sulphuric acid, instead of by ice alone. With the aid of occasional shaking of the absorption bulb the last of the condensed hydrochloric acid was finally absorbed by the water. With cautious manipulation the pressure in the apparatus did not rise above one-half atmos-

phere during the transfer and remained at a few millimeters at the close. The stopcock, I, between K and L was then closed, and the apparatus was cleaned, rinsed, wiped, and set aside to be weighed. The determinations of the second series required about ten hours from beginning to end. About two hours were consumed in transferring the solidified hydrochloric acid from the condensation to the absorption bulb. The solidified hydrochloric acid resembled snow in appearance and the greater part of it was absorbed by the water without passing into the liquid state. Towards the end of the transfer, when the absorption became less rapid, the remaining hydrochloric acid would liquefy, attended by a sudden rise in pressure. Shaking of the absorption tube immediately reduced the pressure and caused the liquid hydrochloric acid to solidify to a glassy solid.

ERRORS AND CORRECTIONS.

Hydrogen.—If there were any errors due to a contamination of the hydrogen, they are not apparent. The hydrogen was prepared and purified with all possible precautions. It was repeatedly tested for impurities in the work on hydrogen and none or only negligible quantities found.¹¹

Two corrections on the weight of hydrogen were found necessary. The first of these was the amount of hydrogen remaining in the chloroplatinate tube at the end of the determination and pumped out with the residual gases to be analyzed. In eight cases out of the twelve this correction was applied, the maximum being 0.11 mg, the minimum 0.01 mg, and the average 0.047 mg. The determinations affected are 1, 2, 4, 5, 6, 7, 11 and 12. The second correction was due to hydrochloric acid in the palladium tube. Under certain conditions, when the current of hydrogen from the palladium tube was not sufficiently rapid, hydrochloric acid found its way back into the palladium tube, either by diffusion or by being drawn back. Correction for this was readily applied. After having been weighed the hydrogen apparatus was charged for the succeeding determination. The tube was

¹¹ Noyes, this Bulletin, 4, p. 179.

J. Amer. Chem. Soc., 29, Dec., 1907.

fitted with a stopcock at both ends, as may be seen in the photograph. After the palladium had been saturated with hydrogen the stopcock farthest from the hydrogen generator was connected to a Liebig flask containing water colored by a drop of methyl orange. The stopcock at A was opened while the hydrogen generator was kept going and the hydrogen bubbled through this water before escaping. If the presence of hydrochloric acid made itself known, the palladium was heated to 160° to expel the larger part of the hydrogen it had absorbed and with it any hydrochloric acid present. It was then allowed to cool with a current of hydrogen passing through it and it was finally filled again at normal temperatures. The hydrochloric acid absorbed by the water was then titrated by $\frac{N}{10}$ sodium or barium hydroxide. The real weight of the hydrogen was greater than the apparent loss of the tube by the weight of the hydrochloric acid found. This correction was applied in determinations 1, 3, 4, 10 and 11, varying between 0.35 and 9.26 mg and averaging 4.8 mg.

Chlorine from the Loss of the Platinum Tube.—The purity of the chlorine entering into the composition of the chloroplatinate has been spoken of. The maximum amount of bromine in the ingredients used seems to have been 1 in 50,000. The resultant error in the weight would be half of this, since 35.5 g are replaced by 80, or 1:100,000. There seems to be ample justification for considering this point beyond question.

The only other impurities which could possibly affect the results were volatile products in the platinum or elements capable of producing volatile products, such as ammonia, water, hydroxyl, or oxygen. The method of preparation seems to preclude the possibility of the presence of ammonium salts. The potassium chloride was especially treated to free it from these. Following this there was repeated treatment with chlorine in hot solution. The final heating of the chloroplatinate to 400° over an extended period of time must have caused the destruction and removal of any ammonium compounds which had escaped previous treatment.

The three following impurities, water, hydroxyl, or oxygen, in the platinum salt were somewhat more difficult of treatment. It

is known that potassium chloroplatinate hydrolyzes in aqueous solution. With this point in view the precipitation of potassium chloroplatinate was carried out in concentrated solutions with both an excess of potassium chloride and hydrochloric acid and the mother liquor was removed as soon as practicable. It was shown that by heating sufficiently long at 400° all occluded hydrochloric acid could be removed, the chloroplatinate being perfectly neutral after this treatment. This served as indirect evidence that the water was also removed. The agreement between series 1 and 2 may be considered as additional evidence on this point. In the first series the chloroplatinate had opportunity to become saturated with water vapor at the temperature of 350° during the course of the experiment. In the second series it was in equilibrium with solid hydrochloric acid at -180° . Now hydrochloric acid with its well known affinity for water vapor may be considered as a very perfect drying agent. Yet the difference between the two series is 1 in 10,000. One hundred and twenty grams of potassium chloroplatinate were used. The presence of 0.05 per cent moisture in this salt would have raised the atomic weight of chlorine found from 35.184 to 35.244.

Further, in the second series it was noticed that if the condensation tube for the hydrochloric acid was allowed to contain a trace of moisture before the determination was begun this would remain as a trace of aqueous hydrochloric acid after the gas had been transferred from this part of the apparatus to the absorption bulb, the temperature during this transfer remaining, of course, below zero. If, however, the condensation tube was perfectly dry to start with, no such trace of aqueous hydrochloric acid remained; that is, no water had been carried over from the chloroplatinate.

A final test of the chloroplatinate was made for water, hydroxyl, or oxygen. One hundred grams of potassium chloroplatinate were treated exactly as for a determination. The chloroplatinate was heated to 400° in a current of dry air, transferred to the apparatus used in the determinations, and then evacuated at 350° . Next pure hydrogen was led over the chloroplatinate until it was completely reduced. The hydrochloric acid formed was led through a narrow U-tube surrounded by a mixture of solid carbon dioxide and alcohol (-78°). Finally the whole apparatus, including the U-tube, was

evacuated, the final conditions being 350° in the chloroplatinate tube, -78° in the U-tube and several thousandths millimeters pressure. Upon weighing, the U-tube had shown an increase in weight of 0.9 mg, or 1 in 30,000, on the hydrochloric acid formed.

A number of corrections on the weight of the chloroplatinate tube were necessary. The first of these were necessary on account of the traces of air or nitrogen found in the chloroplatinate tube at the end of the determination. These corrections were found necessary in experiments 4, 5, 6, and 7, their magnitude being 0.35, 0.76, 0.25, and 2 mg. This air may have been occluded by the chloroplatinate, or it may have come from the water of the absorption tube. It was immaterial how this correction was applied as it lay within the limits of the experimental errors, the maximum correction being 1:10,000.

In the second series it sometimes happened that with the most cautious manipulation chloroplatinate was blown over into the condensation bulb at the beginning of the experiment. After the hydrochloric acid had been absorbed by the water and the apparatus weighed, this chloroplatinate was removed by dissolving in water and weighing first as potassium chloroplatinate and then again as potassium chloride + platinum, after reduction. This correction was applied in experiments 8, 9, and 11, the amounts being 2.03, 15.0, and 36.5 mg. The apparent loss of the chloroplatinate tube was, of course, diminished by these quantities.

Hydrochloric acid.—From the agreement of Series I and II it seems reasonable to assume that no errors were introduced by using water as the absorbing agent for the hydrochloric acid.

The corrections on the apparent gain of the absorption tube were of the following nature: First, the apparent gain was increased by the hydrochloric acid found in the palladium tube and by the hydrochloric acid pumped out from the chloroplatinate tube. The former has been spoken of under hydrogen. In experiments 1, 6, and 7 there were found 1.27, 0.8, and 0.15 mg of hydrochloric acid, respectively, in the gases pumped out.

In 8, 9, and 11 the gain of the absorption tube was diminished by the amount of the chloroplatinate blown over. The corrections were 2.03, 15.0, and 36.5 mg.

The final check on correctness of manipulation, freedom from leaks, and other losses was found in the checking of the sum of the weights of the hydrogen and chlorine with the weight of the hydrochloric acid. In the preliminary experiments of both series discrepancies were found until the details of the determination had been mastered. On this account and on account of other known errors the preliminary determinations were rejected entirely.

RESULTS.

The values obtained were as follows: The weights represent those found after all corrections had been applied. The second part of experiment 5 was lost.

	Hydrogen.	Chlorine.	Hydrochloric acid.	At. Wt. Cl.	Mol. Wt HCl.
1	0.25394	8.93293	9.18695	35.177	36.178
2	0.28004	9.85590	10.13259	35.195	36.183
3	0.51821	18.23468	18.75359	35.188	36.189
4	0.67631	23.79587	24.47123	35.186	36.185
5	0.58225	20.48158	-----	35.177	-----
6	0.47989	16.88423	17.36310	35.184	36.182
7	0.64132	22.55816	23.20054	35.175	36.176
Average of Series I, 35.183 and 36.181.					
8	0.81608	28.71691	29.53167	35.188	36.187
9	0.83194	29.28055	30.11207	35.195	36.195
10	0.29074	13.74926	14.14078	35.187	36.188
11	0.75560	26.58427	27.33926	35.183	36.182
12	0.77518	27.26746	28.04110	35.177	36.175
Total average				35.184 (3)	36.183 (7)
Average of Series II, 35.186 and 36.185.					

The value as found for the atomic weight of chlorine is therefore 35.184 with a probable error of ± 0.0013 . The value obtained for the molecular weight of hydrochloric acid is 36.184 with a probable error of ± 0.0012 . The combined average of both sections of the two series is 35.184, with a probable error of ± 0.0008 . This is, of course, on the basis of hydrogen = 1. On the oxygen

basis this value becomes 35.452 if $H=1.00762$ ¹³ and 35.461 if $H=1.00787$.¹³ The values for silver calculated from Richards' ratio and these two values are, respectively, 107.865 and 107.893.¹⁴

The mean values must be considered as most probable at the present time. These are 35.457 for chlorine and 107.88 for silver.

In eleven experiments 6.41925 g of hydrogen were united with 225.86017 g of chlorine and yielded 232.27288 g of hydrochloric acid. The values obtained from these figures are 35.1846 and 36.1838.

It may be interesting to note the discrepancies between the weights of the hydrogen and chlorine and the hydrochloric acid formed in the individual experiments.

	H + Cl.	HCl.	Difference.
1	9.18687	9.18695	+0.00008
2	10.13594	10.13259	—0.00335
3	18.75289	18.75359	+0.00070
4	24.47218	24.47123	—0.00095
6	17.36412	17.36310	—0.00102
7	23.19948	23.20054	+0.00106
8	29.53299	29.53167	—0.00132
9	30.11249	30.11207	—0.00042
10	14.14000	14.14078	+0.00078
11	27.33982	27.33926	—0.00056
12	28.04264	28.04110	—0.00154
	232.27942	232.27288	—0.00654

In these eleven experiments there were seven with an apparent loss of weight and four with an apparent gain, the total loss being 1 in 35,000.

WASHINGTON, October 1, 1907.

¹³ Morley's value.

¹³ Noyes' recent value.

¹⁴ The values calculated from the results of Dixon and Edgar in the same manner are 35.463 and 35.472 for chlorine and 107.90 or 107.93 for silver. The mean values 35.467 and 107.91 must be considered for the present as the most probable which can be calculated from their work. The values for silver are calculated from the results of Richards and Wells, who found that 100 parts of silver give 132.867 parts of silver chloride. J. Amer. Chem. Soc. 27, p. 525; 1905.

THE PREPARATION OF CHLOROPLATINIC ACID BY ELECTROLYSIS OF PLATINUM BLACK.

H. C. P. Weber.

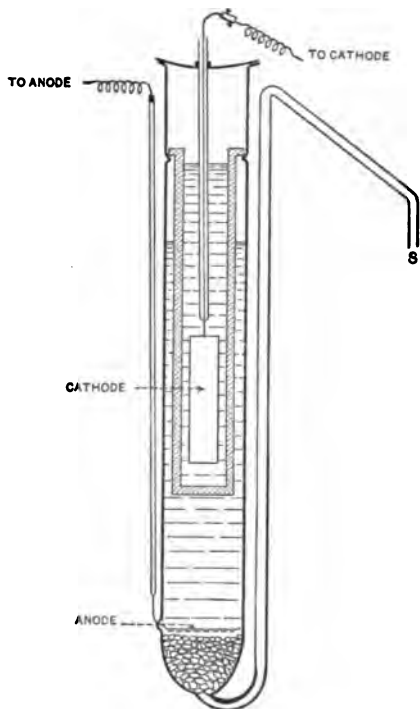
In the work on the atomic weight of chlorine¹ it was necessary to prepare considerable quantities of chloroplatinic acid free from nitric acid. When using aqua regia to dissolve platinum, considerable difficulty was experienced in removing the last traces of nitric acid by evaporation. When working with as much as 100 g of platinum, the oft-repeated evaporation to dryness of the solution becomes exceedingly tedious, and even at best yields uncertain results. If the evaporation is carried on with strong hydrochloric acid, considerable quantities of material become necessary, while with the use of water there is danger of hydrolytic decomposition of the chloroplatinic acid and consequent contamination of the chloroplatinate with hydroxychloroplatinates.

The process here described overcomes these difficulties and yields a pure solution of chloroplatinic acid. The platinum is prepared for electrolysis by dissolving platinum scraps or platinum sponge in aqua regia. The excess of acid is removed either by neutralization or evaporation and the platinum solution is reduced by zinc or an alkaline formate, preferably the latter. The solution is decanted from the precipitated platinum, which is then warmed with a little dilute hydrochloric acid to remove iron. The platinum is then transferred to the electrolytic apparatus, the washing of the precipitated platinum being completed in this apparatus, which is constructed as follows.

It consists of a cylindrical tube about 4 cm in diameter and 35 cm long, which ends in a narrow glass tube, about 4 mm bore,

¹ Noyes and Weber, this Bulletin, 4, p. 345, 1908.

which is given the form of a siphon. The anode is a thin disk of sheet platinum which just fits into the tube and is perforated with numerous small pin holes. A small piece of platinum wire is welded to the disk and carried through the glass tube by means of sealing glass. The other end of the platinum wire ends in a glass tube which is carried to the top of the apparatus and is filled with mer-



cury to make connection for the current. The platinum disk should be about 30 cm from the top of the apparatus at that point where the tube commences to narrow. After the anode has been sealed into the tube the space below it is filled with glass beads to support the platinum disk, which should rest firmly and evenly upon them. About 5 cm from the top of the tube three notches are pressed into the glass. From these the cathode chamber is suspended. This consists of a porous porcelain filter about 18 cm long and 25 mm diameter. It is well that the rim of the filter fit snugly in the glass tube so that the filter cup hangs in the tube fairly rigidly.

The cathode consists of a sheet of platinum 4 to 5 cm long and 2 to 3 cm wide. To it is connected a platinum wire passing through a glass tube. It is suspended from a perforated watch glass, which serves as a cover for the apparatus.

The whole apparatus is suspended in a long cylinder, by means of a large cork, for the purpose of cooling it by running water when necessary. This is not shown in the diagram, and may be dispensed with when low currents are used. The apparatus has been used with a current up to 10 amperes. With a current of this strength the cooling jacket is essential, as the apparatus gradually becomes hot.

The platinum is transferred to the tube, being dropped on the anode plate, and is here washed with dilute hydrochloric acid until clean. The wash waters are drawn off by gentle suction at the siphon end, S. The platinum is then covered with concentrated hydrochloric acid. There should be such a quantity of hydrochloric acid that the liquid stands on a level with S when the porous cylinder is inserted. The porous cylinder is then inserted and filled to the top with hydrochloric acid. After the cathode is inserted the apparatus is ready for electrolysis.

The current may be taken from a 120-volt direct-current lighting circuit with a number of incandescent lamps in parallel with each other and in series with the cell. The cell may be run continuously on 8 to 10 amperes. The current is used quantitatively in dissolving platinum. During a run of $4\frac{1}{2}$ hrs at 8 amperes 64 g of platinum were dissolved. The theoretical quantity for 36 ampere-hours is 65 g. While the apparatus is in operation the hydrochloric acid travels from the cathode cell to the anode under the influence both of gravity and electric endosmosis. With the proper adjustments of height of hydrochloric acid in the anode cell, the heavy layer of chloroplatinic acid solution is delivered at the tip of the siphon S, drop by drop. If the flow of concentrated solution ceases for any reason it may again be started by gentle suction at S. For this purpose it is best to have the siphon tip S connected with a receiving flask by means of a double perforated stopper. The acid in the cathode chamber is replenished from time to time as it becomes necessary.

If toward the end of the operation, when the amount of platinum remaining upon the perforated disk becomes small, bubbles of chlorine commence to rise through the liquid, it is an indication that the current density is becoming too great. In this case, bringing fresh acid into the neighborhood of the platinum black and decreasing the current will remedy the chlorine evolution.

In concentrating the solution of chloroplatinic acid after it is so prepared, chlorine is passed through it for a short while. This insures freedom from platinous compounds in case any have been formed during the electrolysis.

WASHINGTON, October 8, 1907.

THE SELF-INDUCTANCE OF A COIL OF ANY LENGTH WOUND WITH ANY NUMBER OF LAYERS OF WIRE.

By Edward B. Rosa.

The self-inductance of a coil or short solenoid wound with any number of layers of wire is given by the formula of Weinstein, or Stefan's modification of Weinstein's formula, when one applies the proper corrections for the thickness of the insulation and the shape of the section of the wire. But for a long solenoid this formula is not very accurate. I have elsewhere shown how to obtain the self-inductance of a long solenoid wound with a single layer of round covered wire or with bare wire wound at any given pitch. I propose now to show how one may obtain accurately the self-inductance of a solenoid of any length having a uniform winding of any number of layers; this will include the case of short coils as well as those where the length is too great to be calculated by the formulæ of Weinstein or Stefan.

Mr. Cohen gives elsewhere in this Bulletin an approximate formula for the self-inductance of relatively long coils of more than one layer. His formula is convenient in calculation when the number of layers is not large, and is accurate enough for most practical cases, notwithstanding it assumes the current to be distributed in current sheets, taking no account of the shape of the cross section of the wire or the thickness of the insulation. I shall now show how to calculate accurately the self-inductance of a coil of any length and any number of layers, wound with insulated round wire, taking account of the shape of the section as well as the thickness of the insulation of the wire.

Let Fig. 1 be the section of such a winding of mean radius a , length l , and depth of winding t , and having m layers. If n is the number of turns per centimeter,

$$m = nt$$

$$N_1 = nl = \text{number of turns per layer}$$

$$N = mnl = \text{whole number of turns.}$$

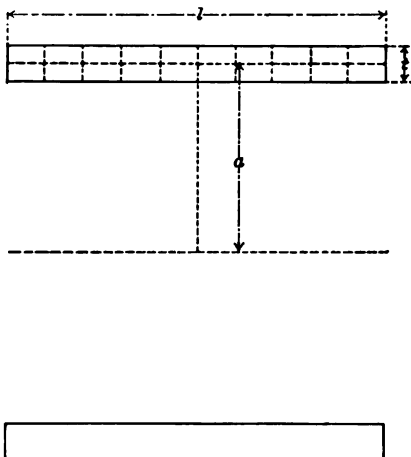


Fig. 1.

Suppose the section is divided into squares, as shown in Fig. 1, of which there will be $l/t = n'$. We shall find first the self-inductance of a single layer winding of n' turns of square wire, which completely fills the section; that is, when the insulation between turns is of infinitesimal thickness. To do this we first find the self-inductance L_s of a thin current sheet of n' turns, radius a and length l , by Lorenz's formula, which is as follows:

$$L_s = \frac{4\pi n'^2}{3l^2} \left\{ d(4a^2 - l^2)E + dl^2F - 8a^3 \right\} \quad (1)$$

where $d = \sqrt{4a^2 + l^2}$, a , l and n' having the meanings explained above, and F and E are the complete elliptic integrals to modulus k ,

$$\text{where } k = \frac{2a}{d} = \frac{2a}{\sqrt{4a^2 + l^2}}$$

The next step is to find the difference between the self-inductance L_s for the thin current sheet and the self-inductance L_t for the thick current sheet, of thickness t and n' turns. Following the method given in a previous paper¹ for a single layer winding of

¹This Bulletin, 2, p. 161, 1906.

round wire, we have to find

$$\Delta L = \Delta L_1 + \Delta M \quad (2)$$

where ΔL_1 is the difference in the self-inductances of the n' turns of square wire making up the thick current sheet and the n' turns of thin tape making up the thin current sheet; and ΔM is the corresponding difference in the mutual inductances. The correction ΔL is to be subtracted from L_s .

CORRECTION FOR SELF-INDUCTANCE.

The self-inductance of one turn of thin strip of width t is given approximately by the expression

$$L'_s = 4\pi a \left\{ \log \frac{8a}{R_s} - 2 \right\}$$

and for one turn of wire of square section

$$L'_t = 4\pi a \left\{ \log \frac{8a}{R_t} - 2 \right\}$$

In the above equations R_s is the geometrical mean distance of a line of length t from itself; R_t is the geometrical mean distance of a square of side t .

The difference between these two expressions is

$$L'_s - L'_t = 4\pi a \left(\log \frac{R_t}{R_s} \right) \quad (3)$$

$$R_t = .44705 \quad R_s = .22313$$

$$\therefore \frac{R_t}{R_s} = 2.0035 \text{ and } \log \frac{R_t}{R_s} = 0.6949$$

Hence the difference in the self-inductances of the n turns of the thick and the thin current sheet is

$$\Delta L_1 = 4\pi a n [0.6949] = 4\pi a n A \quad (4)$$

We may derive the value of the constant A somewhat more accurately otherwise as follows:

The self-inductance of one turn of thin tape of width b by Rayleigh's formula is

$$L'_s = 4\pi a \left\{ \log \frac{8a}{b} - \frac{1}{2} + \frac{b^2}{32a^2} \left(\log \frac{8a}{b} + \frac{1}{4} \right) \right\} \quad (5)$$

The self-inductance of one turn of square wire (of side b) by Weinstein's formula is

$$L'_t = 4\pi a \left\{ \log \frac{8a}{b} - 1.1949 + \frac{b^2}{24a^2} \left(\log \frac{8a}{b} + 0.8777 \right) \right\} \quad (6)$$

The difference between these two is

$$L'_s - L'_t = 4\pi a \left\{ 0.6949 - \frac{b^2}{96a^2} \left(\log \frac{8a}{b} + 2.76 \right) \right\}, \text{ nearly.} \quad (7)$$

The first term of this expression is the same as found above (4); the second term, depending on the dimensions of the coil, is small and may often be neglected.

If $a = 5$, and $b = 1$, this is

$$\begin{aligned} L'_s - L'_t &= 4\pi a (0.6949 - .0027) \\ &= 4\pi a (0.6922) \end{aligned}$$

$$\therefore \Delta L_1 = 4\pi a n A, \text{ where } A = 0.6922$$

Thus the value of the constant A is 0.6949 when b/a is very small, and decreases slightly (to 0.6922 in this particular case) as b/a increases. Equation (7) may be used to find its value accurately for any given case.

TABLE I.

Value of the Constant A as a Function of t/a , t being the depth of the Winding and a the Mean Radius.

t/a	A
0.	0.6949
0.10	0.6942
0.15	0.6933
0.20	0.6922
0.25	0.6909

CORRECTION FOR MUTUAL INDUCTANCE.

The correction for mutual inductance is nearly the same as for a single layer winding of round wire. It would be exactly the same if the geometrical mean distance of square areas from each other were the same as for circular areas. This is true almost exactly except for very near areas, as the adjacent turns. Table VIII of my previous paper^{*} gives the values of the constant B for round wires, and these values may be used in this case without serious error. The correction depending on mutual inductances is

$$\Delta M = 4\pi anB \quad (8)$$

To obtain somewhat more accurate values of B we should use the values of the geometrical mean distances of square areas, as shown in column 2, Table II. These are sensibly the same as for round areas beyond a distance equal to five times the diameter of the squares. Hence only the first five values of δ in Table I differ from those of Table III^{*} of the previous paper, referred to above.

TABLE II.

Table of Geometric Mean Distances and Corrections Depending upon Them.

G. M. D. R for Tapes	G. M. D. R' for Squares	Ratio, $\frac{1}{k} = \frac{R'}{R}$	$\delta = \log \frac{1}{k}$
$R_1 = 0.89252$	1.00655	1.12776	0.12025
$R_2 = 1.95653$	2.00102	1.02274	.02249
$R_3 = 2.97171$	3.00030	1.00962	.00958
$R_4 = 3.97890$	4.00013	1.00531	.00530
$R_5 = 4.98323$	5.00007	1.00337	.00337
$R_6 = 5.98610$	6.00003	1.00233	.00233
$R_7 = 6.98806$	7.0000	1.00171	.00171
$R_8 = 7.98957$	8.0000	1.00131	.00131
$R_9 = 8.99076$	9.0000	1.00103	.00103

From these values of δ and the succeeding values as given in Table III^{*} we can calculate the table of values of B , corresponding

^{*}This Bulletin, 2, p. 161.

to Table VIII for round wires. The constant B is given by

$$B = \frac{2}{n} \sum_{m=n-1}^{m-1} \left(m \log \frac{1}{k} \right) \quad (9)$$

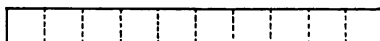
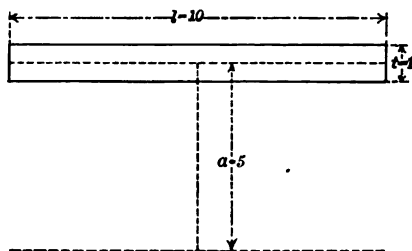
TABLE III.

Values of Correction Term B , Depending on the Number of Turns of Square Conductor on Single Layer Coil.

No. of Turns	B	No. of Turns	B
1	0.0000	16	0.3017
2	.1202	17	.3041
3	.1753	18	.3062
4	.2076	19	.3082
5	.2292	20	.3099
6	.2446	21	.3116
7	.2563	22	.3131
8	.2656	23	.3145
9	.2730	24	.3157
10	.2792	25	.3169
11	.2844	26	.3180
12	.2888	27	.3190
13	.2927	28	.3200
14	.2961	29	.3209
15	.2991	30	.3218

EXAMPLE 1.

As a first example let it be required to find the self-inductance of a coil of mean radius $a = 5$ cm, length $l = 10$ cm, depth of winding $t = 1$ cm, and uniformly wound with 10 turns of wire per cm, having



therefore 10 layers of 100 turns per layer, or a total of 1,000 turns. We first find the self-inductance L_s of a current sheet of 10 turns, radius $a = 5$, $l = 10$.

Using formula (1), we have the following values:

$$\begin{aligned} d &= \sqrt{4a^2 + l^2} = \sqrt{200} & k &= \frac{2a}{d} = \frac{1}{2}\sqrt{2} = \sin \gamma \\ 4a^2 - l^2 &= 0 & \therefore \gamma &= 45^\circ \\ n' &= 10 \\ n'/l &= 1 \end{aligned}$$

Hence

$$L_s = \frac{4\pi}{3} \left\{ 1000\sqrt{2} F - 8 \times 125 \right\} = \frac{4000}{3} \pi \left\{ \sqrt{2} F - 1 \right\} \quad (10)$$

The value of F for $\gamma = 45^\circ$ is 1.85407. Substituting in

$$L_s = \frac{4000}{3} \pi \left\{ 1.62205 \right\} = 2162.73\pi \text{ cm}$$

To find the correction ΔL we must have the two constants A and B . By equation (7), or Table I, $A = .6922$. By Table II, $B = .2792$. Hence

$$\begin{aligned} \Delta_1 L &= 4\pi a n (A + B) \\ &= 200\pi (0.6922 + .2792) \\ &= 194.28\pi \text{ cm} \\ L_u &= L_s - \Delta_1 L = 1968.45\pi \text{ cm.} \end{aligned} \quad (11)$$

Thus the thick current sheet (10 cm x 1 cm) of 10 turns has a self-inductance L_u about 9 per cent less than that of a thin current sheet L_s of the same length and radius equal to the mean radius of the thick sheet.

If the coil were wound with 1,000 turns of square wire (the insulation being of infinitesimal thickness), then the current would be uniformly distributed over the entire section of the winding, and the self-induction would be 100^2 times as great as for 10 turns, being proportional to the square of the number of turns. In general, we get the self-inductance for m layers by multiplying the value for one layer by m^4 . This, of course, supposes a uniform winding such that m layers occupy as much space vertically as m turns horizontally on

the coil. In this case the self-inductance will be

$$\begin{aligned} L_u &= 1968.45\pi \times 10^4 \text{ cm} \\ &= 19.6845\pi \text{ millihenrys} \\ &= 61.841 \text{ millihenrys} \end{aligned} \quad (12)$$

The subscript *u* signifies uniform distribution of current over the entire cross section of the coil. We must now find the self-inductance *L* for this coil when wound with 1,000 turns of *round insulated* wire. Suppose the bare wire to be 0.8 mm in diameter, the covering being 0.1 mm thick; there will be three correction³ terms to apply, *C*, *F*, and *E*, each of which gives an increase in *L*. The first expresses the difference in *L*, due to the wire being round instead of square, as previously assumed; the second expresses the difference in *L*, due to the wire being of smaller diameter than if there was no appreciable thickness of insulation; the third expresses the corresponding difference in the mutual inductances.

The sum $\Delta_s L$ is then as follows:

$$\begin{aligned} \Delta_s L &= 4\pi na(C + F + E) \\ C &= 0.1381 \text{ for the case of round wire.} \\ F &= 0.2231 = \log_e \frac{D}{d} = \log_e 1.25 \text{ in this case.} \\ E &= 0.0172 \text{ for this size and shape of section.}^3 \\ C + F + E &= 0.3784 \\ \therefore \Delta_s L &= 4\pi \times 5000 [0.3784] \\ &= 7568\pi \text{ cm} = 23776 \text{ cm} \\ &= 0.0238 \text{ millihenrys} \end{aligned}$$

This correction $\Delta_s L$ amounts in this particular case to about 4 parts in 10,000 of *L*. We have now finally

$$L = L_u + \Delta_s L = 61.865 \text{ millihenrys} \quad (13)$$

The same method may be used for a coil of any length and any number of layers.

EXAMPLE 2.

As a second illustration of this method of calculating the self-inductance of a coil of any number of layers let us take a very short

³This Bulletin, 8, pp. 34 and 37, 1907.

coil and check the result by the formula of Stefan. Let the mean radius be 10 cm and the cross section of the winding 1 x 1 cm. For the self-inductance L_s of the current sheet of radius a and length b we will this time use Rayleigh's formula, which is very accurate where b/a is as small as 1/10. Rayleigh's formula is

$$L_s = 4\pi a N^2 \left\{ \log \frac{8a}{b} - \frac{1}{2} + \frac{b^2}{32a^2} \left(\log \frac{8a}{b} + \frac{1}{4} \right) \right\}$$

Here $\frac{8a}{b} = 80$ $\log_e 80 = 4.382027$

$$\frac{b^2}{32a^2} = \frac{1}{3200} \therefore \frac{b^2}{32a^2} \left(\log_e 80 + \frac{1}{4} \right) = \frac{0.001448}{4.383475} = \frac{-.50}{3.883475}$$

$$\therefore L_s = 4\pi a N^2 \times 3.883475$$

This is the self-inductance if the current is condensed into a ring of zero thickness; that is, $a=10$, $b=1$, $c=0$. By equation (7) the correction term $A=0.69415$ (see also Table I) and $B=0$, since $n'=1$ (Table III).

$$\begin{aligned} \therefore \Delta_1 L &= 4\pi a N^2 (.69415) \\ \therefore L_u &= L_s - \Delta_1 L = 4\pi a N^2 \times 3.18932 \text{ cm.} \end{aligned} \quad (14)$$

If $N=400=20$ turns per cm and 20 layers, we have for L_u , a being 10 cm,

$$\begin{aligned} L_u &= 6400000 \times 3.18932 \text{ cm} \\ &= 64.125 \text{ millihenrys.} \end{aligned} \quad (15)$$

This is the self-inductance if the wire is square and fills the entire section. The corrections C , F , and E must be applied as in example 1 to reduce to the actual case of a winding of round, insulated wire. In this case, supposing the bare wire is 0.3 mm in diameter, and hence $D/d=5/3$.

$$\begin{aligned} C &= 0.1381 \\ F &= 0.5108 \\ E &= 0.0176 \\ \hline C+F+E &= 0.6665 \end{aligned}$$

$$\begin{aligned}
 \therefore \mathcal{A}_s L &= 4\pi n a (C + F + E) \\
 &= 16000\pi \times 0.6665 \\
 &= 33,502 \text{ cm.} \\
 &= .0335 \text{ millihenrys} \\
 \therefore L &= L_u + \mathcal{A}_s L = 64.1585 \text{ millihenrys.} \quad (16)
 \end{aligned}$$

We can check the value of L_u by the formula of Weinstein or that of Stefan. These formulae assume the current to be uniformly distributed over the section of the coil; that is, that the wire is square and fills the entire section.

Stefan's formula is as follows:

$$L_u = 4\pi a n^2 \left\{ \left(1 + \frac{3b^2 + c^2}{96a^2} \right) \log \frac{8a}{\sqrt{b^2 + c^2}} - \gamma_1 + \frac{b^2}{16a^2} \gamma_2 \right\} \quad (17)$$

In this case $a = 10$, $b = c = 1$ $\gamma_1 = 0.84834$ $\gamma_2 = 0.8162$

$$\log \frac{8a}{\sqrt{b^2 + c^2}} = \log \frac{80}{\sqrt{2}} = 4.03545$$

$$\left(1 + \frac{3b^2 + c^2}{96a^2} \right) \log \frac{80}{\sqrt{2}} = 4.03713$$

$$\frac{b^2}{16a^2} \gamma_2 = \frac{0.00051}{4.03764}$$

$$\gamma_1 = -0.84834$$

$$\frac{3.18930}{}$$

$$\therefore L_u = 4\pi a N^2 \times 3.18930 \quad (18)$$

This differs from the value found above (14) for L_u by less than 1 part in 100,000.

It is of course to be expected that these two methods of obtaining L_u would agree, from the method of obtaining the correction term A . But an actual numerical test is nevertheless not superfluous.

When the coil is long, so that Stefan's (or Weinstein's) formula does not apply, the method given above for obtaining L_u from $\tilde{L}_s$ is the same and just as accurate provided the correction term B is accurately known. That term here differs very slightly from its value for single layer coils where it has been already verified, so we

may be sure that L_u and so also L for any winding may be obtained with great precision by the above method.

Let us now use this method to determine the magnitude of the error in results obtained by Stefan's formula when applied to coils longer than those for which this formula is accurate.

EXAMPLE 3.

$$a = 10, b = 10, c = 1$$

By Coffin's formula (extension of Rayleigh's) we have

$$\begin{aligned} L_s &= 4\pi a N^2 (1.57944 + .07280 - .00138 + .00009) \\ &= 4\pi a N^2 \times 1.65095 \end{aligned} \quad (19)$$

$$A = .6942 \quad \text{as before}$$

$$\underline{B = .2792} \quad \text{Table II}$$

$$A + B = .9734$$

$$\Delta_1 L = 4\pi a n' \times (.9734)$$

Here $n' = b/c = 10$, and hence $A + B$ must be divided by 10 to put the correction in the same form as L_s . That is, if $n = 10$,

$$\Delta_1 L = 4\pi a n'^2 [.09734]$$

Therefore, if $n' = N$ we have

$$L_u = L_s - \Delta_1 L = 4\pi a N^2 [1.5536] \quad (20)$$

So far we have assumed the section 10×1 cm to be divided into 10 squares. But N may now be any number, and the above expression L_u gives the value of the self-inductance, assuming the current is uniformly distributed over the section.

Another way of putting it is to put $n = m^2 n'$, where m is the number of layers and mn' is the number of turns per layer. In the above example $n = 10$. It is always b/c . The correction $\Delta_1 L$ is

$$\Delta_1 L = 4\pi a n' (A + B) \text{ for } n' \text{ turns}$$

$$\therefore \Delta_1 L = 4\pi a n'^2 \left(\frac{A + B}{n'} \right) \text{ for } n' \text{ turns}$$

But if the winding instead of being one layer of n' turns is m layers of $m^2 n'$ turns, the self-inductance and also the correction $\mathcal{A}_1 L$ will be m^4 times as great. Therefore

$$\begin{aligned}\mathcal{A}_1 L &= 4\pi a m^4 n'^3 \left[\frac{A+B}{n'} \right] \\ &= 4\pi a N^3 \left[\frac{A+B}{n'} \right] \text{ since } N = m^2 n' \quad (21)\end{aligned}$$

Hence $(A+B)$ is always divided by n' . In example 2, $n' = 1$.

By Stefan's formula we have, since $y_1 = 0.59243$, $y_2 = 0.1325$

$$\begin{aligned}L_u &= 4\pi a N^2 \left\{ \left(1 + \frac{301}{9600} \right) 2.074467 - 0.59243 + \frac{0.1325}{16} \right\} \\ &= 4\pi a N^2 \times 1.55536 \quad (22)\end{aligned}$$

This is 1 part in 900 more than the value obtained above (20) by the more accurate method, showing that Stefan's formula is not seriously in error for a coil whose length is equal to its radius, but it is much less accurate than for short coils, as its method of derivation requires.

For a coil only half as long ($b=5$, $c=1$) the difference between the two methods is only one part in 4,000, while for a coil twice as long ($b=20$, $c=1$) the difference is about 1 per cent. That is, Stefan's very convenient formula is correct to within 1 per cent for a coil whose length is as great as its diameter. Beyond that the error increases rapidly.

SUMMARY.

To recapitulate the method set forth in the preceding pages for obtaining the self-inductance of a coil wound in the ordinary manner with insulated round wire, we have the different self-inductances to consider, L_s , L_u and L , which are related as follows:

$$\begin{aligned}L_s - \mathcal{A}_1 L &= L_u \\ L_u + \mathcal{A}_2 L &= L \\ \therefore L &= L_s - \mathcal{A}_1 L + \mathcal{A}_2 L\end{aligned}$$

L_s is the first approximation to the self-inductance required, and is obtained by the current sheet formula of Rayleigh, Coffin or Lorenz.⁴ It is the self-inductance of a coil of length l , radius a ,

⁴ This Bulletin, 2, p. 186.

having N turns, but the *depth of the winding* t is supposed reduced to zero.

$\Delta_1 L$ is the correction to apply to L_s to obtain L_u , which is the self-inductance of the coil of same dimensions and number of turns, but with *actual depth* t of winding; the current is, however, now supposed to be uniformly distributed over the cross section of the winding, as though the wire were of square section and had only an infinitesimal covering. $\Delta_1 L$ depends on two terms, A and B . A is found from equation (7) and B from Table II.

The actual self-inductance of the coil L differs from L_u by the correction $\Delta_2 L$, which depends on three correction terms, C , F , and E . The determination of these quantities, which depend on the shape of the section of the wire, the thickness of the insulation, the number of turns of wire in the coil, and the shape of the section of the coil, has been fully discussed elsewhere⁵ and their values given. The correction $\Delta_2 L$ is much smaller than $\Delta_1 L$, and can be neglected except when the highest accuracy is sought. The value of L_s and $\Delta_1 L$ can be calculated with accuracy if the dimensions are accurately known, and this is possible if one uses enameled wire of uniform section and takes proper care in winding and measuring the coil. However, such a coil can not be recommended for a standard of the highest precision, and I have given the full theory for the sake of completeness and to show the magnitude of the smaller corrections, rather than because all the corrections are likely to be generally needed in practice.

WASHINGTON, October 12, 1907.

⁵ This Bulletin, 8, pp. 3, 4, 28-37.

SELF-INDUCTANCE OF A SOLENOID OF ANY NUMBER OF LAYERS.

By Louis Cohen.

The only formula heretofore available for the calculation of the self-inductance of a long coil or solenoid of more than one layer is that of Maxwell, which is as follows:

$$L = \frac{4}{3} \pi^2 n^2 l (x - y) (x^2 - y^2) \quad (1)$$

where l is the length of the solenoid, n is the number of turns per unit length, x and y are the external and internal radii of the solenoid. This formula, however, was developed on the assumption of a uniform field within the coil, which means that the end effects can be neglected. The assumption may lead to an error as great as 12 per cent as I shall show later, even in the case of a comparatively long solenoid, when the length is ten times the radius. It is quite evident that such a formula is not of very great value where accurate results are desired. It is, however, a very simple and convenient formula for numerical computations and is useful when only a rough approximation is desired.

I propose to develop in the following pages a formula for the self-inductance of a solenoid of any number of layers, and which will give results accurate to within one-half of 1 per cent even for a short solenoid, where the length is only twice the diameter, the accuracy increasing as the length increases. For most practical cases this degree of accuracy is amply sufficient.

Suppose we have a solenoid of m layers, the mean radius of the first layer being a_1 , and of the last layer a_m , then the self-inductance of the solenoid will be given by the following expression:

$$\begin{aligned}
 L = & L_1 + L_2 + L_3 + \dots + L_m \\
 & + 2[M_{12} + M_{13} + \dots + M_{1m} \\
 & + M_{23} + M_{24} + \dots + M_{2m} \\
 & + M_{34} + M_{35} + \dots + M_{3m} \\
 & + \dots \\
 & + \dots \\
 & + M_{m-1, m}] = A + 2B
 \end{aligned} \tag{2}$$

L_1, L_2, L_3 , etc., are the self-inductances of single layer solenoids whose mean radii are a_1, a_2, a_3 , etc., and the M s are the mutual inductances between the various single layer solenoids. We have various formulae for the self-inductance of a single layer solenoid, and also for the mutual inductance of two coaxial solenoids,¹ and if after substituting the various values for L and M , we can combine them so as to give a simple expression, we shall evidently have a useful formula for the self-inductance of a solenoid of m layers.

In a previous paper² I have developed an absolute formula for the self-inductance of a single layer solenoid, which is as follows:

$$L = 4\pi n^2 \left\{ \frac{l^4 + 4a^2 l^2}{3\sqrt{4a^2 + l^2}} F + \frac{4a^2 - l^2}{3} \sqrt{4a^2 + l^2} E - \frac{8a^2}{3} \right\} \tag{3}$$

a is the mean radius, l is the length of the solenoid, and n is the number of turns of wire per centimeter, F and E are the complete elliptic integrals of the first and second kind to modulus k , where

$$k^2 = \frac{4a^2}{4a^2 + l^2}$$

This formula was first given by Lorenz without, however, giving its derivation. F and E can be put in the form of series, thus:

$$\begin{aligned}
 F &= \frac{\pi}{2} \left\{ 1 + \frac{1}{2^2} k^2 + \frac{1 \cdot 3}{2 \cdot 4^2} k^4 + \dots \right\} \\
 E &= \frac{\pi}{2} \left\{ 1 - \frac{1}{2^2} k^2 - \frac{1 \cdot 3}{2 \cdot 4^2} k^4 - \dots \right\}
 \end{aligned}$$

¹This Bulletin, 8, p. 305.

²This Bulletin, 8, p. 303.

If $l=4a$, then $k^2 = \frac{4}{20} = \frac{1}{5}$, and the term in k^4 will be only about $\frac{1}{200}$ which may be neglected if we are aiming at an accuracy of only one half of a per cent. For longer solenoids the accuracy will of course be higher. Using, therefore, only the first two terms of the above series, and introducing the value of k^2 , we get:

$$F = \frac{\pi}{2} \frac{5a^2 + l^2}{4a^2 + l^2}$$

$$E = \frac{\pi}{2} \frac{3a^2 + l^2}{4a^2 + l^2}$$

Introducing the values of F and E into equation (3) and simplifying we obtain

$$L = 4\pi^2 n^2 \left\{ \frac{2a^4 + a^2 l^2}{\sqrt{4a^2 + l^2}} - \frac{8}{3} \frac{a^3}{\pi} \right\} \quad (4)$$

This is a very simple and convenient approximate formula for the calculation of the self-inductance of a single layer solenoid.

The self-inductance of a solenoid is a function of its radius, hence if we denote by L_0 the self-inductance of the central layer solenoid, the self-inductance of the various other layers will be given by the following formulæ:

$$\begin{aligned} L_{\frac{1}{2}-1} &= L_0 - \int_{a_{\frac{1}{2}}}^{a_0} \frac{dL}{da} da \\ L_{\frac{1}{2}-2} &= L_0 - \int_{a_{\frac{1}{2}}}^{a_0} \frac{dL}{da} da - \int_{a_{\frac{1}{2}-1}}^{a_{\frac{1}{2}}} \frac{dL}{da} da \\ &\dots \dots \dots \\ L_{\frac{1}{2}+1} &= L_0 + \int_{a_0}^{a_{\frac{1}{2}}} \frac{dL}{da} da \\ L_{\frac{1}{2}+2} &= L_0 + \int_{a_0}^{a_{\frac{1}{2}}} \frac{dL}{da} da + \int_{a_{\frac{1}{2}}}^{a_{\frac{1}{2}+1}} \frac{dL}{da} da \\ &\dots \dots \dots \end{aligned}$$

Usually the winding is of comparatively fine wire, say, 0.1 to 0.2 cm., and therefore the value of any of the above integrals will be a small quantity as compared with L_0 , and the difference between any two of the above integrals will be an exceedingly small quantity. In summing up the self-inductances of the various layers, we evidently have as many negative as positive integrals which will nearly cancel each other and therefore:

$$\Sigma L = mL_0 = m4\pi^2 n^2 \left\{ \frac{2a_0^4 + a_0^2 l^2}{\sqrt{4a_0^2 + l^2}} - \frac{8a_0^3}{3\pi} \right\} \quad (5)$$

when a_0 is the mean radius of the solenoid.

It remains now to find the value of B of equation (2). The mutual inductance between two coaxial solenoids is given by the following expression³

$$M = 4\pi^2 n^2 [l - 2Aa]$$

where

$$a = \frac{l-r+A}{2A} - \frac{a^2}{16A^2} \left(1 - \frac{A^2}{r^2} \right) - \frac{a^4}{64A^4} \left(\frac{1}{2} + \frac{2A^2}{r^2} - \frac{5}{2} \frac{A^2}{r^4} \right) + \dots$$

$r = \sqrt{A^2 + l^2}$, A is the radius of the external solenoids, and a is the radius of the internal solenoid. Since, however, we are only aiming at an accuracy of one-half per cent for short solenoids we may put

$$a = \frac{l-r+A}{2A} - \frac{a^2}{16A^2}$$

and neglect the other terms. Introducing this value of a we get:

$$M = 4\pi^2 a^2 n^2 \left\{ \sqrt{A^2 + l^2} - A + \frac{a^2}{8A} \right\} \quad (6)$$

Putting $a_2 = a_1 + \delta a$, $a_3 = a_1 + 2\delta a$, $a_m = a_1 + (m-1)\delta a$ where δa is the distance between two consecutive layers, and also expanding the term under the radical and neglecting small terms, we shall have:

³ Maxwell, Electricity and Magnetism, II, § 678.

$$M_{1,2} = 4\pi^2 a_1^2 n^2 \left\{ \sqrt{a_1^2 + l^2} + \frac{a_1 \delta a}{\sqrt{a_1^2 + l^2}} - a_1 - \delta a + \frac{a_1}{8} - \frac{\delta a}{8} \right\}$$

$$M_{1,3} = 4\pi^2 a_1^2 n^2 \left\{ \sqrt{a_1^2 + l^2} + \frac{2a_1 \delta a}{\sqrt{a_1^2 + l^2}} - a_1 - 2\delta a + \frac{a_1}{8} - \frac{2\delta a}{8} \right\}$$

.

$$M_{1,m} = 4\pi^2 a_1^2 n^2 \left\{ \sqrt{a_1^2 + l^2} + \frac{(m-1) a_1 \delta a}{\sqrt{a_1^2 + l^2}} - a_1 - (m-1) \delta a + \frac{a_1}{8} - \frac{(m-1)\delta a}{8} \right\}$$

$$M_{2,2} = 4\pi^2 a_2^2 n^2 \left\{ \sqrt{a_1^2 + l^2} + \frac{2a_1 \delta a}{\sqrt{a_1^2 + l^2}} - a_1 - 2\delta a + \frac{a_1}{8} \right\}$$

$$M_{2,3} = 4\pi^2 a_2^2 n^2 \left\{ \sqrt{a_1^2 + l^2} + \frac{3a_1 \delta a}{\sqrt{a_1^2 + l^2}} - a_1 - 3\delta a + \frac{a_1}{8} - \frac{\delta a}{8} \right\}$$

.

$$M_{2,m} = 4\pi^2 a_2^2 n^2 \left\{ \sqrt{a_1^2 + l^2} + \frac{(m-1)\delta a}{\sqrt{a_1^2 + l^2}} - a_1 - (m-1)\delta a + \frac{a_1}{8} - \frac{(m-3)\delta a}{8} \right\}$$

$$M_{3,3} = 4\pi^2 a_3^2 n^2 \left\{ \sqrt{a_1^2 + l^2} + \frac{3a_1 \delta a}{\sqrt{a_1^2 + l^2}} - a_1 - 3\delta a + \frac{a_1}{8} + \frac{\delta a}{8} \right\}$$

$$M_{3,4} = 4\pi^2 a_3^2 n^2 \left\{ \sqrt{a_1^2 + l^2} + \frac{4a_1 \delta a}{\sqrt{a_1^2 + l^2}} - a_1 - 4\delta a + \frac{a_1}{8} \right\}$$

.

$$M_{3,m} = 4\pi^2 a_3^2 n^2 \left\{ \sqrt{a_1^2 + l^2} + \frac{(m-1) \delta a}{\sqrt{a_1^2 + l^2}} - a_1 - (m-1) \delta a + \frac{a_1}{8} - \frac{(m-5) \delta a}{8} \right\}$$

$$M_{4,m} = 4\pi^2 a_4^2 n^2 \left\{ \sqrt{a_1^2 + l^2} + \frac{(m-1)a_1 \delta a}{\sqrt{a_1^2 + l^2}} - a_1 - (m-1)\delta a + \frac{a_1}{8} - \frac{(m-7)\delta a}{8} \right\}$$

.

.

$$M_{m-1,m} = 4\pi^2 a^2 n^2 \left\{ \sqrt{a_1^2 + l^2} + \frac{(m-1)a_1 \delta a}{\sqrt{a_1^2 + l^2}} - a_1 - (m-1)\delta a + \frac{a_1}{8} - \frac{(m-2m+1)}{8} \delta a \right\}$$

Summing up we get:

$$M_{1,2} + M_{1,3} + \dots + M_{1,m} = 4\pi^2 a_1^2 n^2 (m-1) \left\{ \sqrt{a_1^2 + l^2} + \frac{ma_1 \delta a}{2\sqrt{a_1^2 + l^2}} - a_1 - \frac{m}{2} \delta a + \frac{a_1}{8} - \frac{m}{2} \frac{\delta a}{8} \right\}$$

$$M_{2,3} + M_{2,4} + \dots + M_{2,m} = 4\pi^2 a_2^2 n^2 (m-2) \left\{ \sqrt{a_1^2 + l^2} + \frac{(m+1)a_1 \delta a}{2\sqrt{a_1^2 + l^2}} - a_1 - \frac{(m+1)\delta a}{2} + \frac{a_1}{8} - \frac{(m-3)}{2} \frac{\delta a}{8} \right\}$$

$$M_{3,4} + M_{3,5} + \dots + M_{3,m} = 4\pi^2 a_3^2 n^2 (m-3) \left\{ \sqrt{a_1^2 + l^2} + \frac{(m+2)a_1 \delta a}{2\sqrt{a_1^2 + l^2}} - a_1 - \frac{(m+2)\delta a}{2} + \frac{a_1}{8} - \frac{(m-4)}{2} \frac{\delta a}{8} \right\}$$

$$M_{4,5} + M_{4,6} + \dots + M_{4,m} = 4\pi^2 a_4^2 n^2 (m-4) \left\{ \sqrt{a_1^2 + l^2} + \frac{(m+3)a_1 \delta a}{2\sqrt{a_1^2 + l^2}} - a_1 - \frac{(m+3)\delta a}{2} + \frac{a_1}{8} - \frac{(m-5)}{2} \frac{\delta a}{8} \right\}$$

$$\begin{array}{cccccccc} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{array}$$

Adding all the summation terms we get for the total mutual inductance between the various layers:

$$\begin{aligned} \Sigma M = 4\pi^2 n^2 & \left\{ [(m-1)a_1^2 + (m-2)a_2^2 + (m-3)a_3^2 + \dots] \left(\sqrt{a_1^2 + l^2} \right. \right. \\ & \left. \left. - \frac{7}{8}a_1 \right) + \frac{1}{2} [m(m-1)a_1^2 + (m-1)(m-2)a_2^2 + (m-2)(m-3)a_3^2 \right. \\ & \left. + \dots] \left(\frac{a_1 \delta a}{\sqrt{a_1^2 + l^2}} - \delta a \right) - \frac{1}{2} [m(m-1)a_1^2 + (m-2)(m-3)a_3^2 + \dots] \frac{\delta a}{8} \right\} \end{aligned}$$

The final formula for the self-inductance of a solenoid of m layers will therefore be as follows:

$$L = 4\pi^2 n^2 m \left\{ \frac{2a_o^4 + a_o^2 l^2 - 8a_o^3}{\sqrt{4a_o^2 + l^2}} - \frac{8a_o^3}{3\pi} \right\} + 8\pi^2 n^2 \{ [(m-1)a_1^2 + (m-2)a_2^2 + \dots] \\ \left(\sqrt{a_1^2 + l^2} - \frac{7}{8} a_1 \right) + \frac{1}{2} [m(m-1)a_1^2 + (m-1)(m-2)a_2^2 + \dots] \\ \left(\sqrt{\frac{a_1 \delta a}{a_1^2 + l^2}} - \delta a \right) - \frac{1}{2} [m(m-1)a_1^2 + (m-2)(m-3)a_2^2 + \dots] \frac{\delta a}{8} \} \quad (7)$$

The last term in equation (7) is only about 1/500 of the total inductance even in the case of a relatively short solenoid, where the length is twice the diameter, and hence we may neglect the last term. We therefore have:

$$L = 4\pi^2 n^2 m \left\{ \frac{2a_o^4 + a_o^2 l^2}{\sqrt{4a_o^2 + l^2}} - \frac{8a_o^3}{3\pi} \right\} + 8\pi^2 n^2 \{ [(m-1)a_1^2 + (m-2)a_2^2 + \dots] \\ \left(\sqrt{a_1^2 + l^2} - \frac{7}{8} a_1 \right) + \frac{1}{2} [m(m-1)a_1^2 + \dots] \left(\sqrt{\frac{a_1 \delta a}{a_1^2 + l^2}} - \delta a \right) \} \quad (8)$$

For long solenoids, where the length is, say, four times the diameter, we can neglect also the last term in equation (8).

The formula for the self-inductance as given by equation (8) is a simple one and very convenient for numerical computations.

Below are two numerical examples to show the per cent of error that Maxwell's formula (1) will introduce:

EXAMPLE 1.

$$a_1 = 5 \text{ cm}, \delta a = 0.1 \text{ cm}, m = 4, a_o = 5.15 \text{ cm}, l = 50 \text{ cm}.$$

Introducing these values in (7) we get:

$$L = 4\pi^2 n^2 \{ 4841.9 + 14134.7 - 45.9 - 4.4 \} = 4\pi^2 n^2 \times 18926$$

It is seen that in this case where the length is five times the diameter, even the third term of equation (7) is only about one four-hundredth part of the total inductance, which may be neglected in approximate calculations.

The same example calculated by Maxwell's formula gives

$$L = 4\pi^2 n^2 \times 21228$$

The error is a little over 12 per cent.

EXAMPLE 2.

$$A_1 = 5 \text{ cm}, \delta a = 0.2 \text{ cm}, m = 5, a_0 = 5.4 \text{ cm}, l = 20$$

By formula (7)

$$L = 4\pi^2 n^2 \{2271.5 + 8795.9 - 161.5 - 18\} = 4\pi^2 n^2 \times 10888$$

By Maxwell's formula (1) we get for this case:

$$L = 4\pi^2 n^2 \times 14622$$

The error in formula (1) is therefore about 35 per cent.

WASHINGTON, October 11, 1907.

INSTRUMENTS AND METHODS USED IN RADIOMETRY.

By W. W. Coblentz.

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I. INTRODUCTION.

There are few fields of experimental investigation so beset with difficulties as the quantitative measurement of radiant energy. This is due chiefly to the fact that the radiation to be measured is generally from a surface, of which it is practically impossible to determine the temperature. The measurement of radiant energy, moreover, generally involves its transformation into some other form, and the receiver used for this purpose is subject to losses by heat conduction within, and by reflection, radiation, and convection losses from its surface.

As a result of inquiry into the development of the various instruments and methods used in measuring radiant energy, viz, the radiometer, the thermopile, the radiomicrometer and the bolometer with its auxiliary galvanometer, the writer has accumulated data, part of which are included here, with the hope that it may be useful to others interested in the subject. An attempt is also made to give the general principles involved in the construction and use of different radiation meters, as well as original experimental data of their relative efficiencies.

Much has been written on the theory and design of sensitive galvanometers and bolometers, and nothing radically new will be attempted in this paper. Not that improvements are impossible, but, as will be noticed presently, the working sensitiveness seems to have attained a fixed value for all of the various designs of galvanometer coils and magnet systems thus far described. It will also be noticed that the "working sensitiveness," at which it is possible to use the instrument with precision and convenience, and the highest attainable sensitiveness are two distinct factors in rating radiation meters. For example, it may be possible at certain hours of the day to read a bolometer-galvanometer deflection to 0.1 mm. and thus detect a rise in temperature of, say, one ten-millionth degree, but for an instrument that is useful at all hours a fair estimate of the sensitiveness attained by various observers is about one-tenth to one-twentieth this value. It is practically impossible to buy instruments as sensitive as this. While galvanometers and thermopiles can be purchased, they often fall short of the specifications of the original, so that it is better to build the



instrument in one's own laboratory. Here again one experiences the difficulty that the descriptions of such galvanometers and bolometers are scattered through so many journals that it becomes a burden to learn of the different improvements that have been made. For example, one may find a galvanometer with large coils, and a heavy magnet system, provided with an excellent magnetic shield. Again, one will find a galvanometer with small coils, and light suspension, built in such a manner that the latter is visible, which is a desirable feature, but the whole is enclosed in a large glass case which renders magnetic shielding very difficult. Furthermore, European investigators have used their bolometers and balancing coils in separate cases, and the bridge arms of equal resistance, while the latest developments in this country show that it is best to have the resistance of the bridge arms several times that of the bolometer strips, and the whole, including the balancing wire, enclosed in a single double walled case. These are some of the facts brought out by the writer's inquiry into the matter; and in designing the instruments to be described, and in the improvements suggested, an attempt was made to include as many as possible of the good points in the various instruments previously described, and to introduce simplifications wherever possible.

Various instruments for measuring radiant energy have been devised, the relative sensibilities of which can be rated without further investigation. That in many cases the sensitiveness has been overestimated will be noticed in the present paper. Four instruments, viz, the radiomicrometer, the thermopile, the bolometer, and the radiometer have been used extensively in radiation work, and in each case the investigators have found qualities which seemed to render each type of instrument superior to the others. But, so far as the writer has been able to learn, all four instruments have not been heretofore studied by any one person. Each instrument requires a special mode of handling, and has peculiarities which can be learned and controlled only after prolonged use. This is particularly true of the radiometer, and of the bolometer with its auxiliary galvanometer. Having already had considerable experience with radiometers, one of which was

the most sensitive yet constructed,¹ the writer has, in this examination, devoted most of his attention to the bolometer. The investigation originated for the most part from the question whether the radiometer was selective in its action in the region of short wave-lengths. In previous work it was found that the radiometer gave small deflections in the violet spectrum of the arc where Snow,² using a bolometer, found large deflections.

In the course of the discussion it will be noticed, as was previously known in a general way, that each instrument has some quality which makes it useful for particular kinds of work. For measuring very narrow emission lines and determining dispersion curves the bolometer is no doubt the best instrument. For measurements requiring a larger receiving surface the linear thermopile is the more sensitive and the more precise. It has the further advantage that there is no permanent current. On the contrary, the bolometer has a current which heats the bolometer strips above the temperature of the surrounding air. This causes air currents which make the zero of the galvanometer unstable. This is not true of the thermopile. Less is known concerning the radiometer, which rivals the bolometer and the thermopile in sensitiveness. Furthermore, the radiometer is not subject to magnetic perturbations. Its window limits its usefulness to the region of the spectrum up to $20\ \mu$. The fact that it is not portable is a minor objection.

An attempt is made in the present paper to discuss all the important details involved in radiometry, so that it will be possible to gain a knowledge of the subject without searching through the already extensive literature.

II. THE MICRORADIOMETER.

Since we are concerned with radiation meters of the greatest sensitiveness, the ingenious device of Weber,³ called the "micro-radiometer," deserves notice. The instrument is not unlike a combination of a differential air thermometer and a Wheatstone bridge. Two arms of the bridge consist of a thin glass tube con-

¹ See "Investigations of Infra-red Spectra," Part II. A still greater sensibility was attained in the present investigation.

² Snow, *Physical Review*, 1, p. 32; 1893.

³ Weber, *Archiv. Sci. phys. et Nat.* (3), 18, p. 347; 1887.

taining a drop of mercury at the center, with a solution of zinc sulphate at the ends, into which dip platinum electrodes. The ends of the glass tube widen out into large bulbs containing air. The ends of the bulbs are covered with rock salt windows. If radiant energy is allowed to enter one of the bulbs, the air expands and pushes the liquids toward the opposite bulb. This will change the relative lengths of the column of mercury and of the solution between the platinum terminals, which means a change in resistance in the bridge arm and a consequent deflection of the galvanometer. The instrument was stated to be sensitive to a temperature change of 0.00001° , and while it is not adapted to spectrum radiation measurements, it might be used in total radiation work where an elaborate installation is not convenient. By making the receiving bulb of opaque nonconducting material and covering the inside with lampblack, or platinum black, this would be as complete an absorber (black-body) as the thermopile or bolometer. Its efficiency would of course depend upon the gas enclosed.

III. THE RADIOMICROMETER.

The radiomicrometer is essentially a moving coil galvanometer having a single loop of wire with a thermo-junction at one end. This instrument was invented independently by d'Arsonval⁴ and by Boys.⁵ The former used a loop, one part of which was silver and the other was of palladium. The latter used a junction of bismuth and antimony, which was soldered to a loop of copper wire.

The sensibility of the Boys instrument was given as 10^{-8} to 10^{-7} of 1° . From subsequent work with other radiation meters in which this high degree of sensitiveness has never been attained, it would appear that the sensibility of the radiomicrometer was overestimated. It certainly has never attained the sensibility of the radiometer, one example of which, used by Nichols (*loc. cit.* Table III), was 12 times as sensitive as the radiomicrometer of Boys. The latter gave a deflection of a

⁴ d'Arsonval, *Soc. Franc. de Phys.*, pp. 30 and 77; 1886.

⁵ Boys, *Proc. Roy. Soc.*, **42**, p. 189, 1887; **44**, p. 96, 1888; **47**, p. 480, 1890; *Phil. Trans.*, **180A**, p. 159, 1889.

little less than 1 cm per mm² of exposed vane for a candle and scale each at a distance of 1 meter. Paschen⁶ attempted to improve the radiomicrometer, but out of about fifty junctions only three were useful, and these were only three times as sensitive as that of Boys, while the period was about forty seconds. The long period is not always detrimental, however, for the radiomicrometer is not subject to magnetic disturbances and is a very useful instrument for work not requiring the highest attainable sensitiveness. The writer⁷ has indicated further improvements in the instrument, and places it in a vacuum, which increases the sensibility by at least 70 per cent. The instrument was about six times as sensitive as that of Boys for a full period of 25 seconds. Para- and dia-magnetism limited the sensitiveness to this value. The work with this instrument brought out the fact that one may use too strong field magnets and that further improvement may be made by using weak magnets, or by using narrow strong magnets situated as far as possible above the thermojunction, so as to avoid the effect of para- or dia-magnetism. The combination of the radiomicrometer and the radiometer is feasible, although the writer found its usefulness as limited as that of the radiomicrometer. When wires can be obtained more free from magnetic material it will be possible to construct a more sensitive instrument. It is doubtful, however, whether it will ever surpass the bolometer used with a galvanometer of the highest sensibility. With the radiomicrometer, Lewis⁸ was able to investigate infra-red emission spectra of the alkali metals, which are weak in energy. Wilson⁹ and Julius¹⁰ have used the radiomicrometer for total and spectrum radiation work, and found the instrument highly satisfactory. The long period (which also obtains in other sensitive radiation meters) and lack of portability, mentioned by some writers, is certainly not to be weighed against its indifference to magnetic perturbations and constancy of the zero reading. Even a slow period is less objectionable than a quick-period instrument with which just as

⁶ Paschen, *Ann. der. Phys.* (3), 48, p. 272; 1893.

⁷ This Bulletin, 2, p. 479; 1906.

⁸ Lewis, *Astrophys. J.*, 2, p. 1; 1895.

⁹ Wilson, *Proc. Roy. Soc.*, 55, 1894; 58, 1895; 60, p. 337; 1896

¹⁰ Julius, *Handlingen*, 5, de Nederlandsch Natuur en Geneeskundig Congres; 1895.

much time is lost by repeating observations, which may be affected by the lack of constancy of the zero. The instrument is self-contained and where the greatest sensitiveness is not required, it deserves a wider application. In Table I are given the various radiomicrometers thus far described and their sensitiveness, expressed in centimeter deflections per mm^2 of exposed vane, for a candle and scale each at a distance of 1 meter.

It will be shown below that the highest efficiency is obtained when the resistance of the thermocouple is equal to the combined resistance of the connecting wires and of the auxiliary galvanometer. Since the resistance of a single couple is much less than that of the galvanometer, it is most advantageous to use several pairs of junctions. On the other hand, in the radiomicrometer the connecting loop of wire has a negligible resistance, and hence there is no advantage in using more than a single pair of junctions; for as we increase the electromotive force by adding junctions the resistance is increased in like proportion, so that the current remains practically constant. For like reasons there is no advantage in using more than one turn of wire in the connecting loop.

TABLE I.

Sensitiveness of Radiomicrometers and Rubens Thermopile.

Observer	Full period	Area of vane	Deflections in cm/mm^2 candle and scale at 1 m
Boys	10 sec.	4 mm^2	0.9 cm.
Phil. Trans., 180 A, p. 159, 1889.			
Paschen	40	-----	3
Wied. Ann., 48, p. 275, 1893.			
Lewis	20	1.4	1.3 (?)
Astrophys. Jour., 2, p. 1, 1895.			
Coblentz	40	3	3.6
This Bulletin.....	25	3	6 (in vacuo)
2, p. 479, 1906.			
Thermopile.			
Rubens	14	16 (?)	16(250cm total deflection)
Wied. Ann., 45, p. 244, 1898.			1 mm= 1.1×10^{-6} C.

IV. THE THERMOPILE.

The thermopile has been in use from the very beginning of radiant energy measurements, and in the hands of Tyndall and other pioneers in this domain has rendered excellent service in spite of its great heat capacity. For spectro-radiometric work, however, only the linear thermopile of Rubens¹¹ is well adapted. This thermopile consists of 20 junctions of iron and constantan wires about 0.1 mm to 0.15 mm diameter (resistance 3.5 ohms), and when used with a galvanometer, having a figure of merit of $i = 1.4 \times 10^{-10}$ amperes (resistance = 3 ohms, period = 14 seconds) a deflection of one scale division indicated a temperature¹² change of 1.9×10^{-6} . A candle at 5 m gave a deflection of about 10 cm or 250 cm at 1 m. The area of exposed face of pile is about 0.8×20 mm. The deflections were as rapid as for a bolometer, and its stationary temperature was reached in less time than the single swing of the galvanometer needle. In other words, its heat capacity was so small that it gave an accurate register of the energy falling upon it. In another experiment, using a galvanometer sensitiveness of $i = 5 \times 10^{-10}$ amperes, and the scale at 1 meter, 1 mm deflection = 2.9×10^{-6} . The sensitiveness is the same as that of the best bolometers yet constructed, while its simplicity commends itself even in spectrum radiation work.

The general experience in this country, however, has been that the commercial instrument does not fulfill all the excellent qualities claimed for the one originally described. The wires are heavier than in the original specifications, which makes the instrument sluggish.

The problem in thermopile construction is to secure a low resistance (equal to that of the galvanometer), a low heat capacity

¹¹ Rubens, *Zs. für Instrumentenkunde*, 18, p. 65; 1898.

¹² If p = the thermoelectric power in microvolts per degree (= 53 microvolts for iron and constantan), n = number of junctions exposed and r = the internal resistance, of the thermopile; and if we combine the pile with a galvanometer, which, with an internal resistance of w ohms, gives a deflection of m millimeters per microampere, then a deflection of 1 mm indicates a change in temperature of the junctions of Δt degrees where

$$\Delta t = \frac{r + w}{n p m}$$

and heat conductivity, and a high thermoelectric power. The latter requirement is fulfilled by using junctions of iron and constantan. The heat capacity can be reduced by using finer wire, say 0.06 to 0.08 mm diameter, and by making the unexposed junctions smaller than the ones to be exposed. The junctions are soldered with quite large beads of silver, which are then flattened to present a large surface. The unexposed junctions do not need this, and the small bead formed by the fusion of the two wires (with a bit of silver solder if necessary) can be hammered thin, in order to have it radiate rapidly. By using finer wires (to reduce heat conduction) the resistance will be increased if the dimensions of the Rubens pile be retained. In the commercial instrument,

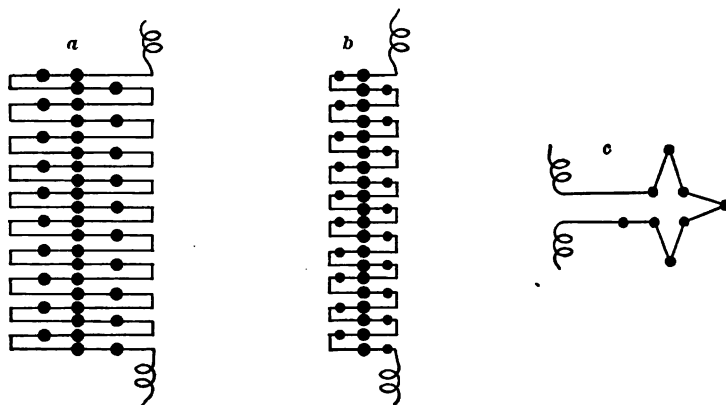


Fig. 1.

at least one-third of the wire is between the unexposed junctions and the binding posts. The greater part of this wire may be eliminated by making the supporting frame narrower, while still retaining the original distance between the exposed and the unexposed junctions. The elimination of this superfluous wire will reduce the resistance by about one-third. In Fig. 1 is shown the original design, *a*, and the suggested improved design *b*. In Fig. 1c, is shown, on an enlarged scale, a thermopile for "point" sources. By using iron and constantan wires 0.06 to 0.08 mm diameter it is possible to place quite a number of junctions within a small area. The combination could be used to advantage in a F ry pyrometer, and for measuring radiation from sun spots,

etc., in which the bolometer, on account of the smallness of the exposed surface, is lacking in sensitiveness. By properly arranging the groups, of four or more junctions each, the combination will take the form of a hollow enclosure, which would tend to make it a more complete absorber of radiant energy.

1. Comparison of Old and New Form of Thermopile.—In order to test these conclusions in regard to the use of finer wire, a new iron-constantan pile of 20 junctions, made of wire 0.08 mm diameter, was ordered from the makers of the original instrument. Although the specifications were not completely fulfilled (the frame was nearly the same size as the original, which increased the resistance to 9 ohms), the sensitiveness was 1.4 times that of the old type, which has a resistance of 4.8 ohms (wire about 0.15 mm). By means of suitable switches the two thermopiles were connected to the same galvanometer, having a full period of 12 seconds ($i = 2 \times 10^{-10}$ amperes) and exposed to the radiation from a Nernst heater. For all deflections, as large as 35 cm, the new thermopile showed no drift greater than ± 2 mm, which may be attributed to the galvanometer. On the other hand, the zero of the old thermopile would drift 0.5 cm in a 10 cm deflection to 2.2 cm in a 27 cm deflection, and it would require 15 to 20 seconds for the deflection to become zero.

The two instruments were then tested in a vacuum. The sensitiveness of the old instrument was increased only 15 per cent, while no change in sensitiveness could be detected in the new one, although two distinct tests were made on different days, the pressure having been reduced to 0.01 mm. The thermopiles are mounted on ivory frames and covered with a sheet of copper, one side having a slit the other a funnel-shaped opening (1×15 mm). The slit was covered and the radiation passed through the funnel. The whole was suspended from a rubber cork in a wide-mouthed bottle, which was exhausted with a mercury or a Geryk pump. The source of energy was an incandescent lamp. With 200 ohms in series with the galvanometer the deflections were about 10 cm. The fact that the sensitiveness of these thermopiles did not increase appreciably in a vacuum is rather remarkable. Brandes¹³

¹³ Brandes, *Physikal. Zs.*, 6, p. 503; 1905.

found that a single junction of 0.02 mm wire became 18 times more sensitive in a vacuum. Lebedew¹⁴ found that a 0.025 mm iron and constantan junction, when black, was 7 times, and when bright was 25 times, more sensitive at a pressure of 0.01 mm than at atmospheric pressure.

Since writing the above an investigation of Moll¹⁵ has appeared, in which he used a modified form of Rubens thermopile. The iron-constantan wires were 0.06 mm diameter, the junctions were 0.2 mm diameter, while the resistance was 12 ohms. The galvanometer sensibility was $i = 1 \times 10^{-8}$ amperes (not very sensitive), and had a full period of 12 seconds. The radiation curves were

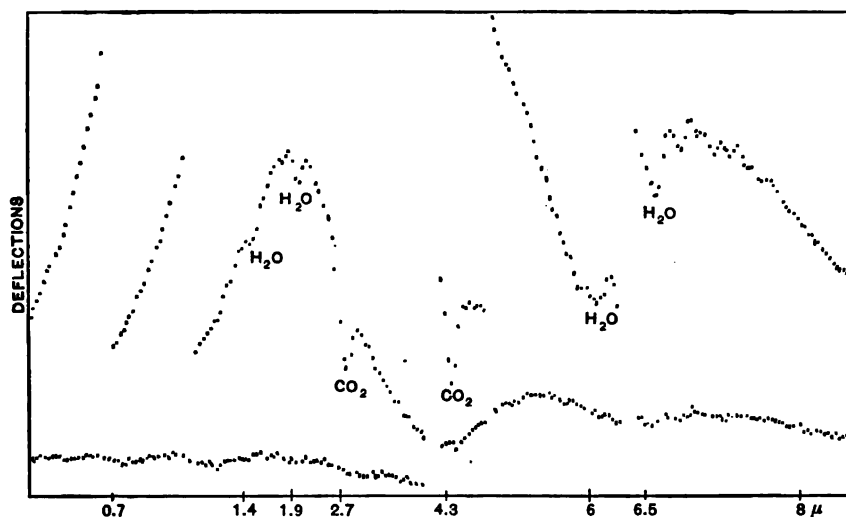


Fig. 2.—Energy curve of Nernst Glower. (Moll.)

recorded automatically by a device that also registered the zero of the instrument after each deflection. The curves must therefore be free from any personal bias, and one is reproduced in Fig. 2 to show that the thermopile of fine (.06 to .08 mm) wire deserves more consideration than it has heretofore received.

2. The Peltier Effect.—The result of the Peltier effect is to lower the temperature of the exposed junction. Consequently, the thermopile does not give an accurate record of the energy

¹⁴ Lebedew, *Ann. der Phys.* (4) 9, p. 209; 1902.

¹⁵ W. J. H. Moll, *Dissertation*, Utrecht. February, 1907.

received. The actual error introduced has never been determined. Since there is a possibility of using the thermopile for quantitative work in place of the bolometer, it is desirable to learn the degree of accuracy of this instrument.

The rate of generation of heat by the Peltier effect is proportional to the current, while the generation of heat on account of resistance is proportional to the square of the current. Jahn¹⁶ has shown that the heat generated by the Peltier effect, determined experimentally, agrees, within experimental error, with the value computed from the observed thermoelectric power. The value for iron-constantan has never been determined experimentally^{16a} but from the work of Jahn it is permissible to compute the heat generated in the thermopile by using the known thermoelectric power, which is about 50×10^{-6} volts.

Using a galvanometer of 5 ohms resistance and having a figure of merit of $i = 3 \times 10^{-10}$ amperes per mm for a scale at 1 m, and an iron-constantan thermopile of 20 junctions, wire 0.08 mm and 5 ohms resistance (see footnote 12);

$$1 \text{ mm} = 2 \times 10^{-6} \text{ degree.}$$

The Peltier effect in calories is computed from the formula:

$$P = \frac{Tit}{J} \frac{dE}{dt}$$

where $T = 274^{\circ}$, $i = 3 \times 10^{-11}$ c.g.s. units, $t = 5$ seconds, $J = 4.2 \times 10^{-7}$ and $dE/dt = 50 \times 10^{-2}$ c.g.s. units,

$$\therefore P = \pm 5 \times 10^{-18} \text{ gr.-cal. (in 5 seconds).}$$

The total weight of the junctions is about 0.01 gr. and the specific heat is about 0.1 gr.-cal.

Hence the temperature change of the exposed junctions is:

$$\Delta t = \frac{5 \times 10^{-18}}{0.1 \times 0.01} = 5 \times 10^{-9} \text{ degree (for 1 mm deflection)}$$

and since the temperature of the unexposed junctions is changed an equal amount in the opposite direction the total change $\Delta t = 1 \times 10^{-8}$ degree. But a deflection of 1 mm = 2×10^{-6}

¹⁶ Jahn, Wied. Ann., **84**, p. 755; 1898.

^{16a} Since writing this it has been found that Lecher, Ber. Akad. Wiss. Wien., **115**, p. 1505, 1906; Sci. Abstracts, 1083, 1907, has recently determined this constant to be 12.24 gr.-cal. per amp.-hr., while the value previously computed was 10.5 gr.-cal. per amp.-hr.

degree, hence the error is 1 part in 200 under the best theoretical conditions. In practice the temperature sensitiveness will not be so great; it will be shown presently to be of the order 5×10^{-6} degree, whence the Peltier effect would cause an error of 1 part in 500, or 1 mm in 50 cm, which is as close as one can read such large deflections. Since the Joule heat depends upon the square of the current, it is negligible. Further consideration of the thermopile as an instrument for quantitative measurements will be found below in connection with the bolometer.

It will be noticed presently, that prior to his construction of the iron-constantan thermopile, Rubens used several very sensitive bolometers, all of which were displaced by the thermopile. For exploring spectra with very narrow lines, the linear bolometer is probably better adapted than the pile which, however, may be covered with a diaphragm, having a narrow slit. For extreme sensitiveness it equals the bolometer, and it is a noteworthy fact that all the investigations in the extreme infra-red and ultra-violet parts of the spectrum, where the energy is weak, have been accomplished by means of the thermopile. Unless one can build up an elaborate bolometric apparatus in a room not exposed to direct sunlight, the thermopile will give the more reliable readings, as far as the constancy of the zero is concerned. Whether or not the thermopile will give a true measure of the energy falling upon it will depend upon the manner in which it is employed. It requires no particular skill to manipulate, and is easier to protect against temperature changes than is a bolometer with its storage battery. The older form of thermopile used by Melloni, Tyndall, and others were subject to drift similar to that observed with the bolometer. This was not due to unequal increments of resistance, as in the bolometer, but to thermoelectric effects at the binding screws, to the connecting wires moving in the earth's magnetic field, and principally to the large heat capacity of the junctions. Most of these disturbances, however, are small and easily avoided in the Rubens type of thermopile. Since the bolometer strips and the balancing coils are of dissimilar material, it is also subject to thermoelectric disturbances.

V. THE RADIOMETER.

The manner in which an interesting scientific toy can be made to serve a useful purpose is well exemplified in the radiometer of Crookes,¹⁷ discovered about 1875. By fastening bits of pith (the one black, the other white) at the ends of a long straw, which was suspended by means of a silk fiber in a long glass tube, he was able to make measurements of radiant energy, even at that early date. Pringsheim¹⁸ simplified the instrument somewhat, suspended the vanes bifilarly with silk thread, and used it to investigate the infra-red spectrum of the sun, produced by means of a glass prism, to about 1.5μ . From this the first really useful radiometer was developed by Nichols.¹⁹

It consists of two similar thin vanes of blackened mica or platinum attached to a horizontal arm, and suspended in a vacuum by means of a fine quartz fiber. The vanes are about 3 mm from the window. The radiation to be measured falls upon one of the vanes, which becomes slightly warmed. This causes the residual gas molecules to rebound with increased velocity from the blackened surface, and the reaction pushes the vane from the window. There is a small mirror attached to the glass staff which supports the vanes, and the deflection is observed by means of a telescope and scale. At certain gas pressures the exposed vane is attracted toward the window instead of being repelled from it. The behavior of the radiometer has been worked out theoretically by Maxwell²⁰ in his paper on "Stresses in Rarefied Gases Arising from Inequalities of Temperature." Among other things, he showed that for two parallel disks very near each other the central points will produce but little effect, because between the disks the temperature varies uniformly, and only near the edges will there be any stress arising from an inequality of temperature in the gas. It has been shown by others, especially by Crookes and by Nichols, that the sensitiveness of the radiometer is a function of the pressure of the residual gas, of the kind of gas surrounding the vanes, and

¹⁷ Crookes, *Phil. Trans.* (II), **166**, p. 325; 1876.

¹⁸ Pringsheim, *Ann. der Phys.* (3), **18**, p. 32; 1883.

¹⁹ Nichols, *Phys. Rev.*, **4**, p. 297; 1897. *Ber. der Berliner Akad.*, p. 1183; 1896.

²⁰ Maxwell, *Collected Papers*, **2**, p. 681. *Phil. Trans.*, Part I, 1879.

of the distance of the exposed vanes from the window. The latter, on account of its absorption, limits the region of the spectrum that can be investigated. If the vanes are not too close to the window, the deflections will be proportional to the energy falling upon one of them.

1. **Comparison of Sensitiveness and Area of Vane.**—For vanes of small dimensions, such as must be used in practical work, the writer has found that the deflections are proportional to the area of the exposed surface of the vane. This is perhaps to be expected, although there seemed to be some doubt. The curve, Fig. 3, of

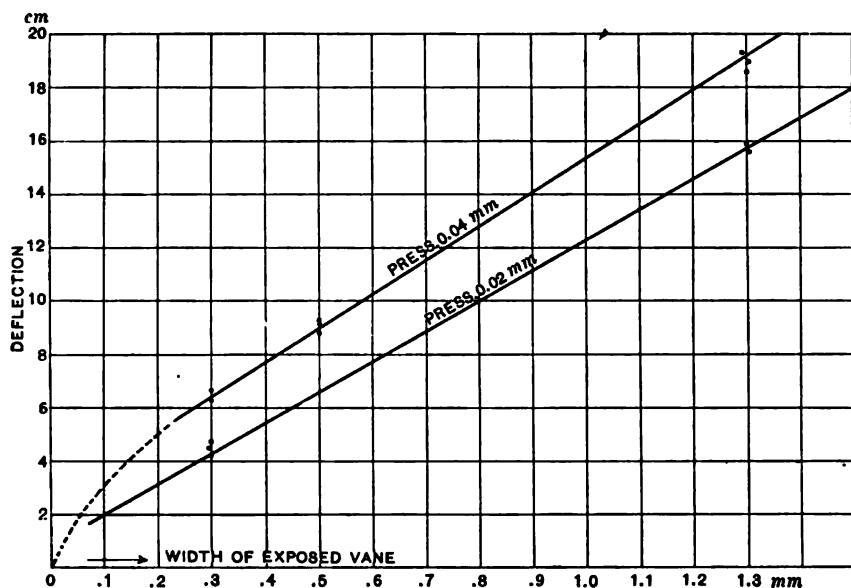


Fig. 3.—Variation of Radiometer deflections with area exposed.

deflections and exposed area of vanes (area 10.5×1.3 mm for constant pressure of 0.02 mm) does not pass through the origin. One explanation may be that for infinitely narrow vanes the graph is not a straight line, but curves as it approaches the origin. Because of the impracticability of suspending vanes of different widths successively from the same fiber suspension, at the same distance from the window, and using the same gas pressure for all vanes, it was necessary to use one wide vane, with a slit before it, and vary the opening of the slit. The source of energy (Nernst

heater) was at a distance of 3 meters, and hence the width of the projection of the slit upon the vane was practically the width of the slit, except for a very narrow slit when diffraction may decrease somewhat the energy incident on the vane. This, however, would displace the graph still farther from the origin. The forces acting in a radiometer are so complex and so little understood that no further examination was made to ascertain the limits within which the above proportionality holds. The test of proportionality was made to reduce the deflections to unit area between the above limits of exposed vane. It is of interest to note in this connection that in a bolometer the sensitiveness varies as the square root of the area of the bolometer strip.

In his earlier communications the writer held to the belief that the weight of the vanes and their size were the most important factors in determining the sensitiveness and period of a radiometer. However, so many factors enter into the problem that it is difficult to decide this point, and the following test may be of interest, showing that the diameter of the quartz fiber suspension is an important factor in determining the sensitiveness.

2. Comparison of Sensitiveness and Diameter of Fiber Suspension.—Using the same vanes (0.5×9 mm area) the sensitiveness and period were found for a heavy and a light quartz fiber suspension. The main difficulty was to insure that the vanes were at the same distance from the window, and that the pressure was the same in the two cases. Hence these quantities are only approximate. In Table II it will be noticed that in changing from a heavy to a light fiber the sensitiveness is increased 4.5 times, while the period was increased almost three fold. It further shows that for the same (light) fiber the sensitiveness was doubled (36 to 71 cm per mm) by changing the pressure and the distance from the window, which was of fluorite and hence opaque beyond 10μ . The sensitiveness of 71 cm per mm^2 of exposed area is the highest on record. This, however, was not the maximum sensitiveness, since the pressure was 0.02 mm, while radiometers have their maximum sensitiveness at a pressure of about 0.05 to 0.1 mm. At this pressure, however, heat conduction would cause annoyance. The vanes of this suspension were of platinum foil 0.01 mm thick, covered on one side electrolytically with platinum black and then

smoked over a candle. It is best to cool the gases from the flame by placing a wire gauze, or sheet of metal full of holes, between the flame and the vanes when smoking them. These vanes were suspended by means of one of the finest workable quartz fibers, and when within 3 mm of the window either one of the vanes would always approach and adhere to it, even at atmospheric pressure. From tests with fluorite windows, which from internal strains might be piezoelectric, and with rock salt windows when bare and also when covered with tinfoil, it was found that this effect is not due to electrification. Starting with the vanes parallel and at a distance of about 5 mm from the window it was found that, as this distance was decreased, one of the vanes (generally the one to be exposed to radiation) would approach the window, and for a distance of about 3 mm would turn until the plane of the vanes was at right angles to the window. The observations extended over several months, and all evidence indicates that this effect is due to gravitational attraction. As a result of this the deflection of such a vane would not be proportional to the energy received. This radiometer had no torsion head to control the zero. However, for general work with very sensitive radiometers a torsion head would be necessary since the best pumps may leak, which will cause a slow drift. It will be shown presently that this is about 5 times the sensitiveness of Snow's bolometer, for which 1 mm deflection (scale at 3 m) indicated a temperature difference of 7.6×10^{-8} . In other words, this radiometer would detect $\frac{1}{100000}$ degree rise in temperature. But the period of the radiometer was 6 times that of the bolometer-galvanometer, which is its weakest point in radiation work requiring a short period.

A comparison can also be made (Table II) between light vanes (0.5×9 mm) and heavy ones (1.3×10.5 mm) at the same pressure but having different quartz fiber suspensions. The results show that while the light vanes are more sensitive than the heavy ones (see Table III), there seems to be no limit to the sensitiveness attainable in either case, without considering the period. The idea of not considering period of vibration with sensitiveness seems reasonable, for by sensitiveness is meant the minutest

quantity of radiation one can detect, assuming one is willing to wait long enough for the deflection to reach a maximum. In Table III are compiled the most notable radiometers used in radiation work. The candle as a standard of comparison is not ideal, but since the sensitiveness of the various instruments varies

TABLE II.
Sensitiveness of Radiometers.

Pressure	Full Period	Deflection per mm ²	Remarks
0.02 mm	45 secs.	7.9 cm	Vanes (area 0.5×9, mm weight 5 mg) 3 mm from window.
.02 "	60 "	11.5 "	Vanes closer to window.
Same vanes (0.5×9 mm), finer quartz fiber.			
Pressure	Full Period	Deflection per mm ²	Remarks
0.02 mm	2 min.	36 cm	Vanes 3 mm from window candle at 3 m.
0.03 "	2.5 "	71 "	Vanes nearer window. This is the greatest recorded sensitiveness. A deflection of 1 mm on scale at 1 m = 2.5×10^{-6} .
0.04 "	-----	63.5 "	
Heavy vanes, area 1.3×10.5 mm, weight 10+mg.			
0.02 mm	40 secs.	5.5 cm	Distance of vanes from window unknown.
.02 "	60 to 64 "	13.7 "	Vanes nearer window, hence longer period.
.026 "	36 to 40 "	8.5 "	Vanes farther from window than in preceding.
.035 "	25 "	3.3 "	Vanes still farther from window, which shortens period and decreases sensitiveness. For a pressure of about 0.05 mm the sensitiveness would be much greater.

by a factor from 2 to 20, it is sufficiently accurate for the present comparison. In this table it will be noticed that for the same period the various radiometers vary in sensitiveness by as much as 50 per cent. Porter's radiometer was the most sensitive of the instruments having a period of 90 seconds. But he gained little on the whole, for the vanes were so light that he could work only

during quiet hours at night. On the other hand, the writer, after trying light vanes, adopted heavy ones,²¹ and was thus enabled to continue his observations at all hours without annoyance even from a large air compressor which was situated in an adjoining basement room.

TABLE III.
Sensitiveness of Various Radiometers.

Observer	Full period	Area of vanes mm ²	Deflections per mm ² area of exposed vane; candle and scale each at 1 m
E. F. Nichols.....	12 secs.	2×15	5 cm (?)
Phys. Rev., 4, p. 297, 1897.			
Astrophys. Jour., 18, p. 101, 1901.	11 "	3.1	12.5 "
Stewart	80 "	30	4.9 "
Phys. Rev., 18, p. 257, 1901.	5 min.		17 "
Drew	90 secs.	7	17.1 "
Phys. Rev., 17, p. 321, 1903.			
Porter	90 "	3.6	27.5 "
Astrophys. Jour., 22, p. 229, 1905.			
Coblentz	90 "	15 (10×1.5)	8 to 10 "
Abs. Spectra.....	50 "	12	10 to 12 "
Phys. Rev., 16, 20, and 22. (Vac. tube.)	100 "	11 (11×1)	52 "
Another vane.....	150 "	4.5	71 "
(This Bulletin, <i>ibid.</i>)	70 "	"	35 "

3. Sensitiveness Compared with Wave-Length of Exciting Source.—In his investigations of emission spectra of the alkali metals, using a prism and lenses of quartz, and a bolometer, Snow²² found that the vapor of the carbon arc had the larger portion of its energy concentrated in one large band in the violet. The writer using a radiometer, a rock-salt prism, and a mirror spectrometer for investigating infra-red emission spectra, found that the radiometer gave small, if any, deflections in the violet. The violet band is far enough from the reflection minimum of silver

²¹ "Investigations of Infra-red Spectra;" 1905.

²² Snow, Phys. Rev., 1, pp. 28 and 221; 1893.

not to be weakened by it, hence it appeared that the radiometer might be selective in its behavior to radiant energy.

To test this point the following experiment was tried: The total radiation from the aluminum spark (with glass-plate condenser) on a 10,000 volt transformer was measured with a very sensitive radiometer (period 65 to 70 seconds, sensitiveness 35 cm per mm²) just described, and with a bolometer to be described subsequently. The window of the radiometer was of white fluorite 2 mm thick, hence transparent to the ultra-violet, but opaque beyond 10μ . A large part of the energy of the aluminum spark lies in the ultra-violet. The maximum energy of the warm electrodes occurs at about 8μ . The radiation from the spark passed through a quartz cell 8 mm thick, containing distilled water which absorbed the infra-red energy. The observations consisted in obtaining the ratio of energy transmitted by a glass plate 8 mm thick (which is opaque to rays shorter than 0.3μ), to the total energy of the spark.

Unfortunately at the high sensitiveness required for measuring ultra-violet radiation the two instruments were not in perfect working order at the same time. During the first test the bolometer-galvanometer had a short period—10 seconds—and caused trouble by the drifting of the zero with changes in the spark, while in the second test the radiometer was leaking slightly, which caused its zero to drift. Then too the spark was by no means constant, but, as will be seen presently, the ratio above referred to is about the same for the radiometer and the bolometer, after correcting for the loss of 4 per cent by reflection at the fluorite window. The agreement is close enough to show that the radiometer is not selective in its action, and hence is adapted to investigations in the ultra-violet.

In the first test the direct radiation from the aluminum spark (with a condenser in parallel) was compared with the part transmitted by glass. There was some infra-red energy in this case, which made the ratio lower than in the second experiment. The direct deflections with the radiometer were about 20 cm. The ratio of the deflection through plate glass to the direct deflection varied from 17 to 19.5 per cent (mean about 18 per cent), while with the bolometer the same ratio varied from 16 to 20 per cent,

the mean being about 19 per cent. The bolometer followed the fluctuations of the spark, hence the greater variations. The spark was 75 cm from the radiometer and 25 cm from the bolometer. In the second test the infra-red radiation was absorbed by the cell of water, with quartz windows. Plate glass was again used to absorb the ultra-violet. In this test the average ratio of the radiation transmitted by the glass to the total radiation was about 65 per cent, while the same ratio for the bolometer was 67 per cent. Correcting for the loss by reflection at the window, the ratio for the radiometer would be about 69 to 70 per cent.

The results as a whole show that the radiometer is not selective, i. e., it is as efficient in the ultra-violet as is the bolometer.

4. The Radiometer Compared with the Bolometer.—In discussing the merits of the radiometer writers have generally emphasized the fact that it is not adapted for quantitative work, since it can not be calibrated. As a matter of fact, in reviewing the work done in radiation it was found that even with the bolometer there are only a few cases where the energy was obtained in absolute measure. Even in the study of the laws of radiation from a hollow enclosure, or Kirchhoff radiator (so-called "black-body"), the galvanometer deflections were observed and reduced to a single standard of sensitiveness, which was in arbitrary units. The same can be done with the radiometer. Its sensitiveness is easier to control, since it can be made to depend only upon the pressure of the residual gas; whereas the constant of a galvanometer varies continually. It is not affected by magnetic variations, and a heavy vane is less affected by earth tremors than is a very light galvanometer suspension. It is sensitive to temperature changes, but less so than the bolometer, and it can be more easily shielded from temperature changes than can a bolometer with its galvanometer, battery, etc. The fact that it is not portable is not a serious drawback, since it is not usually necessary to move the instrument. It has two disadvantages, viz, its window, or preferably double window, is selective in its transmission, and its period is somewhat longer than that of a bolometer and galvanometer of equal sensitiveness. But the latter is nearly always drifting and to repeat one's readings takes as long for an observation as it does with a radiometer. Since the

weight is of minor importance, tremors are avoided by having the suspension weigh about 8 to 10 mg. When used with a good mercury pump it requires no attention after it is adjusted. A delicate galvanometer requires frequent adjustment and, in connection with a bolometer, the investigator's time is occupied principally with the care of the instrument (at least that has been the writer's experience), which should be a secondary matter. The two instruments are of the same order of sensitiveness, with the possibility of the radiometer being the more sensitive. This is well illustrated in the test for their efficiency to ultra-violet radiation, where both instruments were at about their maximum working sensitiveness. The bolometer used was 0.22×10 mm in area, resistance 2.8 ohms, and for a bolometer current of 0.04 ampere, with a galvanometer sensitiveness of $i = 1.5 \times 10^{-10}$ ampere (full period = 16 seconds), had a temperature sensitiveness of $9^\circ \times 10^{-6}$ per mm deflection, on a scale at 1 m. (See Table IV.) A candle²⁴ at 1 m gave a deflection of 45 cm, which, on the assumption that the sensitiveness is proportional to the square root of the area of bolometer strip, is 30 cm per mm^2 . For the radiometer, having a vane 0.5×9 mm, a candle gave a deflection equivalent to 159 cm at 1 m, or, since the deflection is proportional to the area of the exposed vane, 35 cm per mm^2 (Table IV). In other words, the radiometer was 1.2 times as sensitive as the bolometer, or 1 mm deflection corresponded to $7^\circ 5' \times 10^{-6}$. (For a full period of 2.5 minutes its sensitiveness was $3^\circ 8' \times 10^{-6}$.) Its period, however, was 4.5 times that of the bolometer-galvanometer. This examination of sensitiveness is based on the assumption that the radiometer was as complete an absorber of energy as the bolometer. Judging from its period, its efficiency is much lower than that of a bolometer, hence the radiometer must be sensitive to temperature changes less than the value just given.

The sensitiveness of bolometers thus far attained is about $1/100000$ degree per mm deflection. Paschen (loc. cit.) claims a sensitiveness of $1/100000$ degree by reading to 0.1 mm. But the conditions are rare when one can read to 0.1 mm, so that the estimate would seem too high. As will be seen presently, the

²⁴ A Nernst-heater was also used in making the comparison.

working sensitiveness is of the order of $\frac{1}{1000}$ degree, or even considerably less. The maximum sensitiveness is of course needed only in cases where the radiation is very weak.

5. Some Points in Radiometer Construction.—There is room for great improvement in radiometer construction. One must expect some difficulties in controlling instruments of great sensitiveness. There are always inequalities in the two sets of junctions of a thermopile, so that when it is first connected to a very sensitive galvanometer, the deflection must be brought back to its original point by adjusting the control magnets. The same is true of the very sensitive radiometers constructed by the writer.²¹ On exhausting the instrument the deflection will move off the scale (deflection away from the window) and must be brought back by means of a torsion head. If the pump leaks there will be a drift toward the window. The deflection seems to be due to the inequality of the temperature of the window and the metal shield, back of which is the vane that remains unexposed.

By placing the vanes at a greater distance, 6 to 8 mm, from the window, in the sensitive radiometer just described, it was found that the drift due to sudden changes in the temperature of the window was avoided and a torsion head was not needed. The use of a torsion head has been found necessary only in the most sensitive radiometer (deflection = 50 cm per mm²). The drifting of the zero is a far less serious matter than in a bolometer or a thermopile, and is not troublesome in less sensitive instruments.

The dimensions to be chosen for the vane will depend on its distance from the spectrometer slit. The image of the slit can be made to just cover the vane by placing a short focus (say 10 to 12 cm) condensing mirror between the spectrometer slit and the radiometer vane. Since mica vanes are not opaque to all rays, and since the lampblack may rub off, it is better to use platinum vanes, about 0.01 mm thick, blackened electrolytically and then smoked.

In addition to having the radiometer enclosed in a heavy metal case to shield it from temperature changes, it is advisable to cover the instrument with a metal cylinder and to pack wool between it and the inner case. In warm weather it has been found that the vanes become easily electrified from the mercury

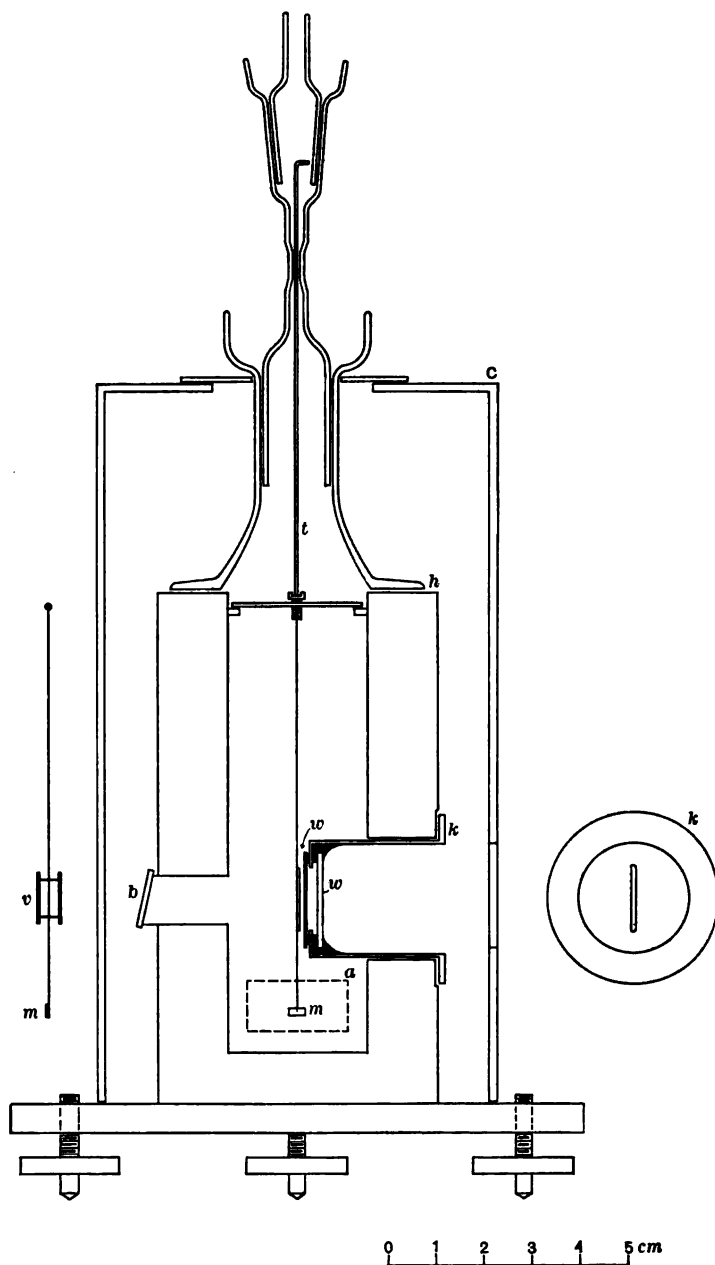


Fig. 4.—Radiometer.

vapor. This is avoided by placing gold foil on cotton between the pump and the radiometer. This electrification effect is not well understood. It seems to be most noticeable when a fluorite window is used next to the vane. This does not always seem to be true, however, for in the ultra-violet radiation test just described there was but one window, with the metal slit on the outside and the vane inside, and no difficulty was experienced with electrification. Without disturbing the vane, a second fluorite window was placed inside the first one, hoping thus to avoid heat conduction. After several days' trial it was found impossible to prevent the vanes from being attracted to the inner window, even when covered with tin foil, and it was necessary to replace this window by rock salt. It is well known that plates of fluorite after being cut from a crystal are frequently under internal stresses; but whether this could produce sufficient polarity, as in the case of quartz, is not known. The freedom from electrification with rock salt may be explained on the assumption that it is hygroscopic and dissipates any electrical charges on its surface.

In spectrum energy work the best method of varying the sensitiveness of a radiometer is to use a sectorized disk of variable aperture. The sensitiveness may also be varied by turning the leveling screw, which is in line with the windows *w. w.* in Fig. 4. The complete instrument is here shown, except the bulb containing gold foil on cotton. The figure is drawn to scale and needs but little explanation. The outer metal shield, *c*, is shown dotted. Between it and the heavy metal case is wool or hair felt. The double windows of rock salt or fluorite are shown at *w. w.* The torsion head, *t*, is self-explanatory, as is also the vane which is also drawn to scale. The viewing window, *b*, of glass, for adjusting the vane on the slit, as well as the glass window, *a*, for viewing the scale, are fastened permanently with melted shellac or "Khotinsky" cement. The joints at the top and at the rock salt window, *k, k*, are made with a mixture of beeswax and tallow, and painted with shellac, which has been found to make air-tight connection.

VI. THE BOLOMETER WITH ITS AUXILIARY GALVANOMETER.

We have now to consider one of the most useful radiation meters yet devised, namely, the bolometer, which is simply a

Wheatstone bridge, two arms of which are made of very thin blackened metal strips of high electrical resistance and high temperature coefficient, one or both of which are exposed to radiation. When thus exposed their temperature changes, thus unbalancing the bridge, and the resulting deflection of the galvanometer gives a measure of the energy absorbed. The maximum sensitiveness of the bolometer is limited by the size of the strip to be exposed to radiation. Any further gain in sensitiveness must be attained by increasing the sensitiveness of the galvanometer, which, for the moving magnet type varies approximately as the square of its period (undamped). The sensitiveness is also directly proportional to the bolometer current, but this is limited by the resistance of the bolometer strips. It will be noticed presently that the working sensitiveness of the various galvanometers thus far used is of the order of 2×10^{-10} ampere per mm deflection, while the working temperature sensitiveness of the bolometer and galvanometer varies from 5×10^{-5} degree to 5×10^{-6} degree for 1 mm deflection.

1. Historical Summary.—The various types of bolometer-galvanometer apparatus will first be noticed, in so far as they relate to spectro-radiometric work.

The first great step in improving the moving magnet galvanometer is due to Kelvin, who decreased the weight of the moving parts to a few milligrams, and introduced the astatic system of magnets.

The main problem in bolometer construction is to use strips of a metal having a high resistance-temperature coefficient, a small specific heat, and a low-heat conductivity. Such metals are nickel, platinum, tin, and iron, but, for various reasons in mechanical construction, platinum is the most commonly used. The manner in which this instrument was developed to its present high sensitiveness is best illustrated by considering the various designs of different investigators.

The pioneers in exact spectro-bolometric work are Langley, Angström, and Julius. While Langley was not really the first to discover the principle of the bolometer, he was the first to

invent a practical instrument²⁶ and demonstrate its superiority to all other radiation meters for accuracy, quickness of action, and adaptability. His improvements of the instrument extend over a long period, and it will be sufficient to say that whereas his first instrument had a temperature sensitiveness of 0°000 02 per mm deflection of the galvanometer, the latest recorded a temperature change of 0°000 001 per mm deflection, when used with a galvanometer having a figure of merit of $i = 5 \times 10^{-10}$ ampere. For his solar radiation work the bolometer strips are about 12 mm long, 0.05 to 0.2 mm wide, and have a resistance of about 4 ohms. The wire is obtained as "Wollaston wire" of 0.1 mm diameter inclosing a platinum core of 0.0125 mm. This is flattened by hammering, after which the silver is dissolved in nitric acid. He found that a thickness of less than 0.002 mm is inadvisable, thinner ones being disturbed mechanically by air currents. The current used was about 0.03 ampere. The galvanometer generally used at the Astrophysical Observatory has 1.6 ohms resistance. The bolometer strips are of platinum, while the balancing resistances are of platenoid, which is practically the same as German silver. It was finally found that the maximum sensitiveness²⁷ of the bolometer circuit is closely approximated when the balancing coils are upwards of 4 times the resistance of the bolometer strips, and the galvanometer resistance is not less than 0.6 or more than 4 times the resistance of the bolometer strip. The bolometer strips, balancing coils, and slide wire adjustment are all enclosed in a double-walled chamber, which eliminates accidental drift of the galvanometer needle. The whole outfit is in a room which can be maintained at a constant temperature. This has reduced the drift to a minimum.

The published details of some of the more important bolometers and accessory apparatus used in radiometric investigations are summarized in the following table (Table IV):

²⁶ Langley, *Proc. Amer. Acad.*, 16, p. 342; 1881. *Chemical News*, 48, p. 6; 1881. *British Assoc. Report*, 1894, (0°000 001) *Annals. Astrophys. Obs.*, 1.

²⁷ Abbot, *Astrophys. Jour.*, 18, p. 1; 1903.

TABLE IV.
Bolometer-Galvanometer Sensitiveness.

Observer	Galvanometer			Bolometer		
	Resistance in ohms	Full period in seconds	Current sensitiveness for 1 mm deflection	Resistance in ohms	Area (mm ²)	Bolometer current in amperes
Langley. Annals Astrophys. Obs.	1.6	20	1 to 5×10^{-10}	4	$0.05 \text{ to } 0.02 \times 12$	0.03
Abbot. Astrophys. J., 18, p. 1; 1903.			5×10^{-11}			
Ångström. Wied. Ann., 26, p. 253; 1885.						
Wied. Ann., 36, p. 715; 1889. Wied. Ann., 48, p. 497; 1893.	8	16	5.7×10^{-9}	5	0.1×12	
Julius. Licht und Wärmestrahlung, p. 31; 1890.	2.7		4×10^{-9}	3	0.3×14	0.133
Helmholtz. Verh. Phys. Gesellsch., Berlin, 7, p. 71; 1888.			8×10^{-9}	8.8		
Lummer and Kurlbaum. Zs. für Instrumentenkunde, 12, p. 81; 1892. Wied. Ann., 46, p. 204; 1892.			1.5×10^{-9}	60	$12 \times 1 \times 32$	0.006
Rubens. Wied. Ann., 37, p. 255; 1889.	5	4 to 5	3.2×10^{-10}	5.2	$7 \times 0.3 \times 35$	0.2
Wied. Ann., 45, p. 238; 1892.	80		3.2×10^{-10}	3	$3 \times 0.2 \times 10$	
	80		3.2×10^{-10}	80	0.09×12	

TABLE IV (continued).

Bolometer-Galvanometer Sensitiveness.

Temperature sensitiveness			Candle test				Remarks
For 1 mm deflection and period given in col. 3	Ibid., for a scale at 1 m	Ibid., for scale at 1 m and a full period of 15 secs.	Distance of candle in meters.	Distance of galvanometer scale in meters	Observed deflection in cm	Equivalent deflection per mm ² area for candle and scale at 1 m	
°C. 1×10^{-4}	°C.	°C.					In practice for solar spectrum work a full period of 3 secs. and $i = 2.2 \times 10^{-3}$ ampere is used. Balancing coils of platinoïd. Bolometer is enclosed in water jacket. In practice drifting is minimized by placing a short copper wire in series with one of the bolometer arms. Surface bolometer; 23 strips of tin foil; blackened with Pt Cl₂ and soot. Balancing coils of copper wire in separate box. Sensitiveness 556×10^{-9} gr-cals/cm²/sec. Single Pt strip 0.1×12 mm in spectrobolometer. Two arms of tin foil in form of grating. Sensitiveness also given as 127×10^{-9} gr-cals/cm² sec. Bolometer strips of nickel 0.002 mm. thick. Balancing coils of platinum wire in oil in separate box. Found deflection proportional to current. Surface bolometer, four similar arms, so no compensating resistances needed; diagonally opposite arms exposed. Sensitiveness also given as 533×10^{-9} gr-cals/cm²/sec. Surface bolometer, four similar arms of platinum foil 0.001 mm thick; two diagonally opposite arms exposed to radiation; the surface consists of 12 strips of platinum, 1 mm wide $\times$ 32 mm long, separated by 1.5 mm, back of which openings are 12 similar strips of the diagonally opposite arm. Bolometer currents as high as 0.04 ampere could be used. Surface bolometer of tin foil 0.01 mm thick; 7 strips in each arm. Two arms of 0.04 mm iron wire hammered flat; three strips in each arm. Two arms of 0.005 mm platinum wire hammered flat. The deflections of the galvanometer were proportional to the current through the bolometer and the rise in temperature of the arms proportional to the incident energy.
9×10^{-5}				2			
8×10^{-5}							
3×10^{-4}			1		800		
2×10^{-4}							
5×10^{-5}	2.5×10^{-5}	4×10^{-5}	1	5	60	5	
8×10^{-5}	4×10^{-5}		1	5	12	2.7	

TABLE IV—Continued.

Observer	Galvanometer			Bolometer		
	Resistance in ohms	Full period in seconds	Current sensitiveness for 1 mm deflection	Resistance in ohms	Area (mm ²)	Bolometer current in amperes
Rubens and Snow. Wied. Ann., 46, p. 529; 1892.	150	20	1.8×10^{-11}	80	0.09×12	
Snow. Phys. Rev., 1, p. 31; 1893.	140	20	1.5×10^{-11}	75	0.05×7	0.025
Aschkinass. Wied. Ann., 55, p. 401; 1895.		20	6×10^{-11}	13		0.04
Donath. Wied. Ann., 58, p. 609; 1896.			6×10^{-11}	5		
Paschen. Wied. Ann., 48, p. 272; 1893.	60*	7	2.3×10^{-11}			0.01 to 0.02
	60*	15	3.3×10^{-12}			
	20	30	1.6×10^{-11}	12	$3 \times 0.5 \times 15$	0.03
		34	8.3×10^{-12}	8	0.25×7	.02
W. W. C.	5.2	10	2.5×10^{-10}			
	5.2	16	1.5×10^{-10}	2.8	0.22×10	0.04
Rubens Thermopile.	5	14	1.4×10^{-10}	3.5	0.8×20	
W. W. C. Radiometer.		70			0.5×9	
		150			0.5×9	

TABLE IV—Continued.

Temperature sensitiveness			Candle test				Remarks
For 1 mm deflection and period given in col. 3	Ibid., for a scale at 1 m	Ibid., for scale at 1 m and a full period of 15 secs.	Distance of candle in meters	Distance of galvanometer scale in meters	Observed deflection in cm	Equivalent deflection per mm ² area for candle and scale at 1 m	
3×10^{-6}					40		For Hefner candle at 1 m.
8×10^{-6}	2.4×10^{-6}	4×10^{-6}	1	3	15	8.3	Two bolometer arms of platinum 0.000 936 mm thick; balancing coils of German silver; 4 coil galvanometer, total of 7200 turns, wound with two sizes of wire. Magnet system of 12 magnets 3 to 4 mm long; total weight of system 80 mg.
3×10^{-6}	1.2×10^{-6}			4			Two bolometer arms of 3 iron wires hammered flat.
				3			Two arms of platinum 0.17 mm wide $\times$ 0.0074 mm thick; balancing coils (of platinum) of elaborate design in separate box. Drifting was serious.
				3			Bolometer arms of platinum 0.001 to 0.0005 mm thick.
1×10^{-6}	2.7×10^{-6}	11×10^{-6}		2.7			Compensation resistances of manganin wire, final adjustment on platinum wire with mercury contact.
84×10^{-7}	2.1×10^{-6}	8×10^{-6}		2.5			*Galvanometer; 4 coils 40 mm external and 5 mm internal diameter, 1,200 turns of graded wire in each coil; moving system, 13 magnets in each group, 1 to 1.5 mm long, on both sides of glass staff, and 0.3 mm apart; mirror 2 mm diameter $\times$ 0.03 mm thick. Total weight of system = 5 mg.
	9×10^{-6}	9×10^{-6}	1	1	45	30	Using an 8-magnet system and a full period of 20 secs. $i = 8 \times 10^{-11}$ ampere, while a 6-magnet system had a sensibility of $i = 2.5 \times 10^{-11}$ ampere for a full period of 36 seconds.
1.1×10^{-6}		1×10^{-6}	5	(1 ?)	10	15	†These values are obtained by comparing the deflections per unit area with the bolometer.
	7.5×10^{-4}		2.3	1	30	35	
	3.8×10^{-4}		3	1	35.5	71	

Lummer and Kurlbaum²⁸ improved the design of the surface bolometer by using thin platinum foil (0.001 mm. thick) and making the arms of high resistance by a special grid construction, by selecting four arms of as nearly the same resistance and temperature coefficient as possible, and by exposing two diagonally opposite arms to the radiation. Snow²⁹ was among the first to give much attention to the possible gain in sensibility by the use of a more sensitive galvanometer. Paschen³⁰ continued the work in this direction and constructed the most sensitive galvanometer used up to that time. By the use of very light magnet systems and a long working period (as much as 30 secs.) a marked increase in sensibility was attained. He adds that for a full period of 40 seconds and a battery current of 0.06 ampere through his bolometer, by reading to 0.1 mm, it would have been possible to detect 0°000 000 1. However, such a long period and large battery current is not practicable, so that the last estimate of temperature change has little meaning. Even for a full period of 20 seconds the magnetic and thermal disturbances are generally sufficient to interfere with bolometric work, and a fair estimate of the sensitiveness attained by Paschen is $\frac{1}{1000000}$ degree for a scale at 3 m. In the *Astrophys. Jour.*, 3, p. 24, 1896, he gives a practical example of the sensitiveness of his bolometer. The complete period of his galvanometer was 34 seconds and for a scale at 2.5 m the sensitiveness was $i = 8.3 \times 10^{-12}$ ampere. With a current of 0.02 ampere through the bolometer (dimensions = 0.25×0.0005 mm; resistance = 8 ohms), a deflection of 1 mm corresponded to a difference in temperature of $84^\circ \times 10^{-7}$, or $2^\circ 1 \times 10^{-5}$ for a scale at 1 m. This sensitiveness was attained in the present work for a very much shorter period but higher current. Paschen appears to have been the first to use manganin wire in his balancing coils and a heavy platinum wire with a mercury contact for the final compensation, all of which were in a separate wooden box.

From this historical record it will be noticed that a fair estimate of the temperature sensitiveness of the various instruments is about $5^\circ \times 10^{-6}$ for 1 mm deflection on a scale at a distance of 1 m.

²⁸ *Zs. für Instrumentenkunde*, 12, p. 81; 1892. *Wied. Ann.*, 46, p. 204; 1892.

²⁹ *Phys. Review*, 1, p. 31; 1893.

³⁰ *Wied. Ann.*, 48, p. 272; 1893.

Most instruments cited fall far below this value; but the cases are exceptional where so great a sensitiveness is needed. For measuring radiation at low temperatures, or for vacuum tubes, or for the examination of the remote ends of the spectrum this great sensitiveness is required. It is a significant fact that the bolometer with its long period galvanometer has not been used for such work. Here the Rubens thermopile is the most serviceable and reliable.

Mendenhall and Waidner²¹ constructed a sensitive galvanometer of 4 coils, having an external diameter of 15 mm, internal diameter 2 mm, wound with six sizes of wire, 500 turns in each coil. The resistance (with coils in parallel) was 3 ohms. They used 3 magnets in each group, lengths = 1.15 mm, total weight of suspension system 1 mg, and attained a sensitiveness of $i = 5.6 \times 10^{-11}$ ampere for a full period of 9 seconds and scale at 2 meters distance, or 1.1×10^{-10} ampere at 1 meter.

Abbot²² built a 16 coil instrument which had a resistance of 1.6 ohms. The sensitiveness was $i = 5 \times 10^{-11}$ ampere for a complete period of 20 seconds and scale at 1 meter. However, to increase the steadiness of the needle for solar-energy spectra the full period is reduced to only 3 seconds. Assuming that the deflection is inversely proportional to the square of the time of single swing (true only for low air pressures) the sensitiveness is $i = 2.2 \times 10^{-9}$ ampere for a full period of 3 seconds.

Ingersoll²³ has recently constructed a sensitive galvanometer similar to the one described by Mendenhall and Waidner (*loc. cit.*). The galvanometer has 4 coils of 20 ohms each, 16 mm outside and 2 mm inside diameter; weight of suspended system less than 2 mg. The highest sensitiveness attained was 4×10^{-11} ampere for 5 ohms resistance and scale at 1.5 m, but lack of steadiness and proportionality of deflection led him to reduce the sensitiveness to 2×10^{-10} ampere per mm deflection with a 10 second period for ordinary usage. His bolometer was 0.5×8 mm, and at highest sensitiveness a candle at 1 m gave a deflection of 200 cm on the galvanometer scale at a distance of 1.5 meters. His balancing coils are of "IA" wire, which is practically the same composition as "con-

²¹ Mendenhall and Waidner, *Amer. Jour. Sci.*, **27**, p. 249; 1901.

²² Abbot, *Astrophys. Jour.*, **18**, p. 1; 1903.

²³ Ingersoll, *Phil. Mag.* (6), **11**, p. 41; 1906.

stantan" or "advance" and hence, like German silver or plate-noid, is subject to thermoelectric disturbances.

2. The Construction of Sensitive Galvanometers.—One of the best known sensitive galvanometers on the market is the du Bois-Rubens³⁴ type. This instrument is magnetically shielded by enclosing the coils within two spherical shells of iron. There are two coils of about 6 cm external diameter. Two magnet systems are furnished with the galvanometer. The heavy system, which has a large mirror, has 14 magnets about 7 mm long arranged on both sides of an aluminum staff. The lighter one has 10 magnets about 4 mm long. The galvanometer tested by the writer was found to give deflections proportional to current for deflections as large as 30 cm, which is often an important item. The large weight of the moving system is of some importance in places where there are severe mechanical disturbances. In this galvanometer with the two coils in parallel, resistance = 2.85 ohms, using the complete magnetic shield and the heavy system of magnets, the sensitiveness was $i = 4 \times 10^{-10}$ ampere for a full period of 6 seconds and circular scale at 1 meter. Using the light system and a full period of 6 seconds, the sensitiveness was only $i = 7 \times 10^{-9}$ ampere, even after it was remagnetized. But in both cases, and especially with the light system, the zero kept shifting back and forth continuously, so that its maximum working sensitiveness was limited to this value. On the other hand, the galvanometer with short magnets, to be described presently, shielded with only one cylinder of iron, situated close beside the du Bois-Rubens galvanometer and tested at the same time, had a perfectly steady zero, even for three times this period, when its sensitiveness was $i = 1.5 \times 10^{-10}$ ampere. In the latter instrument, however, the coils are small and the proportionality between current and deflections does not hold for deflections greater than 10 to 12 cm; but, since a shunt must be provided anyway, the deflection can be kept within these limits so that this is not a serious objection. A four-coil galvanometer, also due to du Bois and Rubens,³⁵ which has been much used abroad, may be obtained on the market.

³⁴ Du Bois and Rubens, *Ann. d. Phys.* (4) 2, p. 84; 1900.

³⁵ Du Bois and Rubens, *Wied. Ann.*, 48, p. 236; 1893.

This is a much larger instrument, of the Thomson astatic type, than the one described above and is not adapted to magnetic shielding. The instrument is furnished with two sets of four coils, of 20 ohms and 2000 ohms each, respectively, and three magnet systems of 1500, 250, and 100 mg, respectively.

In this country the type of small-coil galvanometer just mentioned is being very generally adopted, although it is not to be obtained in the market. The question of the best form of coils, size of wire, kind of magnets, etc., has been thoroughly discussed by Mendenhall and Waidner,³⁶ and by Abbot,³⁷ so that little need be said on that subject. However, the manner of assembling the different parts into a small space for convenience in magnetic shielding and at the same time leaving the suspended system open to view (but shielded from air currents) seemed to admit of further improvement. The following design of an easily shielded galvanometer is the result of a study of previous types, and while it can not claim any new principles, the simplifications may appeal to the reader. Further simplifications are possible by changing the position of the binding posts to the base of the instrument, when the shield of Swedish iron (Fig. 6) can be brought closer to the coils. Labor in construction may be saved by imbedding the coils in paraffin instead of mounting them on insulated supports. Since the proximity of the upper and lower pairs of coils increases the deflection of the needle, they should be close together, and the control magnets should be on a long rod extending above or below the coils.

(a) *Form of Coils.*—The proper form and method of winding galvanometer coils to secure a maximum effect from a given weight or resistance of copper has been thoroughly discussed by Maxwell.³⁸ He shows that the greatest effect is obtained by winding the coil with different sizes of wire, beginning with the smallest size, and by winding each layer so that it lies within the surface the polar equation of which is $r^3 = d^3 \sin \theta$, where r is the length of the radius making an angle θ with the axis of the

³⁶ Mendenhall and Waidner, *Amer. Jour. Sci.*, **12**, p. 249; 1901.

³⁷ Abbot, *Astrophysical Jour.*, **18**, p. 1, 1903. *Annals Astrophys. Obs.*, **1**, p. 246, etc.

³⁸ Maxwell, *Electricity and Magnetism*, **II**, p. 360.

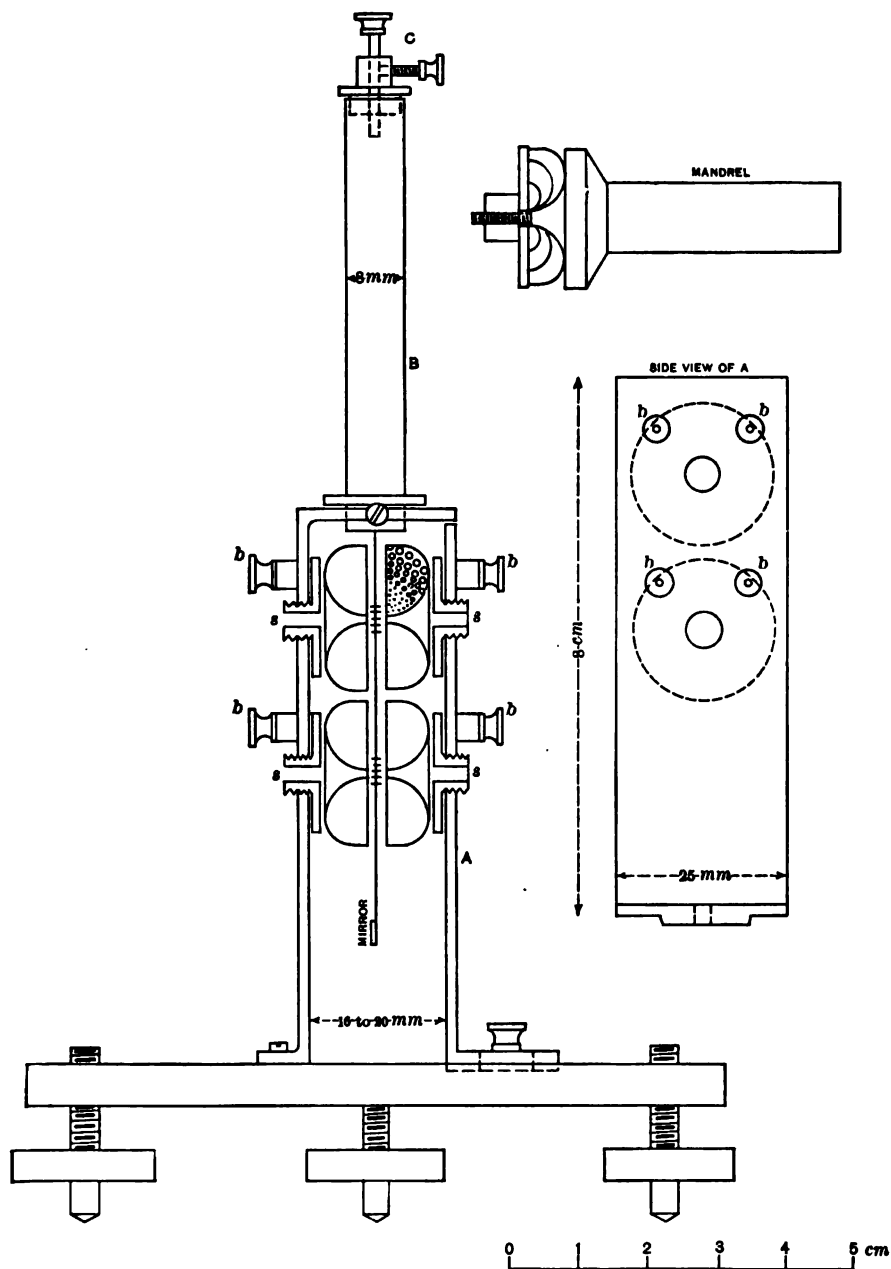
coil, and d the value of r when $\theta = 90^\circ$. Abbot (loc. cit.) has computed the most efficient coils satisfying these conditions, and gives a large table of results for coils wound with a single wire and for coils wound with three sections of wire of different diameters. He found that the total force exerted at the center is closely proportional to the 0.45 power of the total resistance and that coils composed of three sections of best sizes of wire give about 1.4 times the force of a coil of the best single size of wire of the same total resistance. In his best 25-ohm coil, wound in three sections, the diameters of the wires are 0.08, 0.16, and 0.32 mm (B and S size 40, 34, and 28). The lengths are 256, 1031, and 4144 cm, respectively, while the external diameter of the complete coil is 3.3 cm.

In the present design the coils (20 ohms each) were computed for three sections of B and S gauge, Nos. 40, 36, and 30 wire, but were finally constructed of Nos. 40, 36, and 32, diameter of wire 0.07, 0.127, and 0.199 mm, and lengths 1.9, 4.1, and 12.3 m, respectively. The external diameter of the coils is 18 mm, the internal diameter 2 mm. The wire was covered with a single layer of white silk insulation.

The mandrel described by others for winding the coils did not prove strong enough and a new one was devised. It is given full size in Fig. 5; the dotted^{38a} lines indicate the method of winding the sections. To prevent the wires from slipping it is necessary to apply a dilute solution of shellac while winding. To make the coils rigid they are boiled in carnauba wax until the air bubbles are all expelled.³⁹ Some of the shellac will come out in boiling. After the coil has cooled the nut is taken off and the plate of the mandrel is warmed and removed. On warming the heavy metal end of the mandrel the coil drops off. However, there is a tendency for the shellac to adhere to the plate and to the cone of the mandrel, and it is well to cover these parts with a thin layer of carnauba wax or paraffin before winding the coil. In winding the coils the ends of all the sections are brought out and soldered

^{38a}By accident the lines are drawn full instead of broken ones.

³⁹ This wax has a high coefficient of expansion and cracks on cooling, so that the writer prefers refined paraffin, which is easier to work and there is less likelihood of charring the silk insulation.



together (in series) at the back of the coil. It is important to use pure copper wire covered with a single layer of white silk. To avoid static charges of electricity it is customary to cover the coil faces with tin foil or gold leaf. A free space of at least a millimeter between the coil faces is necessary. (See Nichols, "The Galvanometer.")

(b) *The Needle System.*—The best dimensions and construction of the needle system has been extensively investigated by Paschen, by Mendenhall and Waidner, and by Abbot. In any magnet system the greatest sensibility is attained when the ratio of the magnetic moment to the moment of inertia of the system is a maximum. For two systems of magnets otherwise equal, if the magnets of one are n times shorter than the other then its moments of inertia is n^2 times smaller than the other. The result is that the one with the shorter magnets will give n^2 times the deflection of the first,⁴⁰ if the two systems be astaticised to the same period. However, Mendenhall and Waidner (loc. cit.) have shown that it is possible to use too short magnets. In their graphs of sensitiveness and length of magnets there is a decided maximum for a length of 1.1 mm. It has been found that magnets about 1 to 1.2 mm long, 0.2 mm wide, and 0.08 to 0.1 mm thick are very efficient.

During the course of the present investigation it was suggested that a more sensitive instrument would be produced (irrespective of period) by using a suspended system composed of many magnets. A system was constructed containing 20 magnets (10 in each group, 5 on each side of the staff) of the same size and kind of material as a 10-magnet system (5 magnets in each group) previously employed. This system, although made with the greatest care, proved to be far less satisfactory than those having less magnets. It was found that it made but little difference whether the system was composed of 6 (weight 5 mg), 10, or 20 (weight 10 mg) magnets, and, as will be noticed in the references just quoted, this appears to be the general experience; that is to say, one can deduce theoretically the best proportions for the magnet system, but their realization is limited by mechanical

⁴⁰ If the magnetic moment of the magnets varied directly as their length, which is not the case.

difficulties. Of course, in the heavier systems the period will be longer for a given sensitiveness; but the better the astaticism the higher the attainable working sensitiveness, while the fewer the magnets the easier it is to get them into the same plane. This is most easily done by fastening them in place with a sugar solution on a piece of ground plate glass on which the dimensions of the system are marked in pencil. The glass staff is then attached by means of shellac, which is allowed to dry (touch with a hot wire), when the system is loosened from the glass by means of hot water. The system is magnetized in a double electro-magnet in the form of a $\square \square$ the intensity of magnetization being 1000 to 2000 lines per cm^2 .

In the present experiment the magnets were made from a bar of tungsten steel, which was cut into thin sections of about $0.008 \times 1 \times 2$ cm. These thin plates were then cut into strips 0.1 to 0.2 mm wide, and hardened, glass hard, by heating them to a red heat on an iron plate and plunging them into water. Although they were not tempered afterwards, they showed no marked change in astaticism on standing for several months. The lengths of the magnets were from 1.0 to 1.2 mm. The mirror was of thin microscope cover glass about 1.5×2 mm.

Astaticising the system is apparently a simple operation, but in practice it is found otherwise. The system is suspended in a glass tube and by means of a magnet of known polarity it is determined which group in the system is the stronger. The latter is then weakened by bringing a like pole (use a weak magnet or a steel knitting needle) near it. In the same manner the weaker group of magnets is strengthened by bringing a strong unlike pole near to it. It is quite impossible to make the two sets of magnets so perfect that their planes will be parallel, so that in astaticising the two sets of magnets will be of equal strength when the system points east and west. The complete period may then be as long as 6 to 10 seconds, although the latter is rarely attainable with such light systems. Since the magnets are mounted with shellac, which is hygroscopic, it is possible that the change in astaticism is due to a variation in humidity. This might be avoided by using refined paraffin in mounting the magnets. The finest quartz fiber must be used, viz, such that the

suspension head may be turned through several revolutions without deflecting the astaticised needle by more than a few scale divisions.

(c) *The Assembled Galvanometer.*—The galvanometer without the shields is drawn to scale in figures 5 and 6, and needs but little explanation. The side, A, is adjustable for admitting the magnet system. The coils are fastened to the ebonite supports by means of soft wax (a mixture of Venice turpentine and beeswax which hardens on standing). This permits an easy adjustment of the faces of the four coils to parallelism by simply inseting a piece of plate or microscope glass between them. The coils are then made vertical by means of the tripod screws. The needle system, shown at right angles to the coils, is then centered by tilting the tube, B, which is held to the frame by means of a screw, and can be lifted off at will. The top, C, can also be removed for mounting the fiber. The sides of the galvanometer are covered with microscope section glass, secured by soft wax. This prevents disturbances due to air currents, and at the same time admits excellent illumination of the interior. The ebonite supports and the binding posts may be omitted entirely by way of further simplifications. The coils are then mounted in paraffin, which provides the insulation.

(d) *Sensitiveness of Galvanometer.*—Using the 10-magnet system (weight about 5 mg), 5 magnets in each group, separated 0.4 mm, with the mirror between the coils, the sensitiveness for a full period of 9 to 10 seconds was $i = 2.5 \times 10^{-10}$ ampere, and for a full period of 18 seconds $i = 1.5 \times 10^{-10}$ ampere, the scale being at 1 meter. The sensitiveness of the 20 magnet system was only $i = 3 \times 10^{-10}$ ampere for a full period of 20 seconds, while an 8-magnet system (4 in each group) had a sensitiveness of $i = 8.9 \times 10^{-11}$ ampere per mm deflection for the same period, 20 seconds. A light, 6-magnet system had a sensitiveness of $i = 2.5 \times 10^{-11}$ ampere for a full period of 36 seconds. The deflection was aperiodic (i. e., there was only one turning point) for a full period greater than 8 seconds. The resistance for the four coils in parallel was 5.23 ohms at 20°.

This is not the highest attainable working sensitiveness, since the faces of the coils were separated about 2.5 mm to avoid the

effects of static charges which may be troublesome in a newly mounted suspension. However, after the magnetic shields were in place and the instrument was adjusted, it was not disturbed in order to make the final test. By placing the mirror above or below the coils, and thus bringing the upper and lower pair of coils closer together (1 mm is about the limit) a greater sensitiveness would (theoretically) have been possible. In a subsequent test, however, it was found that bringing the upper and lower pairs of coils closer together had but little if any effect in increasing the sensibility. In fact a high sensibility, for a given period, depends more upon the lightness and the perfection of the needle system. The glass (or quartz, which, however, is more brittle) staff must be straight, to avoid an increase in the moment of inertia of the suspension system. Mendenhall and Waidner (*loc. cit.*) show that for a badly constructed system, in which the axis of rotation departs 0.2 mm from the staff at its upper extremity, the sensibility will be decreased by 25 per cent. By using a galvanometer having small coils the glass staff will be short and easily drawn straight, while the complete system will be light.

The glass rod may be straightened, after being drawn, by suspending it with a weight at one end, and heating it with a gas flame, or by simply heating a heavy glass rod until it is melted, and permitting one end to draw out the fiber by means of gravity.

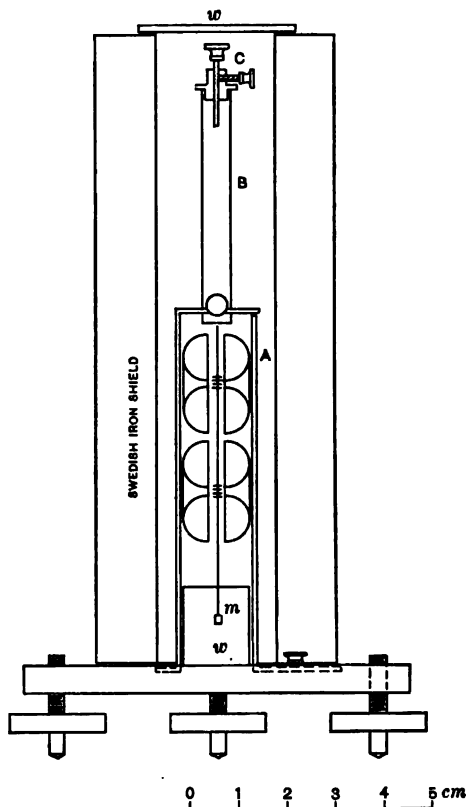


Fig. 6.—Galvanometer with inner magnetic shield.

Where there are serious mechanical disturbances it is advantageous to use heavy systems. Since such disturbances are often encountered and elaborate galvanometer supports have to be provided, it may be noted that frequently the disturbance (tremors) is local, and that by selecting a different part of the same room the ill effects may be avoided. As a practical example the present galvanometer was very unsteady on a massive wall bracket; but by placing the galvanometer on a heavy box in the same place upon this bracket, the tremors were entirely eliminated.

The sensitiveness of a galvanometer (undamped) is approximately proportional to the square of the period. In the present tests the sensitiveness was found to be quite approximately proportional to the period, due to the air damping of the light magnet systems.

(e) *Proportionality of Galvanometer Deflections.*—It is desirable to have the proportionality of the galvanometer deflections to hold throughout a long range, and an attempt was made to attain this end.

From the various sizes of galvanometer coils mentioned in which about the same sensitiveness was attained, irrespective of the size of the coil, the writer concluded to use coils larger than 2 cm diameter, e. g., 3 cm to 4 cm diameter, in which the proportionality of current to galvanometer deflections ought to hold for deflections as large as 15 to 18 cm. The magnets would be the same length, 1 to 1.2 mm, as in the small coils (cf. Paschen, who used coils 4 cm diameter, magnets 1.5 mm long, and found the proportionality to hold for large deflections). To test this conclusion two additional galvanometers, having coils of 6 and 20 ohms, respectively, were constructed. The 6-ohm coils were wound with three layers of 2 ohms each, using B and S gauge Nos. 38, 30, and 26 wire, single covered white silk insulation, diameter of bare wire 0.101, 0.255, 0.405 mm, lengths 92, 595, and 1375 cm, respectively. The complete coil was 2.8 cm diameter and 7 mm through its thickest part. The 20-ohm coils were wound in three sections of equal resistance using B and S gauge Nos. 40, 34, and 28, diameters of wire 0.080, 0.160, and 0.321 mm, lengths 190, 770, and 3080 cm, respectively, and were 3.2 cm in diameter. Using B and S gauge

Nos. 38, 34, and 30, lengths 310, 770, and 2000 cm, a coil 2.6 cm diameter is produced.

The relations between the currents through the galvanometers and the corresponding deflections are shown in Fig. 7. Curve *a*, for the small coil (four 20-ohm coils in parallel, 18 mm diameter, magnets 1 mm long) galvanometer shows that the deflections are

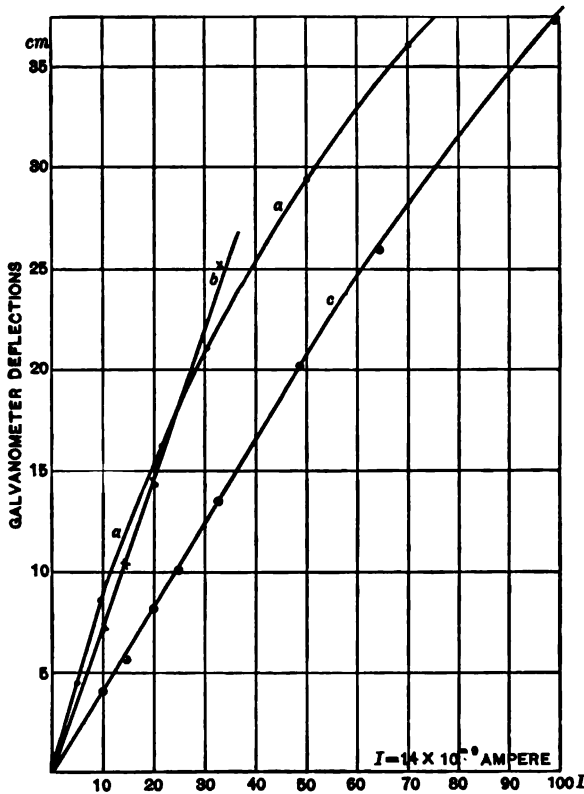


Fig. 7.—Proportionality of Galvanometer Deflections.

proportional to the current up to about 8 cm. Using a 10 magnet system (magnets 1.4 mm long), curve *b*, the proportionality of deflections with current holds for values as large as 16 to 20 cm, depending upon the sensitiveness, which in this case was about 2×10^{-10} ampere. The proportionality is of course greater for the less sensitive instrument.

The deflections observed when using the 6-ohm coils (four coils in parallel) were proportional to the current up to 30 cm. This is the only instrument tried in which the period was affected by varying external resistance, which indicates that in the other galvanometers the damping is due to the air.

Curve *c* gives the deflections for the 20 ohm, 32 mm diameter coils (in parallel), with a system of 12 magnets (6 at each end) 1.6 to 1.8 mm long. Here the proportionality holds for deflections as large as 30 cm. The magnet system weighed about 8 mg which reduced the sensitiveness to 3.5×10^{-10} ampere for a full period of 12 seconds.

Of all these galvanometers, in which about a dozen different styles of magnet systems were used, the one having the smallest coils (18 mm diameter) proved the most satisfactory except for proportionality. As has just been shown, by a judicious selection of longer galvanometer magnets, the proportionality of deflections with current may be made to hold true for large deflections with but slight loss in sensitiveness for a given period.

(*f*) *Magnetic shielding*.—In places where the magnetic field is subjected to great variations it is necessary to surround the galvanometer with an iron shield, which is most efficient when it is in the form of spherical or cylindrical shells of different thicknesses^{40a} of iron. For convenience in working, it is desirable to have the system of shields occupy a small space, which necessitates the use of metal shells of small diameter. This is a desirable feature for efficiency in shielding, but, since the shells usually become permanent magnets in handling, there is great difficulty in lengthening the period of the galvanometer needle. It is therefore a matter of trial in placing the control magnets in such a position that they weaken the combined fields of the shields. In fact, it was found easier to place a short magnet (steel file) so that it controlled the suspended system and then weaken its field than to overcome the complex field of the shields, by means of the control magnets.

^{40a} Wills, *Phys. Rev.* **24**, p. 243, 1907, finds that for both spherical and cylindrical systems the best conditions are obtained when the radii of the shells are in geometrical progression.

For magnetically shielding the small coil galvanometer used in this work four sections of annealed soft steel pipe, about 30 cm long, diameter 7, 10, 15, and 20 cm, respectively, and 4 to 6 mm thick, were provided. In practice only the two middle tubes were used, the inner one being discarded on account of the unusual difficulty experienced in controlling the galvanometer, as just explained.

The mirror was viewed through slits 1 cm high cut into the shields. Although this is supposed to produce an asymmetrical field it made no appreciable difference, since the slits were far below the coils. An inner shield was made from Swedish iron (Fig. 6), which had a circular hole, bored vertically to admit the suspended system and the coils, which were mounted in paraffin. This inner magnetic shield was found to be superior to the others just described, since it is easier to anneal and does not become a permanent magnet in handling. The shield with the glass windows, *w, w*, serves as a further protection against air currents. By taking the leads out through the bottom of the galvanometer, they will not interfere with the outer shields. In the present work the Swedish iron proved as efficient as the system of soft-steel pipes and no outer shields were provided.

It is customary to provide a symmetrical pair of control magnets (30 to 40 cm above the coils) placed upon a contrivance which permits an independent horizontal rotation and a vertical motion.⁴¹ This is desirable in work extending over a long time, for the astaticism of the needle system is constantly changing, which requires a frequent adjustment of the control magnets in order to keep a fairly constant galvanometer period.

3. The Construction of Sensitive Bolometers.—In the bolometer the sensitiveness is closely proportional to the square root of the surface, so that in spectrum energy work, where the bolometer strip is narrow, the sensitiveness attainable through the bolometer is limited.

The general complaint against the bolometer is its drift, which is of two kinds, viz, (1) a slow shift of the zero scale reading due to an unequal change in resistance in the bolometer strips and

⁴¹ See Wiedemanns "Electricität."

(2) a fluctuation of the readings due to air currents and magnetic perturbations. This ought to be the most pronounced for bolometers having large surfaces, although some writers are of the opinion that the narrow strip bolometer is subject to the greatest variations from mechanical vibrations and air currents. There are so many possible sources of trouble in a bolometric outfit that it may be worth while to notice them. (1) The auxiliary galvanometer is subject to magnetic perturbations, and, if exposed to great temperature changes, its sensitiveness is changed, due to a variation in the resistance of the coils. The sensitiveness and zero reading is also constantly changing, due to variations in the magnetic field. (2) The bolometer strip is affected by air drafts and, if very thin, by mechanical vibrations. (3) The electric circuits are subject to temperature (resistance) changes which vary the bolometer current. (4) The storage-battery current is irregular, due to changes in temperature and to polarization. The gases surrounding the bolometer may affect the reading. Lummer and Pringsheim⁴² found that variations in the amount of moisture in the air changed the sensitiveness of the bolometer.

If, therefore, one is working in a well-lighted (southern exposure) and well-ventilated room, a drift is to be expected. As an illustration of this, Abbot (*loc. cit.*) found "that the difference of radiating power between an observer's black coat, distant 2 m from the balanced bolometer, and the walls of the room which his coat momentarily hid, threw the spot of light off the scale some 40 cm, while the observer's naked hand within a meter of the bolometer turned the galvanometer needle round and round." The writer found that his balanced bolometer (not in use for observations, however) would not shift 1 mm in half a day when it was cloudy, but on bright days, with every passing cloud, or on windy days the scale readings would fluctuate back and forth through several centimeters. It is therefore advisable to work in a basement room located on the north side of a building. In this connection it may be noticed that the radiometer is more self-contained, and all its parts are concentrated in a small space in which it is easier to control the temperature. The same is true of the thermopile.

⁴² Lummer and Pringsheim, *Ann. der Phys.* (3) **68**, p. 398, 1897.

(a) *Best Resistance of Bolometer and Balancing Coils.*—In all the bolometers that have been used by foreign investigators the four arms were chosen of equal resistance. Lummer and Kurlbaum⁴³ consider the bolometer a simple Wheatstone bridge which is at its maximum sensitiveness when the four arms and the galvanometer are of equal resistance, and the e. m. f. is constant. In reality we vary the e. m. f.; and Reid⁴⁴ has shown that consequently the resistance of the balancing coils should be considerably larger than that of the bolometer. Child and Stewart⁴⁵ have shown experimentally that the sensitiveness is increased by having the resistance of the balancing coils several times that of the bolometer strips. Abbot⁴⁶ has also shown that the maximum sensibility is very nearly attained when the balancing coils are upward of four times the resistance of the bolometer strips, and the galvanometer resistance is not less than 0.6 or more than four times the resistance of the bolometer strip. This has led to the general adoption (in America) of a resistance for the compensation coils which is from 10 to 20 times that of the bolometer strips.

The equation⁴⁷ showing the relation between bolometer sensitiveness S , the bolometer current I , the temperature coefficient, e , of the part of the strip exposed, a , the resistance of the bolometer strips, r , the absorption coefficient of the surface exposed to radiation, A , the emissivity of the whole surface, E , the area of the surface, F , the heat capacity, W , and the galvanometer constant, k , is:

$$S = \frac{I}{8} e a k \sqrt{r} \frac{f_1(A) f_3(F)}{f_2(E) f_4(W)},$$

From this it will be noticed that the sensitiveness is increased by decreasing the heat capacity and the emissivity; and by increasing the bolometer current, the temperature coefficient, the resistance, the absorption coefficient, and the surface. Since for a given thickness and width of strip the resistance is proportional to the surface, it follows from the above equation that the sensitiveness is proportional to the square root of the exposed surface.

⁴³ Lummer and Kurlbaum, *Wied. Ann.*, **46**, p. 204; 1892.

⁴⁴ Reid, *Amer. Jour. Sci.*, **35**, p. 160; 1888.

⁴⁵ Child and Stewart, *Phys. Rev.*, **4**, p. 502; 1897.

⁴⁶ *Annals of the Astrophys. Obs.*, Vol. I.

The resistance of a wide bolometer may be increased by placing several strips side by side. For n such strips in series the resistance will be increased n times, and for the same current the sensitiveness will be increased by $\sqrt{n}$. If the breadth of one strip is increased n times, the current can be increased n times; but since the resistance of the strip is reduced n times, the sensitiveness remains as before. It follows, therefore, that the greatest sensitiveness is attainable in a bolometer having a large surface. This is well illustrated in the work of Lummer and Kurlbaum⁴⁷ who were thus able to use a galvanometer having a sensitiveness of only $i = 1.5 \times 10^{-9}$ ampere. For the same reasons the sensi-

tiveness of a linear bolometer is very limited, and recourse must be had to a sensitive galvanometer.

4. Design of a Bolometer.—

Langley's bolometer⁴⁸ is the only one described in which the complete instrument is enclosed in a metal case to shield it from temperature changes and drift. The instrument was intended for use in a vacuum, and hence it was complicated in construction. He used a bridge with a sliding metal contact to

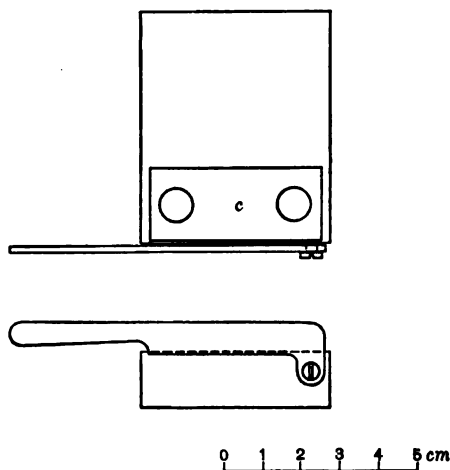


Fig. 8.

balance the bolometer arms. In the instrument used by the writer the friction contact gave so much trouble that a mercury contact was devised. The ideas contained in the present design were incorporated after an examination of all the important bolometers previously described, and hence are not new. The instrument as a whole, however, contains simplifications which it is hoped may be of use to others. It is possible to place the bolometer in a vacuum, but since the sensitiveness attained

⁴⁷ Lummer and Kurlbaum, *Wied. Ann.*, **46**, p. 208; 1892.

⁴⁸ Langley, *Annals of the Astrophys. Obs.*, 1.

is not commensurate with the difficulties arising in the use of the instrument, no attempt has been made to do so. This form of bolometer is of course less portable than the one in which the balancing coils are in a separate box, but the cases are rare where it is not possible to keep the bolometer stationary and rotate the prism table or the other spectrometer arm.

As above stated, the mechanical difficulties involved in making a thin bolometer strip are such that platinum is the best adapted. Platinum in silver wire may be hammered or rolled into flat strips. In an accurately adjusted jewelers' roll it is possible to press bare platinum wire 0.025 mm thick into strips 0.003 mm thick. To do this the rolls are screwed together quite tight (not as tight as possible as one would suppose) and the wire is passed back and forth many times, in the meantime applying heavy lubricating oil,

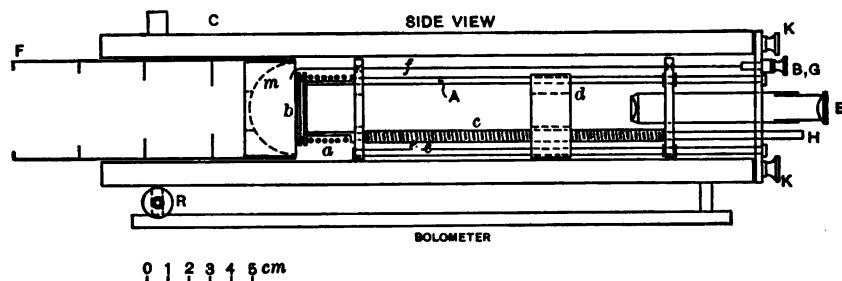


Fig. 9.

which seems to aid in producing a thin strip. During the operation the wire must be annealed frequently. Platinum in silver foil of any desired thickness can now be purchased (Sy and Wagner, Berlin), from which strips of any desired width can be cut. The strips can be cut accurately to the desired dimensions on a dividing engine, but there is danger of tearing the foil unless a suitable cutting tool is made. The steel shears shown in Fig. 8 were constructed for the purpose of cutting strips of the same width with a smooth edge, and have proved very serviceable. A pair of bolometer strips is cut at one stroke from a double sheet of the platinum in silver foil. In this figure, *c*, is a clamp with screws to secure the foil while cutting. The rest of the apparatus is self-explanatory. The difference in the width of a pair of strips was found to be of the order of 0.01 mm.

To construct a bolometer in which the resistances of the strips are equal within 1 per cent, to have the surfaces as nearly equal as possible, and, after this adjustment, to cover these strips electrolytically with platinum black and then with soot is a difficult operation. In cases where an absolute measure of

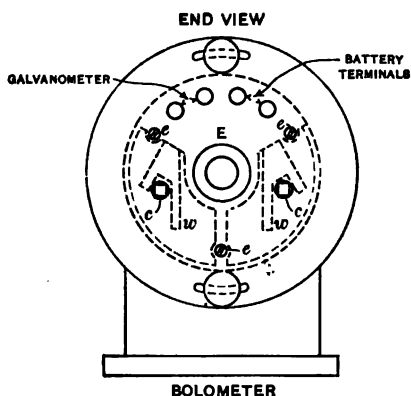


Fig. 10.

radiation is not desired, it is unnecessary to use the electrolytic bath, candle smoke being sufficient, in which case there is but little difficulty in blackening the bolometer. Linear bolometers which are less than 0.001 mm thick are torn by the surface tension of the electrolytic bath, or the nitric-acid bath used in removing the silver from the platinum, and are extremely difficult to adjust.

Paschen overcame this difficulty

in part by stroking the platinum in silver strip (mounted on an ebonite holder) with a drop of nitric acid, held in the end of a small glass tube, until the silver was dissolved and the two arms were of equal resistance. In the present work the silver was removed from the strips before mounting them.

In blackening the bolometer strips it was found that the resistance would sometimes change. The platinum strips were, therefore, annealed, before mounting, by heating them electrically, to a red heat, which served the additional purpose

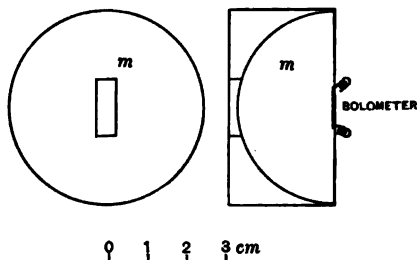


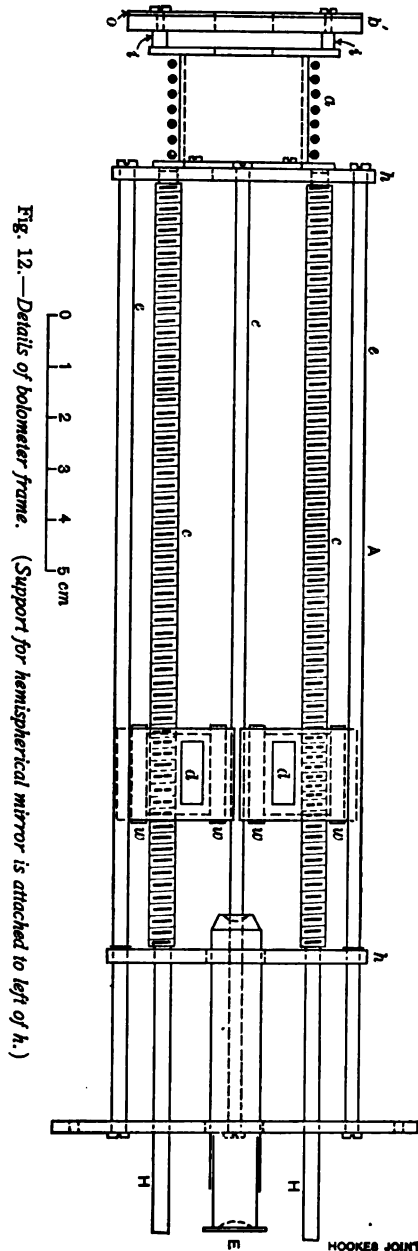
Fig. 11.—Hemispherical mirror.

of burning off a black residue which remains after dissolving the silver from the platinum and which is liable to make a poor contact. In mounting the bolometer strips on the holder, the one to be exposed to radiation is first soldered in place. The unexposed strip is made a little longer, so that it will have a slightly

greater resistance, which by subsequent resoldering is made equal to that of the exposed strip.

The present bolometer consists of three separate parts, viz, the funnel, *F*, for admitting the energy to the bolometer, Fig. 9, the outer double-walled case, *C*, which may be filled with water (running water has a variable temperature), and the frame, *A*, holding the bolometer strips, the compensation coils, and the balancing wires with sliding mercury contact, *d*, Figs. 12 and 14. The frame *A*, Figs. 9 and 12, can be easily removed from the case, which is an important factor. The concave hemispherical mirror *m*, Fig. 11, is used to reflect back to the bolometer strip, *b*, Fig. 13, any energy which has not been absorbed by it. The support and adjusting screws for this mirror is attached to the bolometer frame *A*, but is not shown in these illustrations.

In Fig. 10 is shown an end view of the bolometer, in which it will be noticed that the bolometer strip is adjustable about a vertical axis by means of the screws, *k, k*. The horizontal adjustment, *R*, is not shown in detail. The two pairs of binding posts, *G* and *B*, connect the galvanometer and battery to the bolometer by means of heavy



twisted copper wire. The frame itself for supporting the screws, *c, c*, Figs. 9 and 12, is composed of three rods, *e, e, e*, attached to two brass plates, *h, h*. The small telescope, *E*, is used to adjust the bolometer in the spectrum.

In Fig. 12 is given a more detailed drawing of the frame, *A*. The threads of the screws, *c, c*, for adjusting the slide wires, have a pitch of 1 mm. In this figure are shown the bobbin, *a*, for carrying the two balancing arms of the bridge, the manganin wire being wound noninductively. The metal disk, *b*, for supporting the bolometer strips is attached to the bobbin, *a*, by insulating material, *i*, and is covered with a thick (0.3 mm) piece of mica, as indicated at *o*, Fig. 12.

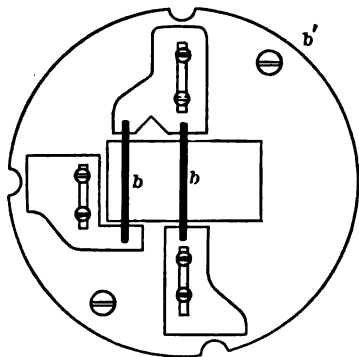


Fig. 13.—Support for bolometer strips.

The details of the bolometer strips, *b, b*, and their support are shown in Fig. 13. The bolometer strips are of platinum 0.001 to 0.003 mm thick, 0.2 to 0.5 mm wide, 10 mm long, and have a resistance of 0.7 to 3 ohms; and are soldered to adjustable copper terminals. Other platinum foil in silver, purchased abroad, had a thickness of 0.0005 to 0.003 mm (of Pt).

The heavy copper wires, *f*, Fig. 9, are soldered directly to these terminals, using rosin.

In Fig. 14 are shown the two hollow fiber sliding contacts. The holes for the wires are shown at *w, w*. It will be noticed that these sliding contacts are made hollow by drilling into them, in two directions, from the top. In this manner, a mercury holder is produced that is open only at the top.^{42a} Since the mercury column surrounding the wire, *w, w*, is 10 to 12 mm long and at least 1.5 mm thick around the slide wire, there is no difficulty whatever with bad contacts.

In previous designs at least three different methods were employed in connecting the bolometer, galvanometer, slide wire

^{42a} The writer is indebted to Mr. Haring, of the instrument division, for his ingenuity in fulfilling this condition.

and storage battery (see Figs. 15 and 16). The following is a modification of the method used by Julius (*loc. cit.*). He moved the conducting wire leading from the bolometer to the battery. In the present case the wires are stationary and the mercury contact piece, *d*, Fig. 12, mounted on a screw, *c*, is moved. In Fig. 16 is shown the method of wiring the bolometer. The heavy copper lead wires are not shown in the figures, except *f*, Fig. 9. The bolometer strips, *b, b*, in Fig. 16 are also to be noticed in Fig. 13, while the balancing coils *a, a* (20 ohms each) of manganin are wound noninductively on the bobbin, *a*, Fig. 12. The sliding contacts, *d, d*, are also shown in Fig. 12, where the wires are indicated at *w, w*. The latter pass through insulated bushings in the frame, *h, h*. For testing the sensitiveness two fine insulated wires are soldered to the ends of the exposed bolometer strip, Fig. 16. By connecting these wires with a high resistance, say 100,000 ohms, the deflection obtained by thus unbalancing the bridge is a measure of the sensitiveness of the galvanometer. For prolonged routine observations, a

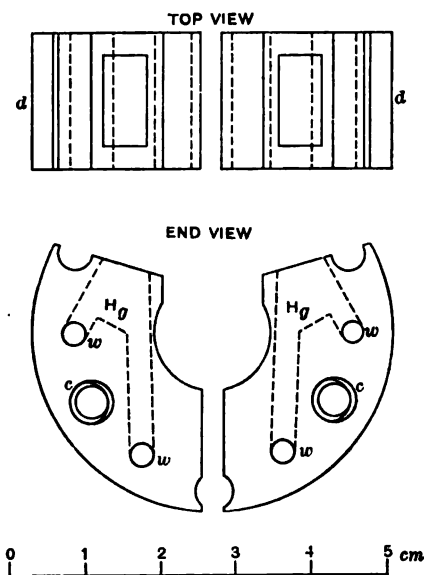
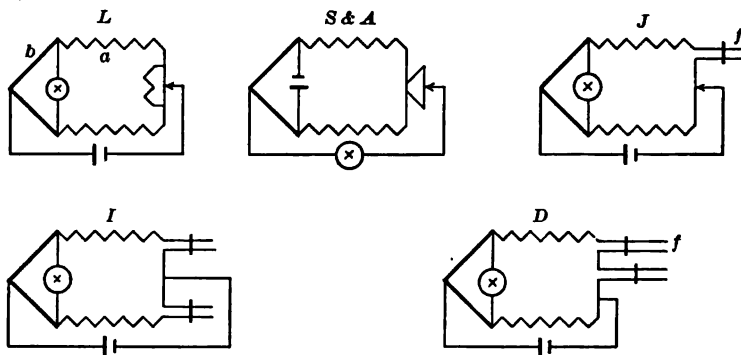


Fig. 14.—Sliding contact.

shunt box for reducing the sensitiveness of the galvanometer by $\frac{1}{5}$, $\frac{1}{10}$, $\frac{1}{25}$, $\frac{1}{50}$, and $\frac{1}{100}$, a reversing switch and a switch to shunt the bolometer, as just indicated, may be mounted in one box, which greatly facilitates the work. The largest wires, *f', f'*, Fig. 16, of constantan are 1.4 mm diameter and have a resistance of 0.3 ohms per meter. By moving the mercury contact 0.01 mm the change in resistance is 0.000 03 ohm, or by joining the wire as shown by the dotted lines, Fig. 16, the change in resistance is 0.000 015 ohm. For the coarse adjustment, constantan wires *f', f'*, having a diameter of 0.63 mm and a resistance of 1.4 ohms

per meter were used. With this combination it was possible to balance the bolometer so close (using the coarse adjustment) that on permanently closing the galvanometer circuit the deflection would be only from 2 to 3 cm, which was then brought back to zero by means of the fine adjustment. By connecting the sliding contact and balancing coils, as indicated, it is possible to balance the bridge with but little adjustment on the slide wire, f' , f'' . An increase in resistance on one side of the bridge corresponds to a decrease on the other side. This permits the use of less wire, and the whole can be confined in a smaller space than would have been possible in some of the previous instruments if an attempt had been made to place the bolometer and balancing coils and slide wire in one case.



L=LANGLEY, SNOW, ASCHKINASS, JULIUS, INGERSOLL, DONATH.

Fig. 15.—Various bolometer arrangements.

The sliding contact, Fig. 14, containing the mercury is made of fiber, which is elastic and makes a close contact, so that no mercury works out in sliding it along the wire, w , w . The openings at the top are closed with cork and there is no danger of spilling mercury. It is important to use flexible wire (twisted cord) between the bolometer and the galvanometer to reduce thermal effects. In the original design it was intended to use manganin for the balancing coils and slide wires, because of its low temperature coefficient and low thermoelectric power against copper. It was found that the manganin amalgamated so seriously with the mercury and became so brittle that constantan

was used for the slide wires in spite of its high thermoelectric power, which, however, is of minor importance, since the wires are situated in the interior of the case away from the bolometer arms, where there are air currents.

The use of copper for a slide wire (as one writer has suggested) is objectionable on account of its high temperature coefficient. If the ends of the bolometer arms, where they are soldered to the balancing coils, are kept at the same temperature by shielding them from radiation, there will be no thermoelectric currents developed, and if they are equally warmed the currents developed at the ends will be in opposite directions and will neutralize each other. The result of the Peltier effect and the Thomson effect has never been considered in radiation instruments except in the radiomicrometer where Boys (*loc. cit.*) calls attention to the fact

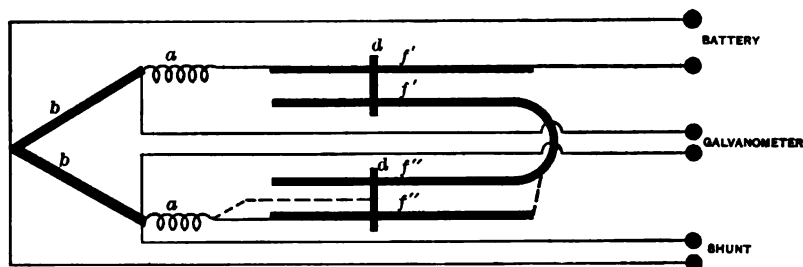


Fig. 16.—Arrangement of present bolometer circuit.

that the exposed junction will not become so warm as it otherwise would on account of heat conduction and Peltier action, which tends to send a current in the opposite direction to that producing it. In a platinum bolometer these two effects as well as the thermoelectric power are small⁴⁹ and, since they are opposed to each other must have little effect on the quantities measured. In iron and nickel the Peltier effect is much larger, and when used in a thermopile (Rubens) it is important that the junctions have as small heat capacity as possible.

5. Comparison of Sensitiveness of Various Bolometer-Galvanometer Combinations.—Important data on sensitive radiation meters, and particularly that relating to bolometers, are given in Table IV. It will be noticed that the thermopile is as sensi-

⁴⁹ See Gray, *Magnetism and Electricity*, and Landolt and Börnstein Tables.

tive as the bolometer. By using iron-constantan wire 0.06 to 0.08 mm thick the high thermal capacity is reduced. The resistance will be somewhat higher, but that is not objectionable, since the galvanometer resistance may also be made higher. The sensitiveness of the radiometer is obtained by comparing it with the bolometer. Although the radiometer is less efficient than the bolometer, it probably absorbs as much of the incident energy. Since the radiometer deflections were larger per unit area than the bolometer-galvanometer deflections, it is safe to assume that the radiometer was just as sensitive as, if not more so, than the bolometer. The long period is of course a serious objection in certain classes of work. In Table IV it will be noticed that the high temperature sensitiveness of the various instruments has been attained by the use of a highly sensitive long-period galvanometer, by using a large bolometer current, and by placing the scale at a great distance from the galvanometer. Assuming that the sensitiveness is proportional to the square of the period for a scale at 1 m, and a bolometer current of 0.04 ampere, it will be seen from column 10, Table IV, that the temperature sensitiveness of the various instruments falls in two groups. To the first group belong the earlier instruments of Rubens, of Snow, and of Paschen, with a sensitiveness of about $5^{\circ} \times 10^{-5}$ per mm deflection. To the second group belongs a more sensitive combination of Paschen's, and the writer's instrument, in which 1 mm deflection corresponds to a rise in temperature of $11^{\circ} \times 10^{-6}$ and $9^{\circ} \times 10^{-6}$, respectively. In other words, the instruments of the latter group have the same sensitiveness, and any increase is to be attained by lengthening the scale distance; the bolometer current of 0.04 ampere is about the maximum limit for accuracy. The sensitiveness of the writer's instruments could have been further increased by lengthening the scale distance to 2.5 m, when the temperature sensitiveness would have been 3.6×10^{-6} , and by doubling the galvanometer period, when the sensitiveness would have been $9^{\circ} \times 10^{-7}$ against Paschen's $1^{\circ} \times 10^{-6}$. Such a computation is of course illusory, on account of damping in the galvanometer. On actual trial (but not for the magnet system quoted) for a full period of 30 seconds the sensitiveness of the galvanometer was $i = 7 \times 10^{-11}$ ampere.

6. Comparison of Bolometer with Thermopile.—The efficiency of the bolometer and the thermopile is reduced by losses due to reflection and radiation from the receiving surface and to heat conduction to the unexposed parts. The loss of energy in the thermopile due to the Peltier effect has been considered in discussing that instrument. The loss of energy due to reflection is about 4 per cent (Kurlbaum⁵⁰). Assuming the bolometer to be made of platinum 0.5×0.002 mm cross section, and the thermopile of 20 junctions of iron and constantan wire 0.06 mm diameter, it can be readily shown that the cross section of the thermopile is about 56 times that of the bolometer, and from their heat conductivities, for the same temperature gradient, that the loss of heat by conduction in the thermopile is about 100 times that of the bolometer. But the temperature gradient at the ends of a bolometer strip carrying an electric current may be 50° to 100° , so that the heat lost by conduction may be about the same for both instruments. Since the temperature of the bolometer is from 50 to 100° higher than the thermopile, the loss of heat per second due to radiation in the former is from 2 to 3 times that of the latter. But the mass of the thermopile is 5 times, while its specific heat is 3.3 times, that of the bolometer. Hence, to raise the temperature of thermopile and the bolometer to the same extent, 16 (5×3.3) times as many heat units must be applied to the former. Since the loss by radiation is 3 times as great from the bolometer, it will require about 5 times as long for the thermopile to reach a steady temperature. In practice, however, on account of the blackening of the surface, which modifies its emissivity, the bolometer is not so quick in its action as here computed.

From these considerations, as well as from the mechanical difficulties in constructing a thermopile of wire less than 0.05 mm in diameter and keeping the resistance low, it appears that the thermopile can not be made so quick in its action as the bolometer, and hence is not so well adapted where a quick automatic registration of the galvanometer deflections is desired. But, as will be shown presently, since it is difficult to read large deflections

⁵⁰ Kurlbaum: *Ann. der Phys.* (3) **67**, p. 846; 1899. *Ann. der Phys.* (4) **2**, p. 555; 1900.

accurately in less than a 4 to 5 seconds swing of the galvanometer system, a thermopile of 0.06 to 0.08 mm wire, which attains a steady temperature in this interval of time, is not objectionable, and, since it is less disturbed by air currents (being at room temperature), it may be the more reliable instrument (see Table V). Neither instrument, however, compares with the radiometer in steadiness. The amount of work done on emission, absorption, and reflection spectra, as well as the accuracy attained, in the infra-red to 15μ , where the radiometer deflections were again and again only a few tenths of a millimeter, would not have been possible with these instruments. In a recent examination of reflection spectra of minerals, using a thermopile, the accuracy attainable without repeating the readings several times was far from that of the radiometer, although the actual deflections were larger.

The present experimental comparison of the thermopile, of 0.08 mm iron and constantan wire, and the platinum bolometer was undertaken in order to determine the accuracy attainable in measuring a constant source of radiant energy, and hence to learn the feasibility of substituting the thermopile for the troublesome bolometer. Within experimental error it has been established by Langley, by Rubens, and by Julius that the bolometer (galvanometer) deflections are proportional to the current flowing through the bolometer and also to the amount of energy falling upon the bolometer strip. It remained, therefore, to determine whether the present bolometer behaved likewise and also whether the same accuracy is attainable with the thermopile.

To this end a bolometer was constructed with the greatest care. It was annealed before adjusting the resistance of the strips, covered electrolytically with platinum black after the method of Kurlbaum⁵⁷ and then smoked over gauze wire over a paraffin candle. The resistances of the bolometer strips were 1.782 and 1.797 ($\Delta=0.015$) ohms, respectively. After blackening them they were 1.766 and 1.818 ($\Delta=0.052$) ohms, respectively. The width of the bright strips was 0.50 mm, which was increased to 0.56 mm after blacking. The length was 11 mm and thickness less than 0.002 mm. The bolometer current was 0.04 ampere and throughout the following experiments there was no difficulty due to air currents. Magnetic disturbances were at a minimum and

conditions for accurate measurements were as perfect as one could expect.

TABLE V.
Comparison of Bolometer and Thermopile.

Direct deflection (mean value)	Deflection with disk. $180^\circ = 49.9$ (mean value)	Ratio	Remarks
Bolometer			
(May 24, '07)			
12.22	6.14	50.24	Nernst heater on 98 volts at 1 m. from bolometer. Galvanometer (full) period 8 seconds undamped; 50 ohms in series. Total deflection about 80 cm. Bolometer is perfectly steady, and comes to rest abruptly. Readings vary from 0.1 to 0.7 per cent from mean of about 10 in each set. Temperature sensitiveness = $6^\circ \times 10^{-5}$.
12.16	6.11	50.24	
Thermopile.			
(May 24, '07)			
13.14	6.61	50.26	Conditions are the same as for the bolometer. Thermopile deflections "creeps," and does not come to rest in the same time as the galvanometer (on open circuit or with the bolometer), due to its larger heat capacity. Galvanometer single swing of 4 seconds is increased to 6 seconds and is fully damped due to lag of thermopile; 50 ohms in series with galvanometer. Temperature sensitiveness = $2^\circ \times 10^{-6}$.
14.22	7.21	50.7	Source and disk nearer the screen. Difficult to read the deflection on account of the "creeping," which amounts to 1 to 3 mm. Readings made at end of 6 seconds vary by 0.4 per cent from mean; 20 ohms in series with the galvanometer. Temperature sensitiveness = $5^\circ \times 10^{-6}$.
14.30	7.24	50.6	

The thermopile of 0.08 wire (20 junctions covered with a slit 0.5 mm wide), already described, showed a slight lag in registering the energy received. Although this was not marked, there was a

tendency for the deflection to creep instead of stopping abruptly, as in the case of the bolometer. This was most marked in large deflections and necessitated exposing the thermopile to radiation for a definite time (6 seconds) and taking the zero at the expiration of an equal interval of time. The results are given in Table V. The last two values for the thermopile are vitiated by radiation from the rotating disk, to be explained later. The results show that there is no great difference in the two instruments. It was necessary, however, to note the time of exposure of the thermopile, which is not convenient for large deflections. The estimation of the relative merits of the bolometer and the thermopile is, therefore, a personal one, and from the experience gained it may be said that for measuring intense sources the bolometer is the more accurate (when working to 0.5 per cent) unless great precautions be taken in making the thermopile readings.

The theoretical temperature sensitiveness of the thermopile was considerably greater than that of the bolometer, as was found on subsequent computation. It may be added, therefore, that if the bolometer sensitiveness had been increased by increasing the current through it, there would have been greater unsteadiness in the galvanometer readings.

7. Bolometric Usage.—The use of the null method in bolometric measurements has been suggested, i. e., to actually measure the change in resistance rather than to read the direct deflections. Wadsworth⁵¹ has shown, however, that the deflection method is quicker and better than the zero method, since it involves no disturbance of the contacts in any part of the bridge. At the highest sensitiveness the zero method is practically excluded for accurate measurements of very feeble radiations.

For the sake of completeness, the method of determining the temperature sensitiveness of the bolometer strip is included here. The method⁵² consists in unbalancing the bridge by placing a high resistance, e. g., 100,000 ohms, in parallel with one of the bolometer strips and noting the resulting deflection. Then, knowing the resistance and the temperature coefficient of the bolometer

⁵¹ Wadsworth, *Astrophys. Jour.*, **5**, p. 268; 1897.

⁵² Rubens and Ritter, *Wied. Ann.*, **40**, p. 62; 1890.

strip, one can compute the change in temperature, Δt , corresponding to 1 mm deflection from the equation:

$$\Delta t = \frac{r}{s} \frac{1}{\delta d}$$

where r = the resistance of the bolometer strip, s = shunt resistance (100,000 ohms), δ = the temperature coefficient (about $0.003 +$ for platinum), and d = the galvanometer deflections in mm. Snow⁵³ placed a large resistance permanently in parallel with the bolometer strip and unbalanced the latter by closing a shunt around a part of the former. This method was also employed to determine the sensitiveness from day to day. Its use is open to one objection, viz, the change in current in the bolometer strip introduced by placing a high resistance in parallel is sufficient to disturb the temperature equilibrium in the bolometer case, and it is necessary to wait a few seconds for the galvanometer to become steady. However, since this test is not very frequent, it is more convenient than the radiation test. Lummer and Pringsheim⁵⁴ compared the daily variation in sensitiveness by exposing the bolometer strip to the radiation from a "black-body" heated with boiling water. The latter has the disadvantage that the amount of atmospheric water vapor as well as the barometric pressure varies from day to day, and hence the amount of energy received by the bolometer is not perfectly constant. Angstrom,⁵⁵ and Kurlbaum⁵⁶ have given methods for determining the sensitiveness in absolute value.

The change in absorption and emission of platinum black and of soot with thickness was investigated by Kurlbaum.⁵⁷ He found that a deposit of soot weighing 25 mg per dm² emits 94 per cent as much energy (at 100° C.) as does a complete radiator, while

⁵³ Snow, *Phys. Rev.*, **1**, p. 32; 1893.

⁵⁴ Lummer and Pringsheim, *Wied. Ann.*, **63**, p. 399; 1897.

⁵⁵ Angstrom, *Acta. Reg. Soc. Upsala*, June, 1893.

⁵⁶ Kurlbaum, *Wied. Ann.*, **51**, p. 591, 1894; **61**, p. 420, 1897; **65**, p. 746, 1898.

⁵⁷ Kurlbaum, *Wied. Ann.* (3), **67**, p. 846, 1899.

The electrolyte used in depositing platinum black consisted of 1 part of platinum chloride, 30 parts of water, and 0.008 of lead acetate. He used a current density of 0.03 ampere per cm², and 4 volts for 3 minutes. The lead acetate causes the platinum black to adhere. The writer has found that for narrow bolometer strips the platinum black deposits more rapidly on the edges than on the flat surface.

a deposit of 83 mg per dm² of platinum black is required to have the same emissivity. From this it was assumed the bolometer absorbs a similar amount. Paschen,⁵⁸ however, appears to be the only one who attempted to make his bolometer a more complete absorber of radiation by placing it in the focus of a highly-polished concave hemispherical mirror, as shown in Fig. 11. He used this arrangement in finding the maxima of his energy curves and found that the constant, 2920, in the Wien displacement law was increased about 4 per cent by using the mirror. Another method of "blackening" the bolometer is to have the strip in the form of a hollow blackened cylinder with a slit to admit the energy to be measured. In this form the resistance of the strip is low, and hence there is difficulty in attaining sufficient sensitiveness.

The adaptability of a radiation meter for a particular kind of work seems never to have been considered. The bolometer is well adapted for measuring intense sources, using a short period galvanometer, but it has not been a great success in measuring weak sources of radiation. A very sensitive radiometer behaves more like a photographic plate (which is cumulative in its action, however) and useful in registering feeble sources. The thermopile is the most useful in measuring weak radiation, such as the selective reflection of long heat waves (Rubens, loc. cit.) where a radiometer can not be used on account of the opacity of its window. It is interesting to note that the radiometer has been used successfully in measuring heat from stars (Nichols, loc. cit., Table III). It is perhaps the most efficient instrument which can be used for such work. This is due to the fact that the star image is small. A radiometer with a small vane, the size of the star image, will have a short period, and since its sensitiveness may be made as high as for a large vane, it is well adapted for such work. On the other hand, the number of thermo-junctions that can be covered with a star image must be very limited, hence, gain in sensitiveness must be attained by increasing the period of the galvanometer. For the same reason the bolometer, on account of its large surface, can not be used as efficiently as the radiometer for this particular problem. That is to say, the great sensitiveness

⁵⁸ Paschen, *Berichte Berliner Akad.*, p. 409; 1899.

required would have to be attained through the galvanometer by lengthening its period, thus subjecting it to magnetic perturbations. For spectrum energy work, using a fine linear surface, the bolometer has been used to great advantage, especially in the investigation of the dispersion of fluorite and rock salt.

(a) *Errors Resulting from Lack of Balance of Bolometer.*—The errors introduced into the observed galvanometer deflections as a result of not keeping the bolometer balanced appears never to have been considered. It has been frequently noticed in this paper that the bolometer is subject to a continuous drift, due to a slow unbalancing of the bridge arms. In the present test the bolometer was

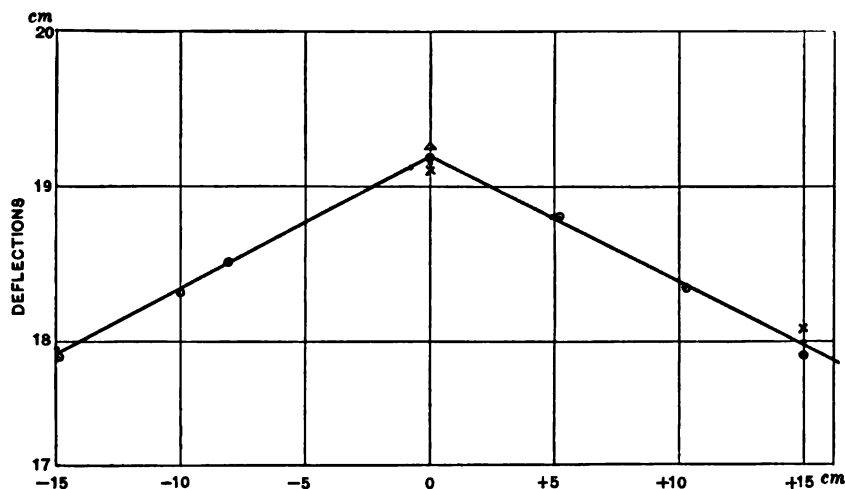


Fig. 17.—Variation insensibility with balancing of bolometer.

unbalanced by moving the mercury contact on the compensating wires. The source of energy was a 110-volt Nernst-lamp heater on a 98-volt storage battery circuit, which supplied a constant current.

In Fig. 17 is shown the change in the galvanometer deflections (ordinates) when the bolometer is exposed to the radiation from the Nernst-heater, after unbalancing the bolometer bridge. The amount of unbalancing is given in galvanometer deflections (abscissæ), positive or negative, depending upon the direction of the unbalancing. In Fig. 17 the zero deflection indicates that the galvanometer deflection was the same on open and on closed

bolometer circuit, i. e., that the bolometer was perfectly balanced. The positive and negative deflections indicate the change in the zero of the galvanometer reading on closing the bolometer circuit after the latter was unbalanced, corresponding to a drift. The different series of observations are indicated by dots, circles, crosses, etc. The lack of symmetry in the two branches of the graph may be due to the galvanometer not being level. The galvanometer sensitiveness was about 3×10^{-10} ampere, and the bolometer current was 0.04 ampere.

The results show that a lack of balance of 2 cm introduced an error of 1 per cent in the resulting galvanometer deflections. From this it will be seen that it is necessary to keep the bolometer balanced to less than 1 cm in order to insure accuracy in the final results.

VII. SELECTIVE RADIATION METERS.

The well-known fact that the resistance of selenium changes, when exposed to light, has been applied to photometric measurements. If an alloy could be produced which has its maximum sensitiveness (i. e., its resistance change greatest) for those wave lengths to which the eye is most sensitive, it would be possible to devise a method for measuring radiant efficiencies which is far superior to present methods of comparing the total radiation to the luminous radiation. In the ultraviolet the photo-electric effect has been applied to measure the radiation from the mercury arc.⁵⁹

Little is known in regard to this class of radiation meters. Since they are highly selective their application is limited, but if the proper combination could be found having a sensibility curve similar to the eye it would have a wide application in photometric work. If the transformation of radiant energy into electrical energy is as complete in the photo-electric effect as in a bolometer it would be a steadier instrument, since the former is not subject to such perturbations as is the bolometer.

⁵⁹ Küch and Retschinsky, *Ann. der Phys.* (4) **20**, p. 563; 1906.

VIII. CHANGE IN SENSITIVENESS OF INSTRUMENTS.

Various observers have tested the bolometer and have found that as a meter of radiant energy it indicates a direct proportionality (in galvanometer deflections), with a change in intensity of the incident energy. Hence, it is unnecessary to reduce its indications by a known amount in order to obtain an accurate measure of the energy falling upon the bolometer strip. Large deflections of the auxiliary galvanometer, however, are not proportional to the current, and hence a shunt or series resistance must be introduced in the galvanometer circuit to reduce the deflections to the region within which proportionality holds, or the necessary corrections must be applied from the calibration curve of the galvanometer. The sensitiveness may also be reduced by changing the current in the bolometer circuit.

On the other hand it has been shown that, on account of the Peltier effect, which may introduce errors of about 1 part in 300 to 400, it is advisable to keep the equivalent deflections less than 30 cm (scale at 1 m) when using an iron-constantan thermopile. Since the galvanometer deflections must also be kept small, the best procedure is to put a large resistance in series with the thermopile. The use of a sectored disk would appear better, for then all the errors are transferred back to the disk. However, the following experiment shows that the motion of the disk may introduce slight errors, which must be determined and eliminated in quantitative work.

(a) *Experiment with a Sectored Disk.*—In comparing the relative merits of the bolometer and the thermopile, the simplest method appeared to be to reduce the intensity of the source by a known amount, by using a sectored disk the angular openings of which are accurately known. It will be noticed that while the ratios of energy transmitted by the sectored disk were in close agreement in any series of measurements (see Tables V and VI), the numerical values were in all cases higher than the true ones. In other words, the disk transmitted too much energy or the apparent opening was larger than the true one. It remained therefore to be shown whether this is due to diffraction (of the very long wavelengths) or to lack of proportionality in the registering of the energy by the bolometer and by the thermopile. The method

TABLE VI.

Reliability of Bolometer Measurements.

Direct deflection (mean of 6 to 10 readings)	Deflection with disk	Ratio	Remarks
<i>Disk opening 240°=66.827</i>			
(May 27, '07)			
9.42 cm	6.33 cm	67.23	Galvanometer period 8 seconds, and vibration is undamped; 20 ohms in series with the galvanometer. Temperature sensitiveness = $6^{\circ} \times 10^{-5}$ C. High values are due to radiation from moving disk, which is 0.5 m from screen. Galvanometer period 14 seconds and vibration is just damped. Temperature sensitiveness = $1^{\circ}.2 \times 10^{-5}$ C.
9.35	6.28	67.25	
11.82	7.95	67.25	
9.11	6.12	67.2	
8.94	6.04	67.5	
13.31	8.99	67.6	
<i>Disk opening 120°=33.406</i>			
(May 28, '07)			
15.82	5.65	35.70	Galvanometer period 14 seconds. Galvanometer period 8 seconds. Disk better shielded than in previous experiments and is closer to screen—6 cm from it. Heater 150 cm from bolometer; screen at 80 cm.
11.56	4.14	35.75	
(May 27, '07)			
12.31	4.28	34.8	14 ohms in galvanometer circuit. Galvanometer period 14 seconds. Nernst heater on 74 volts. No resistance in galvanometer circuit; results show that high value is not due to lack of proportionality of galvanometer deflections. Nernst heater on 95 volts. 20 ohms in galvanometer circuit. Total deflection is about 45 cm.
30.40	10.53	34.7	
13.75	4.70	34.5	
(May 25, '07)			
14.15	4.75	33.56	
17.04	5.68	33.38	
<i>Disk opening 180°=50.125</i>			
(May 24, '07)			
17.17 cm	8.65 cm	50.37	Nernst heater on 98 volts at 1 m from bolometer. Galvanometer period 8 seconds. 30 ohms in series with galvanometer. Total deflection about 80 cm. The individual deflections vary from 0.2 to 0.5 per cent from mean.
17.21	8.64	50.20	

of observation consisted in taking from 5 to 10 readings without the disk, then a similar number with the rotating disk interposed, followed by a number without the disk.

The first test was to determine whether the rotating disk (30 cm diameter, 1.3 meters from the bolometer) affected the instrument, and it was found that the resulting deflections, 1 to 2 mm, were no larger than those due to stray radiation reflected from the stationary disk. A heavy black cardboard shield was then placed between the bolometer and the disk (0.5 m from the disk) and similar screens were placed around the source, which was 2 meters from the bolometer. No radiation was detected from the stationary disk, whether the open or closed part of the disk faced the bolometer; but unfortunately this test was not made for the moving disk. The disk with the 240° opening gave values 0.5 per cent too high (see Table VI, observations of May 27). The results with the 120° disk (6 openings of 20° each) were in still greater error. The space between the bolometer and the shield was then entirely enclosed, and with the disk close to the opening (7×10 cm) in the shield the discrepancy became still greater. It was then found that the increased transmission is due to the moving disk and depended upon the distance of the disk from the screen.

It was further shown that the transmission was proportional to the speed, so that the 240° disk (true transmission 66.827 per cent⁶⁰) gave values from 69.3 to 77.2 per cent. In Fig. 18 are plotted the galvanometer deflections (abscissæ) for different distances of the disk from the screen. The latter was 80 cm from the bolometer, and had an opening in it which was the size of the openings in the sectored disk. No radiation was observed from the stationary disk, nor from the shields back of it when the open sector was before the bolometer. The speed of the disk was such as is used in photometry, and was kept constant for each series of observations. In the lower curve for the 240° disk the speed was slow and there was a flicker on viewing it. The curves show that the maximum radiation occurs when the disk is about 6 cm from the shield, and it disappears immediately on stopping the disk.

⁶⁰ These disks and their constants were supplied by Dr. Hyde. This Bulletin, 2, p. 1, 1906.

The disk was run continuously for a complete series of measurements, and no deflections greater than 1 to 2 mm were observed from it immediately after stopping the rotation. In these tests the motor was shielded from the bolometer. On removing the shield and the disk and on running the motor the deflections were from 1 to 3 mm. The experiment shows that the sectored disk is not as applicable as one would suppose, unless one determines the corrections which in two series of experiments were found to be in very close agreement. In the present curves the galvanometer was at its full sensitiveness (no resistance in series), so

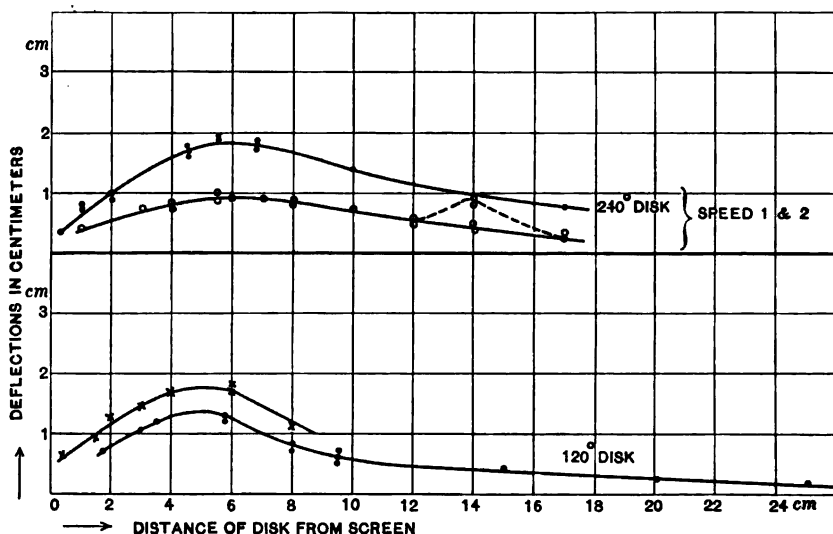


Fig. 18.—Radiation from moving sectored disk.

that in the actual experiments (Table VI) the error was less. For the 240° disk where the error in the deflections was only 1 or 2 mm the correction reduces the observed values close to the true one.

In connection with a bolometer or a thermopile it is possible to run the disk at a very much slower speed than used here; this will reduce the errors in question to a minimum.

From the consistency of the ratios in each series, which is of the order of 0.2 to 0.3 per cent, it will be noticed that the bolometer is a very reliable instrument in spite of its mechanical weakness.

XI. SUMMARY.

The present paper deals with four instruments for measuring radiant energy, viz, the radiomicrometer, the linear thermopile, the radiometer, and the bolometer with its auxiliary galvanometer.

As a result of this historical inquiry and by experiment it was shown that the radiomicrometer is capable of great improvement, by reducing its weight, by lengthening its period, and by placing it in a vacuum. It was further shown, that on account of para- and dia-magnetism the sensitiveness of the radiomicrometer is very limited, perhaps only a fifth of the best bolometers described.

It was also shown that the Rubens thermopile is as sensitive as the best bolometer, and that its heat capacity can be greatly reduced by using thinner (0.06 to 0.08 mm diameter) wires, which are made shorter, thus keeping the resistance low. The computed errors, due to the Peltier effect, are about 1 part in 300. The thermopile is not so well adapted as is the bolometer for instantaneous registration of radiant energy and it does not admit so great a range in variation of sensibility, but on account of its greater steadiness it commends itself for measuring very weak sources of radiation, e. g., the extreme ultra-violet and infra-red region of the spectrum.

By a direct comparison it was shown that the radiometer can be made just as sensitive as the bolometer, but its period will be much longer. It was found that the radiometer is not selective in its action, and hence that it can be used for measuring ultra-violet radiation. The main objection to the use of a radiometer is its long period, but since it is easily shielded from temperature changes, and since it is not subject to magnetic perturbations, this long period is of minor importance so long as we are dealing with a constant source of radiation. In spectrum energy work its usefulness is limited to the region in which the window is transparent, to 20μ . The fact that the radiometer deflections can not be obtained in absolute measure is a minor objection, since in but few cases (thus far at least) has it been necessary to thus obtain the deflections. The action of a radiometer is somewhat analogous to a photographic plate, in that it will detect weak radiation, provided one can wait for it, and, on account of its great

steadiness, is of all the instruments considered, probably the best adapted to search for infra-red fluorescence.

A bolometer installation is so distributed that it is difficult to shield from temperature changes. In spite of its small heat capacity, the bolometer has a "drift" due to a slow and unequal warming of the strips. Air currents which result from the hot bolometer strips also cause a variation in the deflections of the auxiliary galvanometer. Nevertheless, despite these defects, it is the quickest acting of the four instruments considered and is the best adapted for registering the energy radiated from a rapidly changing source. For precision work it is necessary to keep the bolometer balanced to less than 1 cm deflection.

The auxiliary galvanometer is the main source of weakness in measuring radiant energy, and in places subject to great magnetic perturbations a period greater than 5 seconds, single swing, is to be avoided. Hence, although a greater sensitiveness is possible, the working sensibility of the various galvanometers studied is of the order of $i = 2 \times 10^{-10}$ ampere per mm deflection on a scale at 1 m. Under these conditions the various bolometers used were (as a fair estimate of the recorded data) sensitive to a temperature difference of 4×10^{-5} degree to 5×10^{-6} degree per mm deflection, on a scale of 1 meter. The galvanometer sensibility was found to be closely proportional to the period.

A direct comparison was also made of the relative accuracy of the thermopile and the bolometer in measuring intense and weak sources of radiation, and the results show that there is little preference, other than a personal one, in these two instruments.

The manner of reducing the sensitiveness of these instruments is of importance in precision work. The use of a rotating sector disk for reducing the intensity of the source is liable to introduce errors, which must be taken into account.

It may be added that these tests were made in a building which is isolated from mechanical and magnetic disturbances, and hence under the most favorable conditions.

WASHINGTON, October 1, 1907.

A QUARTZ COMPENSATING POLARISCOPE WITH ADJUSTABLE SENSIBILITY.

By Frederick Bates.

All of the polarizing systems so far devised for quartz wedge polariscopes have been defective for one of two reasons; either the sensibility of the instrument can not be varied or it can be used only with monochromatic light. The system which has given the best results and is in general use at the present time is the so-called half-shade. It introduces into polarimetry the photometric principle, inasmuch as the angular position of the analyzing nicol is determined by bringing the two halves of the field of the instrument to a condition of uniform illumination. In any half-shade system the light from the polarizer is plane polarized in two planes which make an angle α , called the polarization angle, with each other. All of the light illuminating either half of the field is thus polarized in one of these planes, and when a setting is made the polarization plane of the analyzer makes approximately a right angle with the bisector of α . Upon the magnitude of α depends the accuracy with which settings can be made; the sensibility for any given light source being an inverse function of that magnitude. Hence it is exceedingly desirable that a polarizing system permit α to be varied as the observer may desire.

A monochromatic light-source of sufficient intensity and suitable for the average work for which quartz compensating polariscopes are used is yet to be obtained. In order to obviate this difficulty the rotation produced by the substance being examined is compensated as completely as possible by the use of oppositely rotating quartz. Since the polarization planes of the different

wave-lengths are thus returned to their original positions, white light can be used. This, however, makes it necessary for the polarizing system and the analyzer to be mounted so as to be immovable. The polarization plane of the analyzer then makes approximately a right angle with the bisector of α .

The systems most used in polariscopes are the Laurent, the Jellet, and the Lippich. In the former a thin plate of quartz cut parallel to the optic axis covers one-half the field of the polarizing nicol. In order that the two rays of the doubly refracting quartz may combine to give plane polarized light in the analyzer they must have an optical difference of path equal to $\frac{\lambda}{2}$. Thus

the thickness of the quartz must be such as to make it a half-wave plate and its use is then limited to a light source giving only that particular wave-length. The advantage of this system is due to its adjustable sensibility, α being twice the angle between the optic axis of the plate and the plane perpendicular to the principal section of the polarizer, can be readily varied by rotating the polarizer. The Jellet system consists of a twin nicol so made that the principal sections form the angle α . Since the different sections are cemented together the sensibility can not be varied. It can, however, be used with white light. In the Lippich system α is formed by two beams of plain polarized light which come from two separate nicols, one of which covers but one-half the aperture of the larger nicol. By rotating either of these nicols α can be varied as in the Laurent polarizing system.

In designing quartz compensating polariscopes the best results so far have been obtained by using a Lippich polarizing system and a white light-source. The greatest weakness has been the lack of an adjustable sensibility. Only one value of α can be used and it must necessarily be large enough to give sufficient light to read for example the darkest colored raw sugar solutions. When polarizing substances having a small coefficient of light absorption, as the better grade of sugars, and the observer has more light than he needs he still has available only the sensibility which corresponds to that value of α which gives sufficient light to polarize substances with a relatively large coefficient of absorption, such as very dark raw sugars. If then it were pos-

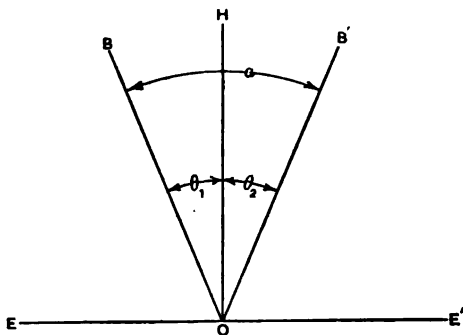




Fig. 3.—*Quartz Compensating Polariscopes.*

sible to retain the white-light source and at the same time have α adjustable, a distinct advance in polariscopes construction would be made.

Let OB and OB' , figure 1, be the traces of the polarization planes of the large and small nicols of a half-shade polarizing system. If the intensities of the light in OB and OB' are equal, the polarization plane EE' of the analyzing nicol will be at right angles with OH , the bisection of α . If α be increased or diminished by displacing OB' about the point O it is evident that when a match is again obtained EE' will have suffered one-half the angular displacement through which OB' has been rotated. However, in the Lippich system, since the smaller nicol covers one-half the field of the larger, we do not have the two beams OB and OB' of equal intensity. If the intensity of OB is A , the intensity of OB' is $A \cos^2 \alpha$. When EE' is set for a match the angle between EE' and OH is therefore never 90° for any value of α except 0.



The condition for equal illumination with a half-shade system is

$$A_1 \sin^2 \theta_1 - A_2 \sin^2 \theta_2 = 0 \quad (1)$$

where A_1 and A_2 are the intensities of their respective halves of the field and θ_1 , and θ_2 are the angles HOB and HOB' .

Let

$$\theta_1 = \phi \pm \delta$$

$$\theta_2 = \phi \mp \delta$$

Equation (1) becomes

$$A_1 \sin^2 (\phi \pm \delta) = A_2 \sin^2 (\phi \mp \delta)$$

From (2) we obtain

$$\tan \delta = \pm \frac{\sqrt{K-1}}{\sqrt{K+1}} \tan \phi$$

or since

$$\begin{aligned}\theta_1 + \theta_2 &= \alpha \\ \tan \delta &= \pm \frac{\sqrt{K} - 1}{\sqrt{K} + 1} \tan \frac{\alpha}{2}\end{aligned}\quad (3)$$

where $K = \frac{A_2}{A_1}$ and δ is the angular difference, for any value of α , between the positions of EE' for a match when $A_2 = A_1$ and when $A_2 = A_1 \cos^2 \alpha$. It is thus evident if α be varied by rotating OB' about the point O , that to obtain a match EE' must be rotated in the same direction as OB' and by an amount such that the normal to EE' always makes an angle δ with OH the bisector of BOB' .

It would seem a difficult task to build a mechanism that would maintain EE' in the proper position to satisfy the theoretical value of δ given by (3). If such a mechanism were obtained the observer could detect no difference in the intensities of the two halves of the field, once the instrument was adjusted, no matter what value α might be given. The limits of accuracy in ordinary polarimetry are such as to make such a mechanism unnecessary. If EE' should always be maintained at right angles to OH , the fixed zero point of the instrument would be in error by the amount δ for any value of α , provided the instrument had been previously adjusted for $\alpha = 0$. If the zero of the instrument be adjusted to read correctly for any particular value of α and then α be changed, the zero will be in error by the difference between the values of δ corresponding to the two values of α . The curve in figure 2 shows the value of δ , obtained by solving (3), corresponding to any value of α between 0° and 15° . From this curve it is at once evident that with an instrument equipped with a Lippich polarizing system, in adjustment for a polarization angle $\alpha = 10^\circ$, α may be varied between the limits of 4° and 12.4 without introducing an error due to a change in the zero point greater than 0.1 S (sugar degrees), provided EE' be constantly maintained at right angles to OH . If the zero point be adjusted for $\alpha = 8.5$, α may be varied from 0° to 11.5 with a maximum error of 0.1 S; or from 5.6 to 10.3 with a maximum error of 0.05 S.

The instrument shown in figure 3 was built¹ for the Bureau of Standards to fulfill the theoretical conditions mentioned above. It is a double quartz wedge compensation instrument with a Lippich polarizing system. The analyzing nicol and the large nicol of the polarizing system are mounted in bearings and are joined by gears with a connecting rod. The milled head of this rod is shown half way between the base of the instrument and the observing telescope. When the milled head is rotated the two nicols are rotated, and the design of the gears is such that the analyzing nicol always receives one-half the angular displacement

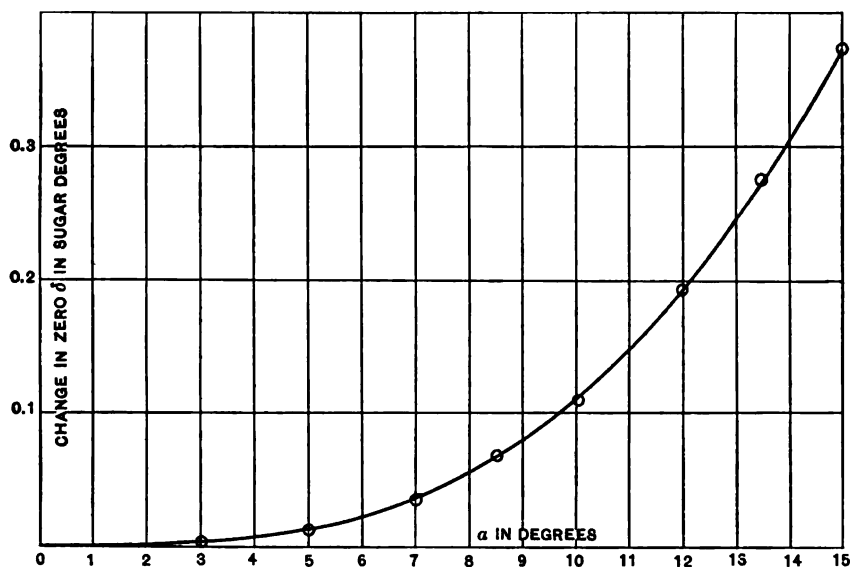


Fig. 2.

of the large nicol of the polarizing system. Around the milled head is a circular scale which shows the polarizing angle for any position of the nicols. The milled heads on the right and left hand sides of the instrument drive the quartz wedges of the compensator, and their position is such as to permit the arm of the observer to rest free from strain while making a setting. The wedges can instantly be clamped rigid for any part of the scale. The scales on the quartz wedges are the type used on regular Frič saccharimeters. Being of glass and read by transmitted light, the scale divisions are exceedingly clear and there is no

¹ The builders were Messrs. Josef and Jan Frič. Prag, Kral. Vinohrady, 233, Austria.

black dividing line between a scale and its vernier. In all research work where small temperature corrections are to be made it is necessary to know accurately the temperature of the quartz wedges. Polariscopes builders seem to have ignored this fact. A thermometer (10° – 40° C., in one-fifth degrees), with a horizontal scale and with its bulb between the quartz wedges, has accordingly been mounted in a brass case on top of the metal box containing the compensator.

For all ordinary sugar testing, where the temperature of the room changes slowly, the reading of this thermometer is practically the temperature of the room. The observer is thus able to take the temperature with the same facility that he reads the scale on his wedge, since the thermometer scale is in a similar position and is illuminated by the same light source. The base of the instrument has been made exceptionally heavy and is mounted on rubber tips to insure against accidental change of position relative to the light source.

The improvements are equally advantageous for all uses to which the polariscope may be put. However, it is in the testing of sugars the new instrument should find its broadest application. With the instrument in adjustment for a polarization angle of 10° the observer can instantaneously adjust the sensibility so as to have sufficient light to polarize the darkest sugars, or he can with equal facility have an instrument far more sensitive than any ordinary saccharimeter. In measuring rotations with the greatest possible accuracy, or when it is desired to make the settings with the least possible strain on the eye, the observer has only to change the polarization angle until he has just sufficient light to bring the two halves of the field to the same intensity. He then has for his eye an instrument so adjusted as to give the maximum sensibility for making the setting, no matter what the character of the substance whose rotation is being measured. With the instrument adjusted for $\alpha=0$ the observer soon learns the exact change in the zero point of the scale for all values of α , and instantly makes the slight correction mentally for each value of α used. He is thus able to determine the polarization of the better grades of sugar to an accuracy of $\pm .01^{\circ}$ S.

WASHINGTON, October 15, 1907.

AN APPARATUS FOR DETERMINING THE FORM OF A WAVE OF MAGNETIC FLUX.

By M. G. Lloyd and J. V. S. Fisher.

The apparatus here described was designed and constructed in the course of measurements upon the magnetic losses of energy in sheet iron. Its object was the determination of the average value of an alternating electromotive force from which the form factor of the emf. wave and the maximum value of the magnetic flux inducing it can be computed. It serves equally well, however, for the tracing of a wave of magnetic induction, and that use of the apparatus is described in the present article with illustrative examples.

DESCRIPTION OF APPARATUS.

The essential part of the apparatus is a rotating commutator which reverses the contacts twice per cycle. An ebonite disk is mounted on a shaft between bearings; to its circumference is fastened a thin conducting strip with gaps at two points 180° apart. Four brushes, equally spaced, bear upon the metal rim of the rotating disk, and their joint action is that of commutation. A carrier, of brass, having four arms, is mounted loose upon the shaft; the arms support the brush holders. Screwed fast to the carrier is a gear wheel which meshes with a worm. A graduated circular scale is attached to the carrier, and is read by means of a fixed index fastened to the base.

By means of the worm and gear the brushes may be set in any desired position. The brushes are thoroughly insulated from each other and from the carrier by means of ebonite strips at the extremities of the arms.

The shaft of the rotating commutator may be coupled directly to the generator shaft. The instrument was designed for use with a four-pole machine, running at 1800 revolutions per minute.

Fig. 1 shows a photograph of the commutator in position for use. The principal dimensions are:

Circumference of disk	56	cm
Thickness of metal rim	1.5	mm
Width of gaps in rim	2	mm
Gear diameter, 96 teeth	10.2	cm

The wheel gearing with the worm on the end of the shaft makes an electrical contact for every hundred revolutions of the shaft. This serves to record the speed of rotation.

Referring to Fig. 1, the brushes *a c* are connected to a secondary coil wound around the flux to be measured. The brushes *b d* are connected to an indicating direct-current instrument, such as a d'Arsonval galvanometer or Weston voltmeter. Any instrument whose deflection is proportional to the first power of the current will answer the purpose.

To plot a curve of magnetic induction, a reading on the Weston instrument is taken for a definite position of the brushes. Then successive readings are taken, the brushes being advanced the same number of degrees each time until they have been shifted 180°. In practice it suffices to shift the brushes only 90°, corresponding to a half cycle, since the readings repeat themselves in the second quadrant when only the odd harmonics of the fundamental frequency are present, and the apparatus is suited only to waves of this character. This condition means that the positive and negative lobes of the wave shall be similar, a condition which will be fulfilled if the magnetization is produced by a well-designed and well-constructed generator.

THEORY OF THE METHOD.

Let Φ = magnetic flux to be determined.

T = its period.

e = instantaneous emf. induced in secondary coil.

N = turns in secondary coil.

t = time of commutation.

V = reading of voltmeter, to which commutated emf. is applied.

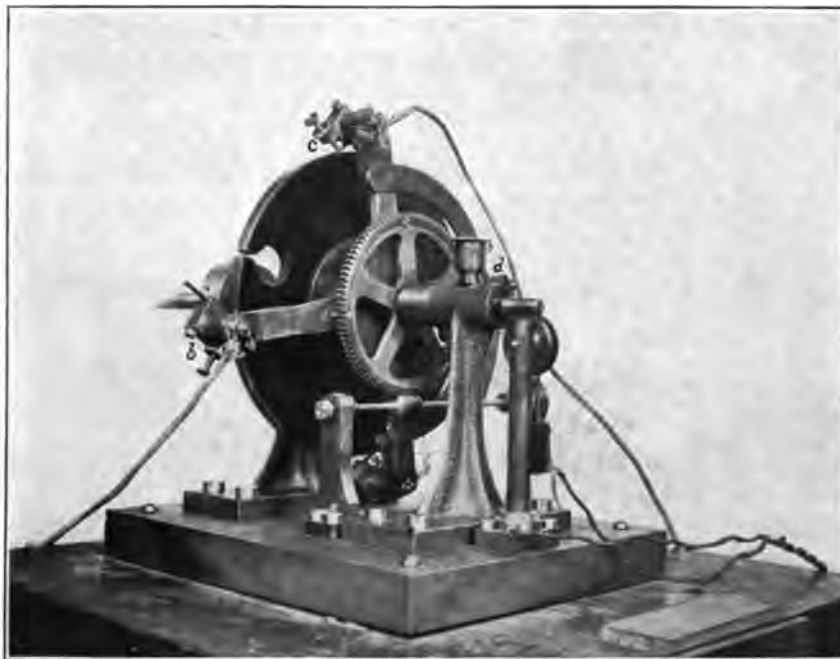


Fig. 1.—*Rotating Commutator with Adjustable Brushes for Tracing Magnetic Waves.*



Then

$$e = -N \frac{d\Phi}{dt}$$

$\Phi = -\frac{1}{N} \int e dt$ is the change in flux during the time expressed by the limits of integration. In the present case the integration is carried on between two reversals or a half period and is repeated during the following half period in the reverse direction.

Hence
$$\Phi_{t+\frac{T}{2}} - \Phi_t = -\frac{1}{N} \int_t^{t+\frac{T}{2}} e dt$$

Now, if positive and negative lobes of the wave are similar,

$$\Phi_{t+\frac{T}{2}} = -\Phi_t \quad \text{Also, } \frac{2}{T} \int_t^{t+\frac{T}{2}} e dt \text{ is the reading of}$$

the voltmeter V . Or $-2\Phi_t = -\frac{VT}{N2}$ and $\Phi_t = \frac{VT}{4N}$

This shows that the value of the magnetic flux at any instant is proportional to the algebraic average of the induced emf. during the succeeding half period. Since the emf. must change sign when the flux passes through a maximum, this maximum will be given by the average value of the emf. during a half period beginning with its zero value. If the wave of magnetic flux has only one maximum during a period, as is usually the case, then the emf. will remain positive during the half period next succeeding the maximum, and the average value of the emf. will be a numerical average. In this case the reading of the voltmeter will give what is commonly known as the "average value of the emf." If to the secondary coil another instrument be directly connected whose indications are proportional to the square of the emf., as a dynamometer or electrometer, this instrument will give the "effective value" of the emf. and the ratio of the two readings will give the form factor.

If, however, the flux wave be so much distorted as to include a dimple between two maxima, the emf. will change sign *during* the half period, the algebraic average will no longer be a numerical

average, and the form factor as determined above will not agree with the customary definition of form factor.

The apparatus can also be used to plot curves of electromotive force or current. For this purpose the emf., or current, is applied to the primary of an air transformer, the secondary of the same being connected through the rotating commutator to the Weston voltmeter, and readings taken as before.

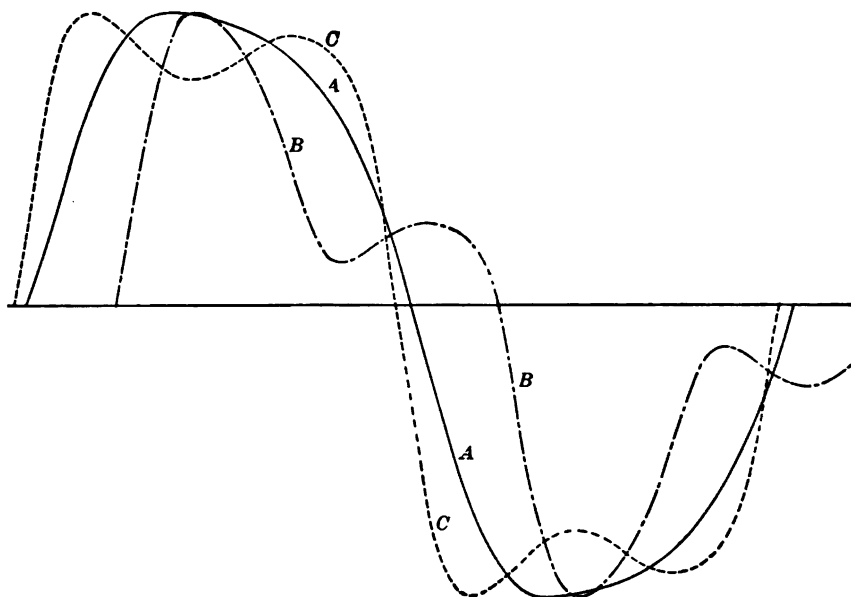


Fig. 2.—Waves of Magnetic Flux. June 5, 1907. $B_{\max}=10,000$ gaussses.
Period = 0.0333 sec.

The principal source of error in the apparatus is in the imperfect contact of the brushes, and the breaking of the contact upon reversing. The contact resistance is a variable and uncertain quantity. It is desirable, therefore, to have a circuit of high resistance, so that the maximum value of the contact resistance is a negligible part of the whole. This was accomplished in the experiments cited by using a very sensitive Weston instrument, giving full scale deflection with about 0.0004 ampere. Resistance sufficiently high to make the inductance error negligible, could be connected in series.

This difficulty can be avoided by using an electrometer as indicating instrument. In this case the two pairs of quadrants are connected to the commutator brushes, while the needle is kept charged to a constant high potential.

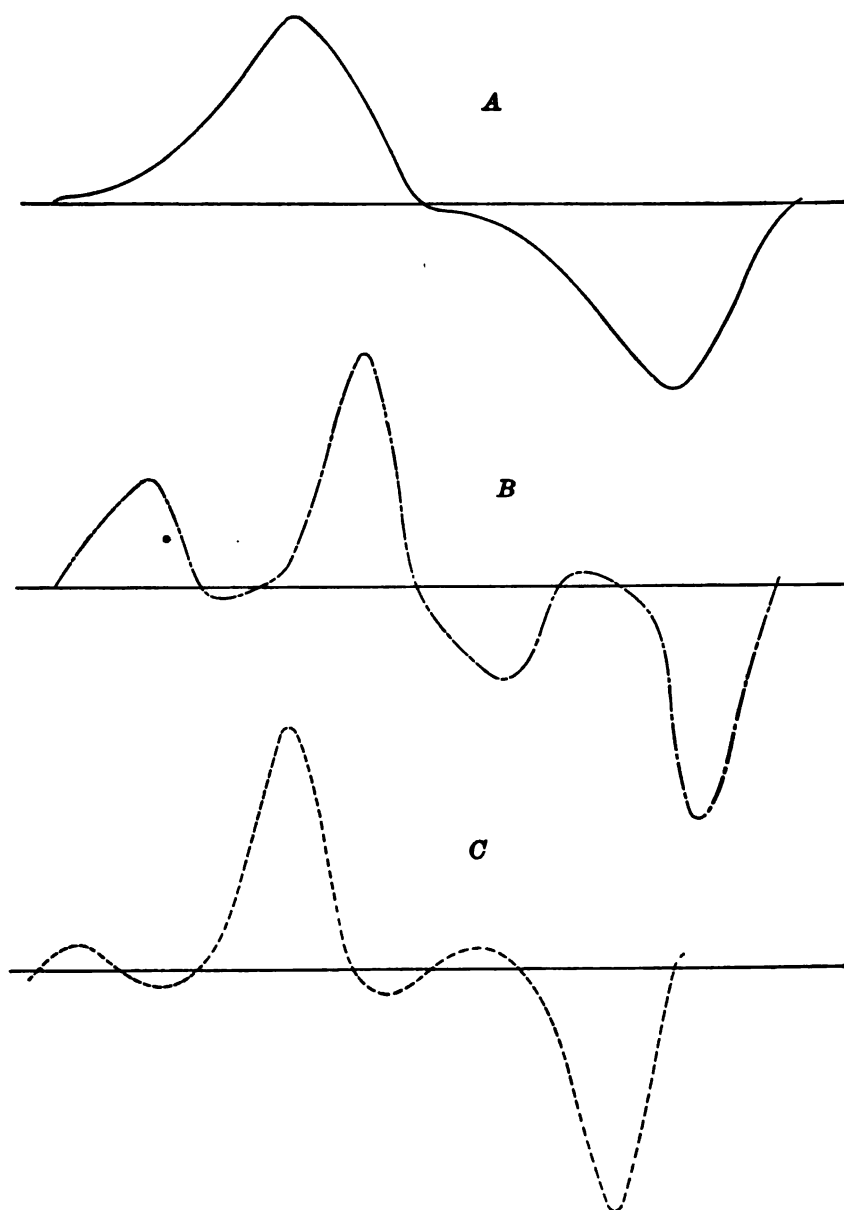


Fig. 3.—Waves of Electromotive Force Corresponding to the Flux Waves of Fig. 2.

If the terminals of the indicating instrument are reversed when the emf. is passing through zero there is no appreciable loss due to reversal; but if the reversal occurs at any other time, the reading will be not quite equal to the integrated value of the emf., since certain elements in the integration are lost during the time the circuit is broken during reversal. If the brush be made wide enough to bridge the gap on the commutator, so that one contact is made before the other is broken, the instrument is short-circuited during the overlap, and again the reading is too low. In the instrument described the breaks occupy 1.4 per cent of the time of a cycle, supposing the brushes to make a line contact. As the brushes are applied tangentially, with pressure, they spring into the air space and make the interval of break much less than this. Its exact value will depend upon the speed and has not been determined. For points near the maximum of the flux curve the elements lost are insignificant, and for points near the zero, where they become important, a larger percentage error may be tolerated.

Error may also arise from unequal spacing of the brushes, but it is not difficult to make this error negligible.

EXAMPLES OF USE.

Several examples of flux waves determined with this apparatus are shown in Fig. 2. The corresponding emf. waves induced in the surrounding coil, as determined with the oscillograph, are given in Fig. 3.

The wave form was altered by taking the magnetizing current from two generators in series, the frequencies being 60 and 180, respectively. The two generators have their shafts coupled together so that the speed and phase relation remains constant during any run. By varying the excitation of the two generators a wide variation in wave form is obtained; by reversing one of them, a dimpled emf. wave is changed to a peaked one, etc. Fig. 4 gives some additional examples of waves of magnetic flux.

In order to test the degree of accuracy obtainable, an iron ring, made up of a number of thin sheets, was used, and the wave form of the flux in the ring plotted out in the manner already described. The coil wound on the ring was then connected through a non-inductive resistance to the primary of a mutual inductance whose secondary in turn was connected to the rotating commutator, and a second set

of readings taken in order to secure the wave form of emf. The readings are given in Table I, the commutator advancing six degrees

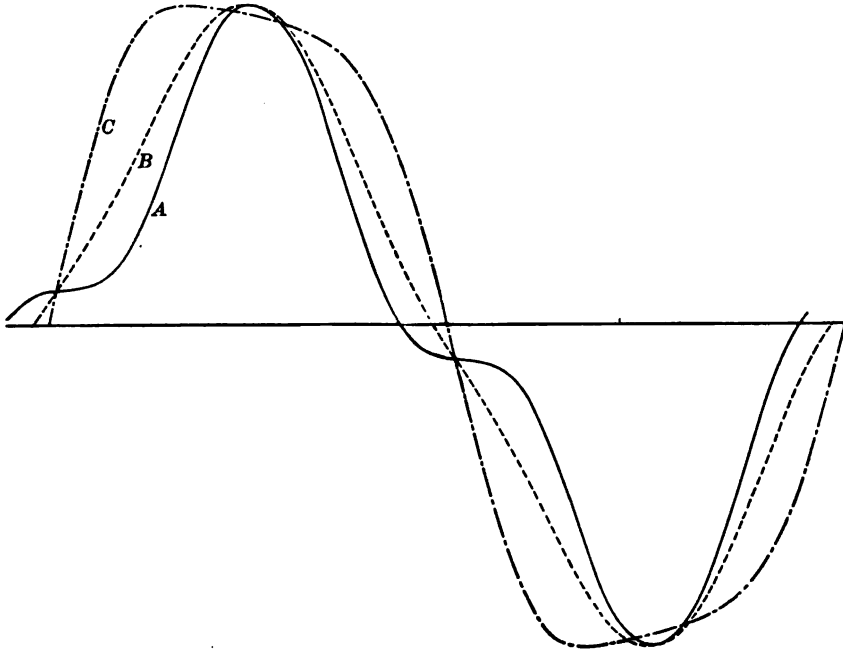


Fig. 4.—Waves of Magnetic Flux. Nov. 11, 1907. Ring No. 31. $B_{\max}=10,000$ gausscs.
Period = 0.0333 sec.

each time; the curves plotted from these 15 equi-spaced ordinates are shown in Fig. 5.

Next each of the waves was analyzed into its component sine waves by a method given by Lyle,¹ and the proportions of the harmonics present, as well as the phase relations, were thus determined.

The constants of the circuits were as follows:

Mutual inductance, 0.0552 henry.

Resistance of primary of same, 212 ohms.

Resistance of secondary with voltmeter, 3560 ohms.

Resistance of voltmeter circuit when connected directly to coil, 33560 ohms.

Turns in coil, 315.

Maximum flux, 23500 maxwells.

Frequency, 30.

¹T. R. Lyle, Phil. Mag., 11, p. 25; 1906.

TABLE I.

Flux		Electromotive Force	
Degrees	Voltmeter Readings	Degrees	Voltmeter Readings
52	-351	-38	-712
46	- 17	-32	-571
40	+306	-26	-300
34	550	-20	- 33
28	689	-14	+104
22	738	- 8	+143
16	742	- 2	+118
10	726	+ 4	+ 39
4	721	10	- 19
- 2	753	16	- 16
- 8	812	22	+ 51
-14	873	28	+181
-20	903	34	+397
-26	843	40	+603
-32	650	46	+719

The equation found for the flux wave is

$$\phi = 944 \sin a + 223 \sin 3 (a - 7^\circ 24') + 14 \sin 5 (a - 14^\circ 3') + 3 \sin 7 (a + 19^\circ 40')$$

and for the emf.

$$e = 430 \sin (a - 90^\circ 20') + 295 \sin 3 (a - 38^\circ 41') + 30 \sin 5 (a - 34^\circ 6') + 4.6 \sin 7 (a + 9^\circ 45')$$

where a is measured from the point where the fundamental of the flux wave crosses the axis, and the amplitudes are in divisions of the voltmeter and involve the constants of the circuits. To find whether the two waves are consistent, we make the amplitudes of the two fundamentals alike and then integrate the emf. wave.

$$\phi = 1000 \sin a + 236 \sin 3 (a - 7^\circ 24') + 15 \sin 5 (a - 14^\circ 3') + 3 \sin 7 (a + 19^\circ 40')$$

$$\begin{aligned}
 e &= 1000 \sin (a-90^{\circ}20') + 686 \sin 3 (a-38^{\circ}41') \\
 &\quad + 70 \sin 5 (a-34^{\circ}6') + 11 \sin 7 (a+9^{\circ}45') \\
 - \int e \, da &= 1000 \cos (a-90^{\circ}20') + 229 \cos 3 (a-38^{\circ}41') \\
 &\quad + 14 \cos 5 (a-34^{\circ}6') + 2 \cos 7 (a+9^{\circ}45') \\
 &= 1000 \sin (a-0^{\circ}20') + 229 \sin 3 (a-8^{\circ}41') \\
 &\quad + 14 \sin 5 (a-16^{\circ}6') + 2 \sin 7 (a+22^{\circ}36')
 \end{aligned}$$

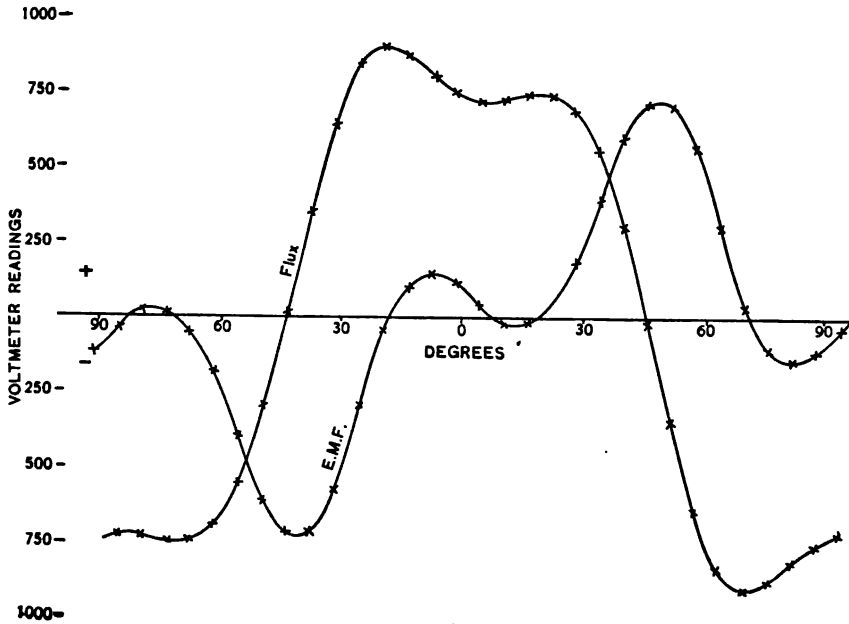


Fig. 5.

The accuracy of determination of the phase angle decreases as the amplitude decreases, so that it may be in error several degrees with the higher harmonics. This is especially true where only fifteen ordinates are used. The discrepancy in phase for the fundamental is only $20'$, and if the fundamentals were brought into agreement, it would be less than one degree for the third harmonic. The principal discrepancy to be noted is in the amplitude of the third harmonic, amounting to seven units, or 0.7 per cent of the amplitude of the fundamental. An accuracy of better than one per cent is not to be expected from the present apparatus, as the brushes can be set only to the nearest 0.1° , and on a steep part of the curve this may cause an error in the reading of an ordinate, which would amount to one per cent of the maximum amplitude. In an

apparatus intended primarily for the accurate plotting of waves, means should be provided for a more accurate setting of the brushes.

Forty-five ordinates are read when greater accuracy is sought. The third, fifth, ninth, and fifteenth harmonics are then easily separated, and the seventh and any higher ones desired are afterward found in the usual way.

COMPARISON WITH PREVIOUS METHODS.

The first recorded waves of magnetic flux were computed by mechanical integration of the curve of secondary emf.

Sahulka³ in 1898 described an apparatus similar to the present and based upon the same principle. He utilized, however, only one-half of the cycle, and his circuit was open seven-eighths of the time, and hence he did not obtain such steady readings on the voltmeter. This becomes of importance at low frequencies. Townsend⁴ constructed a similar apparatus, utilizing half of each wave.

Lyle⁵ has also utilized the same principle in a wave tracer, which may be used to determine the harmonics separately, as well as the total wave. He uses carbon brushes, and while no dimensions are given, the illustrations would indicate that the errors due to losing certain elements in the integration might be large. This source of error is not considered by him.

To Blondel⁵ has been ascribed the idea of using an oscillograph whose galvanometer needle should have no control and no damping. The second differential of its angular position with respect to time would then be proportional to the instantaneous current. A coil surrounding the flux to be measured is connected to the primary of a transformer with air core, the secondary being connected to the galvanometer. In this way a current is secured which is proportional to the second differential of the flux with respect to time.

We have been unable to find any published description of such an apparatus or of its performance.

WASHINGTON, November 15, 1907.

³J. Sahulka. *Zs. f. Elektrotechnik*, 16, p. 4; 1898.

⁴F. Townsend. *American Inst. Electrical Engineers, Trans.* 17, p. 5; 1900.

⁵T. R. Lyle. *Phil. Mag.*, 6, p. 549; 1903.

⁶*Electrician*, 49, p. 172; 1902.

EFFECT OF WAVE FORM UPON THE IRON LOSSES IN TRANSFORMERS.

By Morton G. Lloyd.

It has long been known that the iron loss in a static transformer is dependent upon the form of the wave of electrical pressure which is applied to its primary winding, since the form of this wave, with the constants of the transformer, determines the wave of magnetic flux produced in the iron core. In 1895 Dr. Roessler¹ read a paper before the Verband Deutscher Electrotechniker in which he presented the results of experiments, using two generators whose curves of electromotive force were quite different. With the same emf. applied to the transformer, the iron losses were decidedly less with a peaked emf. wave, corresponding to a flat wave of magnetic flux. He also pointed out that when the form of wave is not a sine curve, it is necessary to modify the formula of Steinmetz for iron losses by introducing the form factor of the emf. wave.

As the Bureau of Standards possesses facilities for supplying emf. waves of any desired form, the present investigation was undertaken, at the suggestion of Dr. E. B. Rosa, for the purpose of shedding additional light upon this question. The problem is first considered from the theoretical standpoint, and the results of experiments are given in which various wave forms were used with commercial types of transformers. The wave forms were obtained by using a series of generators² giving approximately sine waves of frequencies 60, 180, 300, 420, 540, 660, 780, and 900 cycles per second when running at normal speed. The phase relations of the several generators are adjustable, and as they are mounted upon a single shaft, remain

¹G. Roessler. *Electrician*, **36**, p. 124; 1895.

²A full description of this set of generators will be given in a subsequent issue of this Bulletin.

invariable during an experiment. By connecting these generators in series and exciting each to the proper voltage, any desired wave form which involves only the odd harmonics up to the fifteenth may be procured.

1. THEORETICAL.

Let W = power expended in the core of a transformer.

W_H = power expended in hysteresis.

W_E = power expended in eddy currents.

B = maximum flux density in the iron, the flux being assumed uniform.

η and ζ be constants depending upon the iron.

q = ratio of hysteresis to total iron loss.

$n = \frac{1}{T}$ = frequency of applied emf.

f = form factor of induced emf.

e = instantaneous value of induced emf.

E = effective value of induced emf.

E = average value of induced emf. for a half period.

A = amplitude of fundamental component of induced emf.

h = ratio of amplitude of harmonic to amplitude of fundamental.

m = order of harmonic.

θ = phase angle of harmonic.

t = time since fundamental passed through value zero in positive direction.

t_1 = time at which $e=0$.

$\phi = \frac{2\pi t}{T}$

$\phi_1 = \frac{2\pi t_1}{T}$

The subscript zero applies to the case of a sine wave.

DEFINITIONS.

1. The effective value of a periodically varying quantity is the square root of the average value of the square of that quantity.

2. The form factor of a periodically varying quantity is the ratio of its effective value to the algebraic average value during the half period from t_1 to $t_1 + \frac{T}{2}$.

We assume the iron losses in the transformer to be represented by the modified form of the Steinmetz formula,

$$\begin{aligned} W &= W_H + W_E \\ &= \eta n B^{1.6} + \xi f^2 n^2 B^2 \end{aligned}$$

The induced voltage in the primary winding of a transformer differs from the applied voltage by the amount of the ohmic drop, or difference of potential due to resistance. As the ohmic drop is small, and as consideration of it introduces difficulties, we shall consider only the induced voltage. Commercial voltmeters measure the effective value of the voltage. We assume consequently that with different wave forms the effective value of the induced voltage is the quantity kept constant. This approximates closely to practical conditions.

Since the induced voltage depends upon the rate of change of the magnetic flux, its average value at a definite frequency is proportional to B , and its effective value to fB . If the effective value be kept constant, the term in the above formula representing eddy currents will be constant at any definite frequency. The only change in W will arise from an alteration in B , which must accompany any alteration in f .

$$\begin{aligned} W_0 &= W_{H_0} + W_E = \frac{W_{H_0}}{q} \\ \frac{W - W_0}{W_0} &= \frac{W_H - W_{H_0}}{W_0} = q \left(\frac{W_H}{W_{H_0}} - 1 \right) = q \left[\left(\frac{B}{B_0} \right)^{1.6} - 1 \right] \\ &= q \left[\left(1 + \frac{B - B_0}{B_0} \right)^{1.6} - 1 \right] \end{aligned}$$

If $B - B_0$ be small in comparison with B_0 , this may be written

$$1.6 q \frac{B - B_0}{B_0}$$

If B and q be known, the change in power may be calculated.

q may be determined approximately by runs at two frequencies, using sine waves and voltages proportional to the frequencies. One frequency n_1 and voltage should be those used with the varying

wave forms. Under these conditions the total flux remains constant and B is constant. Hence

$$W_1 = \eta n_1 B^{1.6} + \zeta n_1^2 f^2 B^2$$

$$W_2 = \eta n_2 B^{1.6} + \zeta n_2^2 f^2 B^2$$

and by subtraction, after dividing by frequency,

$$\zeta f^2 B^2 = \frac{\frac{W_1}{n_1} - \frac{W_2}{n_2}}{n_1 - n_2}$$

from which

$$q = \frac{n_1 \frac{W_2}{n_2} - n_2 \frac{W_1}{n_1}}{(n_1 - n_2) \frac{W_1}{n_1}}$$

It remains to determine B for various forms of the emf. wave. B is proportional to the average value of the induced voltage during the time in which the flux changes from a positive maximum to a negative maximum. We assume the positive and negative halves of the wave to be alike. This is true of the emf. wave of any well-designed and well-constructed generator, which will therefore contain only the odd harmonics of its fundamental.

Any such emf. may be represented by the equation

$$\begin{aligned} e = & A \sin \frac{2\pi t}{T} + Ah_3 \sin \left(\frac{6\pi t}{T} + \theta_3 \right) \\ & + Ah_5 \sin \left(\frac{10\pi t}{T} + \theta_5 \right) + Ah_7 \sin \left(\frac{14\pi t}{T} + \theta_7 \right) \\ & + \dots \end{aligned}$$

$$E^2 = \frac{2}{T} \int_0^T e^2 dt = \frac{A^2}{2} \left(1 + h_3^2 + h_5^2 + h_7^2 + \dots \right)$$

$$E = \frac{A}{\sqrt{2}} \sqrt{1 + h_3^2 + h_5^2 + h_7^2 + \dots}$$

In our case E is to remain constant. For the sine wave $E = \frac{A_0}{\sqrt{2}}$

Hence
$$\frac{A}{A_0} = \frac{1}{\sqrt{1 + h_3^2 + h_5^2 + \dots}}$$

$$\begin{aligned} E &= \frac{2}{T} \int_{t_1}^{t_1 + \frac{T}{2}} e dt = \frac{1}{\pi} \int_{\phi_1}^{\phi_1 + \pi} e d\phi \\ &= \frac{1}{\pi} \int_{\phi_1}^{\phi_1 + \pi} A \sin \phi d\phi + \frac{1}{\pi} \int_{\phi_1}^{\phi_1 + \pi} A h_3 \sin (3\phi + \theta_3) d\phi \\ &\quad + \frac{1}{\pi} \int_{\phi_1}^{\phi_1 + \pi} A h_5 \sin (5\phi + \theta_5) d\phi + \dots \\ &= \frac{2A}{\pi} \left[\cos \phi_1 + \frac{1}{3} h_3 \cos (3\phi_1 + \theta_3) + \frac{1}{5} h_5 \cos (5\phi_1 + \theta_5) + \dots \right] \\ E_0 &= \frac{2A_0}{\pi} \end{aligned}$$

If the harmonics are in phase with the fundamental

$$0 = \theta_3 = \theta_5 = \theta_7 = \text{etc. and } \phi_1 = 0.$$

$$E = \frac{2A}{\pi} \left(1 + \frac{h_3}{3} + \frac{h_5}{5} + \dots \right)$$

$$\frac{B}{B_0} = \frac{E}{E_0} = \frac{A}{A_0} \left(1 + \frac{h_3}{3} + \frac{h_5}{5} + \dots \right) = \frac{1 + \frac{h_3}{3} + \frac{h_5}{5} + \dots}{\sqrt{1 + h_3^2 + h_5^2 + h_7^2 + \dots}}$$

The form factor has the value

$$f = \frac{E}{E_0} = \frac{\pi}{2\sqrt{2}} \frac{\sqrt{1 + h_3^2 + h_5^2 + \dots}}{\left(1 + \frac{h_3}{3} + \frac{h_5}{5} + \dots \right)}$$

and for a sine wave

$$f_0 = \frac{E_0}{E_0} = \frac{\pi}{2\sqrt{2}}$$

If any of the harmonics are reversed in phase, $\theta = 180^\circ$, and the corresponding term in the expression for E is negative. This case can be included in the case where $\theta = 0$ by considering negative values for h .

One Harmonic.

Let us consider the case where a single harmonic is present.

$$E = \frac{A}{\sqrt{2}} \sqrt{1+h^2}$$

$$E = \frac{2A}{\pi} \left(\cos \phi_1 + \frac{h}{m} \cos (m\phi_1 + \theta) \right)$$

If the harmonic be in phase with the fundamental

$$E = \frac{2A}{\pi} \left(1 + \frac{h}{m} \right)$$

$$\frac{B}{B_0} = \frac{E}{E_0} = \frac{1 + \frac{h}{m}}{\sqrt{1+h^2}}$$

If h be negative, or the harmonic reversed, the loss will always be less than with a sine wave. This corresponds to a peaked wave for $m=3$, $m=7$, $m=11$, etc.

If h be positive, the loss may be greater or less according as $1 + \frac{h}{m}$ is greater or less than $\sqrt{1+h^2}$, and the loss will be unchanged when these two expressions are equal. For small values of h , $1 + \frac{h}{m}$ is greater, and the loss is increased. With increasing values of h the loss reaches a maximum, decreases again to the value for a sine wave, and then goes lower.

The value of h which leaves the loss unchanged is found by equating the numerator and denominator of the expression for $\frac{B}{B_0}$,

$$1 + \frac{h}{m} = \sqrt{1+h^2}$$

from which

$$h = \frac{2m}{m^2 - 1}$$

For maximum loss the derivative with respect to h will be zero.

$$\frac{\partial \left(\frac{1 + \frac{h}{m}}{\sqrt{1 + h^2}} \right)}{\partial h} = \frac{1}{m} \frac{1}{\sqrt{1 + h^2}} - \left(1 + \frac{h}{m} \right) \frac{h}{(1 + h^2)^{3/2}} = 0$$

which gives

$$h = \frac{1}{m}$$

For large values of m , the maximum occurs for a value of h which is one-half the value for no change, and in any case at approximately one-half.

The values of h which give a maximum loss and which give the same loss as a sine wave, are to be found in the adjoining Table I.

TABLE I.

m	h		Maximum Value of $\left(\frac{E}{E_0} \right)^{1.6}$
	For Maximum Loss	For Unchanged Loss	
3	0.333	0.750	1.088
5	.200	.417	1.032
7	.143	.292	1.016
9	.111	.225	1.010
11	.091	.183	1.007
13	.077	.155	1.005
15	.067	.134	1.0035

The maximum value of the loss depends upon $\left(\frac{E}{E_0} \right)^{1.6}$, which may be obtained by substituting the value of h for the maximum and expanding.

$$\begin{aligned} \left(\frac{E}{E_0} \right)^{1.6} &= \left(\frac{1 + \frac{h}{m}}{\sqrt{1 + h^2}} \right)^{1.6} = \left(\frac{1 + \frac{1}{m^2}}{\sqrt{1 + \frac{1}{m^2}}} \right)^{1.6} = \left(\sqrt{1 + \frac{1}{m^2}} \right)^{1.6} = \left(1 + \frac{1}{m^2} \right)^{0.8} \\ &= 1 + \frac{0.8}{m^2} - \frac{0.08}{m^4} + \frac{0.032}{m^6} - \dots \end{aligned}$$

These values also are given in Table I.

In Fig. 1 the values of $\left(\frac{E}{E_0}\right)^{1.6} - 1$ are plotted for various values of h and for the harmonics from $m=3$ to $m=15$. Two sets of curves are drawn, one for $\theta=0$ and one for $\theta=180^\circ$. An ordinate of one of these curves multiplied by q is the change in iron loss for the given case in terms of the loss for sine wave.

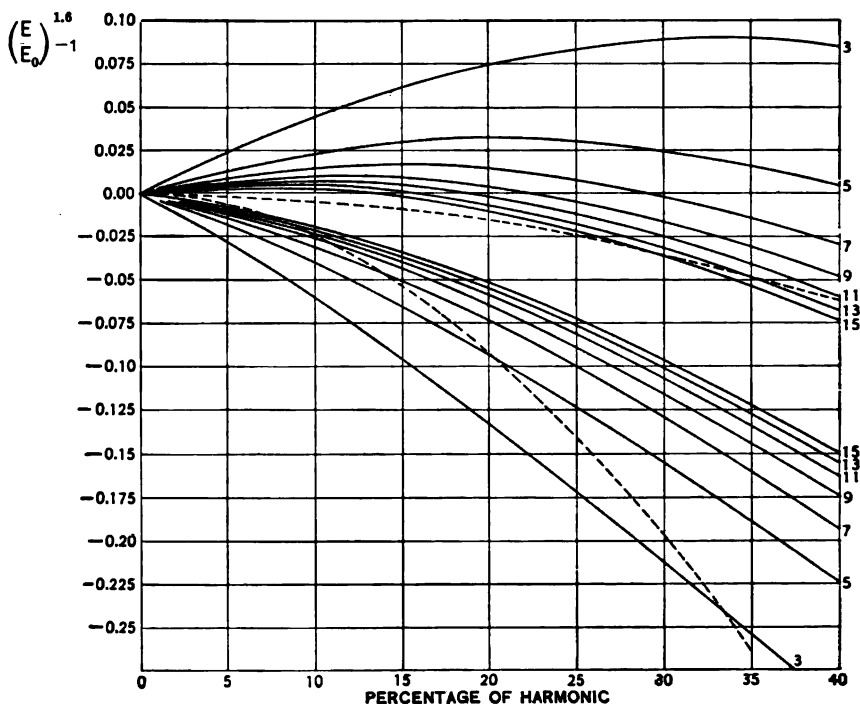


Fig. 1.

If we give h the same value in the case of different harmonics, the loss depends upon m . For $\theta=0$, $\frac{B}{B_0}$ is larger, as m is smaller. But for $\theta=180^\circ$, $\frac{B}{B_0}$ is less, as m is smaller. The two values of $\frac{B}{B_0}$ lie nearer together the higher the order of the harmonic. The curves of Fig. 1 illustrate this very clearly.

If h be proportional to m or $h=km$

$$\frac{B}{B_0} = \frac{1+k}{\sqrt{1+m^2 k^2}}$$

and the loss is less the greater is m . For small values of k the decrease is nearly proportional to m^2 .

If h do not exceed the value for no change in iron loss when $\theta=0$, there will be some value of θ which will leave the loss unchanged; for in changing θ from 0° to 180° the difference in loss changes from an increase to a decrease.

$$\frac{E}{E_0} = \frac{1}{\sqrt{1+h^2}} \left[\cos \phi_1 + \frac{h}{m} \cos (m \phi_1 + \theta) \right]$$

In this case $E = E_0$ and hence

$$\sqrt{1+h^2} = \cos \phi_1 + \frac{h}{m} \cos (m \phi_1 + \theta)$$

By definition $e=0$ when $\phi=\phi_1$ or

$$\sin \phi_1 + h \sin (m \phi_1 + \theta) = 0$$

These two equations are sufficient to determine ϕ_1 and θ . The form of the equations makes it easier to solve for $\cos \phi_1$ or $\cos (m \phi_1 + \theta)$ rather than for $\sin \phi_1$. Since ϕ_1 is a small angle, greater accuracy is attained by solving for $\cos (m \phi_1 + \theta)$.

Let

$$\begin{aligned} \sqrt{1+h^2} &= \mu \\ m \phi_1 + \theta &= v \end{aligned}$$

Then

$$\begin{aligned} \mu &= \cos \phi_1 + \frac{h}{m} \cos v \\ \cos^2 \phi_1 &= \left(\mu - \frac{h}{m} \cos v \right)^2 \end{aligned}$$

Also

$$\begin{aligned} \sin \phi_1 &= -h \sin v \\ 1 - \cos^2 \phi_1 &= h^2 \sin^2 v = h^2 (1 - \cos^2 v) \end{aligned}$$

Eliminating $\cos \phi_1$ and solving for $\cos v$ we get

$$\cos v = \frac{m}{h(m^2-1)} \left[-\mu + \sqrt{h^2(2m^2-1)+1} \right]$$

Substituting numerical values will give v ; ϕ_1 is then obtained from

$$\sin \phi_1 = -h \sin v$$

and finally θ from $m \phi_1 + \theta = v$.

If the proportion of harmonic varies inversely as its order, v is roughly constant.

For let $mh = k$

Then

$$\begin{aligned}\cos v &= \frac{m^3}{k(m^3-1)} \left[-\frac{\sqrt{m^3+k^2}}{m} + \sqrt{\frac{k^3}{m^3}(2m^3-1)+1} \right] \\ &= \frac{1}{k\left(1-\frac{1}{m^3}\right)} \left[-\sqrt{1+\frac{k^2}{m^3}} + \sqrt{2k^3+1-\frac{k^2}{m^3}} \right]\end{aligned}$$

As m increases, the value of the bracket increases and the value of the coefficient decreases, both in small degree; the largest change is in the coefficient, and may amount to about 10 per cent.

We see on inspection also that if h be small, θ is about 90° . For $\theta = v - m\phi_1$ and ϕ_1 is of opposite sign to v , $\cos v$ in this case being a small quantity and v consequently a large angle. The larger is m , the smaller must h be to give the same value of θ .

To illustrate by a numerical example let $m=3$, $h=0.20$

Then

$$\begin{aligned}\cos v &= \frac{3}{1.6}(-1.0198 \pm 1.2962) = 0.51825 \text{ or } -4.34 \\ v &= \pm 58^\circ 47'.5 & \phi_1 &= \mp 9^\circ 51' \\ \theta &= \pm 88^\circ 20'\end{aligned}$$

Again, let $m=5$, $h=0.12$

$$\begin{aligned}\cos v &= \frac{5}{2.88}[-1.0072 \pm 1.3057] = 0.5188 \\ v &= \pm 58^\circ 45' & \phi_1 &= \mp 5^\circ 53'.3 \\ \theta &= \pm 88^\circ 11'.5\end{aligned}$$

For the case of $m=3$ and various values of h , the values of θ have been worked out as shown in Table II and plotted in Fig. 2. This curve shows how θ decreases from 90° to 0° as h increases from zero to the limiting value given in Table I. If h exceed this value, the loss will be decreased for every value of θ . The curve resembles closely an ellipse, but is somewhat fuller than either an ellipse or a cycloid with the same semiaxes. The curve has no simple equation, as may be seen by reference to the expressions used in calculating its coordinates.

Other curves have been plotted in Fig. 2, each connecting points representing the same loss, the value of the loss being different for

each locus. Each line represents a change of two per cent in the value of $\left(\frac{E}{E_0}\right)^{1.6}$; the lines inside the "no-change" curve representing increased loss, those outside decreased loss.

TABLE II.

Calculation of phase angle for unchanged loss with third harmonic.

h	μ	$\cos v$	v	$\sin v$	$\sin \phi_1$	ϕ_1	θ
0.0	1.0000	0.0000	90° 0'	1.0000	0.0000	0° 0'	90° 0'
0.1	1.0050	0.2876	73° 17'	0.9577	0.0958	5° 30'	89° 47'
0.2	1.0198	0.5182	58° 48'	0.8553	0.1711	9° 51'	88° 20'
0.3	1.0440	0.6832	46° 54'	0.7302	0.2191	12° 39'	84° 51'
0.4	1.0770	0.7985	37° 1'	0.6020	0.2408	13° 56'	78° 49'
0.5	1.1180	0.8800	28° 22'	0.4750	0.2375	13° 44'	69° 34'
0.55	1.1413	0.9117	24° 16'	0.4108	0.2260	13° 4'	63° 27'
0.6	1.1662	0.9388	20° 9'	0.3445	0.2087	11° 56'	55° 57'
0.65	1.1927	0.9622	15° 48'	0.2723	0.1770	10° 12'	46° 24'
0.7	1.2207	0.9824	10° 46'	0.1869	0.1308	7° 31'	33° 19'
0.75	1.2500	1.0000	0° 0'	0.0000	0.0000	0° 0'	0° 0'

So long as the emf. passes through the zero value only twice per cycle, there is little difficulty in determining the value of ϕ_1 . It is only necessary to find where $e=0$. If, however, the emf. cross the axis more than this number of times, it becomes necessary to discriminate between the different values of ϕ for which $e=0$. This takes place only in cases where there are secondary maxima in the flux curve, and it is of course the highest maximum which must be located. For any half wave of emf. the number of crossings will be an odd number, and they are alternately in the same direction as the fundamental wave and in the opposite direction. Those in the opposite direction represent minima, and this class includes the case $\phi=0$ for $\theta=180^\circ$ with a large component of harmonic. In the previous work we have taken $\phi_1=0$ for $\theta=180^\circ$, but this holds only when there is but one value of ϕ for which $e=0$ in that half wave. The limiting values of h to avoid secondary maxima in the wave of magnetic flux are given in Table III for $\theta=0$ and $\theta=180^\circ$. For other values of θ , the limiting values of h may be taken from the curves of Fig. 3.



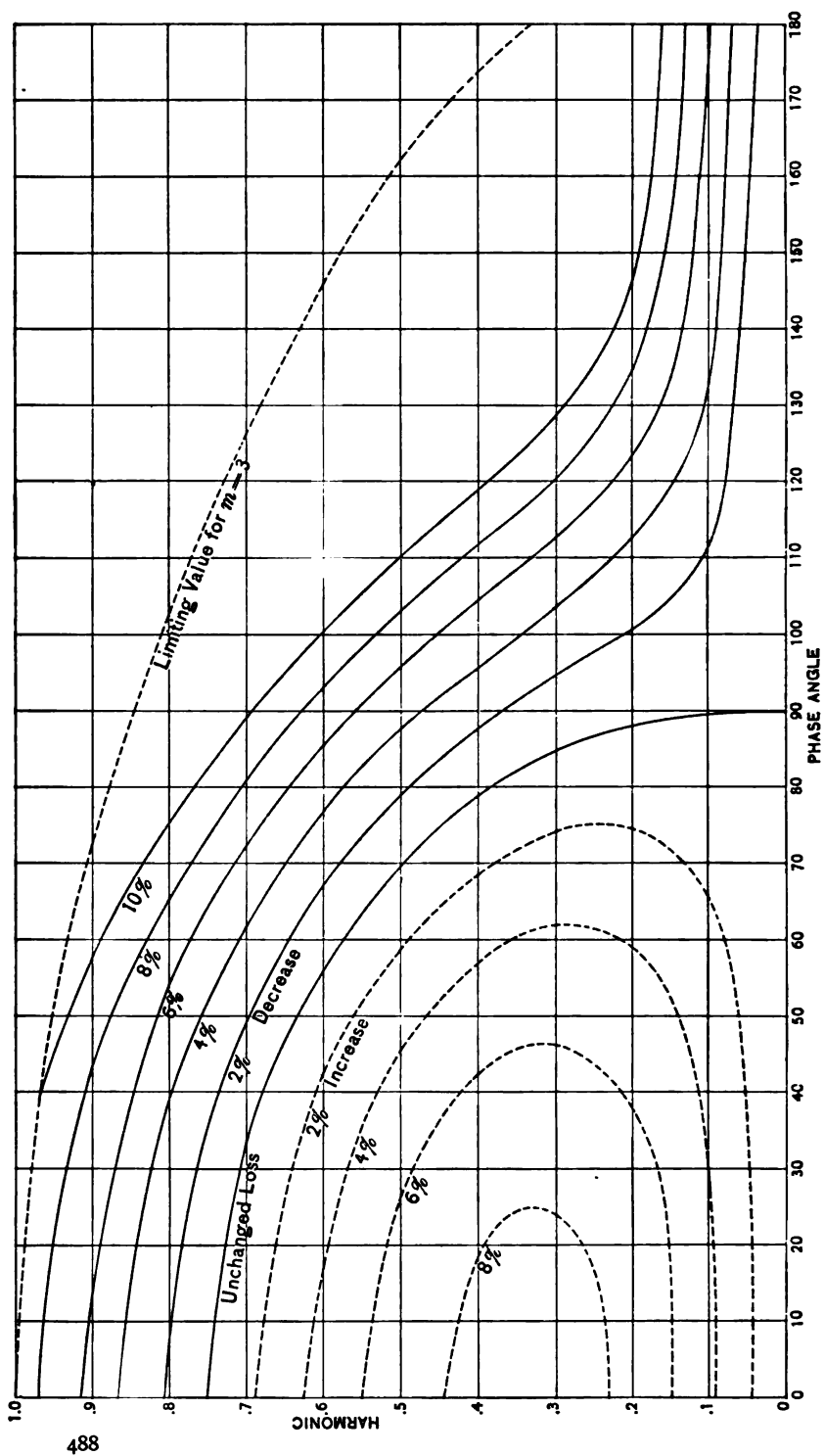


Fig. 2.—Showing the change in hysteresis loss for the third harmonic, w th different percentages and phase angles.

TABLE III.

m	Limiting Values of h	
	$\theta = 0^\circ$	$\theta = 180^\circ$
3	1.000	0.333
5	.800	.200
7	.613	.143
9	.490	.111
11	.407	.091
13	.347	.077
15	.302	.067

It will be noticed that the limiting values of h for $\theta = 180^\circ$ are the same values which give a maximum loss when $\theta = 0$; viz, $h = \frac{1}{m}$.

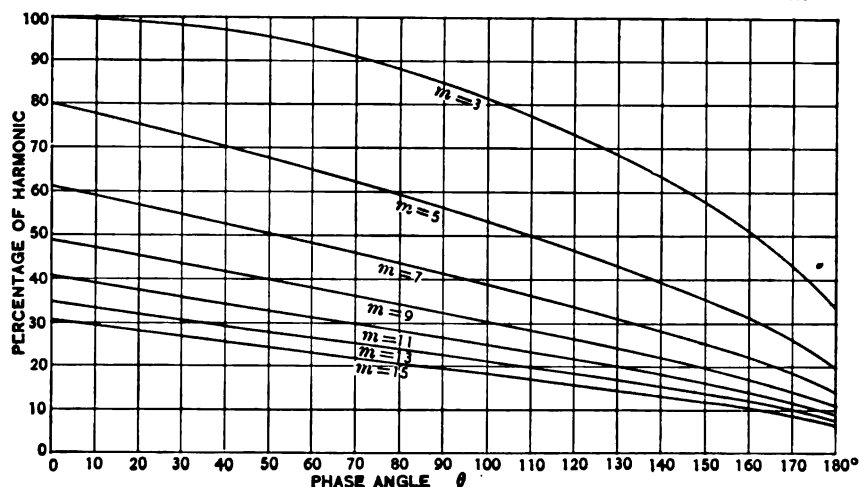


Fig. 3. —Limiting Values of Harmonic.

The value of ϕ_1 is to be chosen from those values of ϕ for which the emf. curve crosses the axis in the same direction as the fundamental, and that value of ϕ should be taken which will give the largest average emf. If the curve of the emf. has been plotted, it can often be seen from inspection which is the desired value. Thus in Fig. 4 the average should be taken between a and d , which gives two larger lobes positive and the smallest lobe negative, since it is obvious that this will be larger than the average between c and

f , which includes a negative lobe of medium size. The average between b and e is yet smaller and represents a minimum in the curve of magnetic flux.

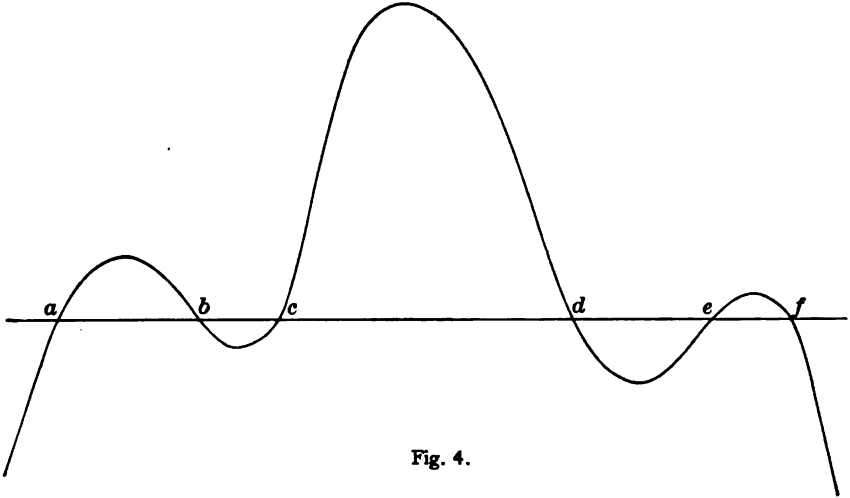


Fig. 4.

In the case of a large component of high harmonic the value of ϕ_1 can be determined approximately from inspection. The harmonic will pass through zero every time ϕ changes by $\frac{\pi}{m}$, and one of these zero values will be at $-\frac{\theta}{m} = \phi$. This will be very close to the value of ϕ for which $e=0$ and for which $\phi=\phi_1$, but will be slightly further from $\phi=0$. This results because the fundamental has relatively small values in this neighborhood and the harmonic is changing very rapidly.

We may say then

$$\begin{aligned} E &= \frac{2A}{\pi} \left[\cos \frac{\theta}{m} + \frac{h}{m} \cos \left(-m \frac{\theta}{m} + \theta \right) \right] \\ &= \frac{2A}{\pi} \left[\cos \frac{\theta}{m} + \frac{h}{m} \right] \end{aligned}$$

The extreme values of E occur for $\theta=0$ and $\theta=180^\circ$, and this approximation includes the extreme values of the true E . ϕ_1 is always zero for $\theta=0$, and in that case

$$E = \frac{2A}{\pi} \left(1 + \frac{h}{m} \right)$$

as in the original derivation. The range of variation is from 1 to $\cos \frac{\theta}{m}$, and this is smaller, the larger is m .

If we let $m=15$, $\theta=180^\circ$

$$E = \frac{2A}{\pi} \left(\cos 12^\circ + \frac{h}{m} \right) = \frac{2A}{\pi} \left(0.978 + \frac{h}{m} \right)$$

or the range of variation in E is about 2.2 per cent.

If $m=13$ $\theta=180^\circ$

$$E = \frac{2A}{\pi} \left(\cos 13^\circ 51' + \frac{h}{m} \right) = \frac{2A}{\pi} \left(0.971 + \frac{h}{m} \right)$$

a range of nearly three per cent.

We see, then, that with a large percentage of high harmonic the loss is always decreased, and the amount of decrease varies only slightly with the phase angle.

There is another effect which must be considered when the emf. passes through zero more than once per half cycle, that is, when the wave of magnetic flux has more than one maximum. We have assumed that the hysteresis is proportional to the 1.6th power of the maximum flux density. The assumption is at least approximately true when the iron is put through a simple cycle, but no validity is claimed for it in cases where the magnetic flux does not progress continuously from a positive maximum to a negative maximum. If there be more than one maximum to the half cycle, there will be an extra or secondary loop in the hysteresis curve, and greater power will be consumed. The formula worked out does not strictly apply to such cases. A quantitative examination of actual cases shows, however, that this extra power will seldom amount to as much as one per cent of the whole, and consequently may usually be neglected. Only when one of the harmonics enters to the same order of magnitude as the fundamental does this source of error become important. Thus with fifty per cent of the fifth harmonic the extra loop is through a range of only four per cent of B , and this would not add more than one per cent to the hysteresis loss.

Figure 1 is based upon values of E taken for $\phi_1=0$, and consequently the curves do not represent $\left(\frac{B}{B_0} \right)^{1.6}$, except within the limits

given by Table III. These limits are shown on the figure by the dotted lines. For values of h only slightly in excess of the limiting values, however, the effect upon E and the extra loops of hysteresis are both negligible, and the curves may be used somewhat outside of these limits.

For higher values of h the actual loss would be greater than that calculated by use of the curves, and the discrepancies will be greatest for $\theta = 180^\circ$.

Numerical Example with one Harmonic.

When q has been determined for a given frequency, the change in iron loss may be determined for $\theta = 0^\circ$ or $\theta = 180^\circ$ by reference to the curves of Fig. 1. For other values of θ the value of ϕ_1 must be determined before B can be calculated. This can be done by a graphical method, or by successive trial and calculation.

Thus let $m = 3$, $h = 0.20$, $q = 0.60$

For $\theta = 0$, or a flat voltage wave,

$$\left(\frac{1 + \frac{h}{m}}{\sqrt{1 + h^2}} \right)^{1.6} = 1.075$$

$$\frac{W - W_0}{W_0} = 0.045$$

an increase of 4.5 per cent.

For $\theta = 180^\circ$, or peaked voltage wave

$$\left(\frac{1 - \frac{h}{m}}{\sqrt{1 + h^2}} \right)^{1.6} = 0.867$$

$$\frac{W - W_0}{W_0} = -0.080$$

a decrease of 8.0 per cent.

The results for intermediate values of θ are given in Table IV.

TABLE IV.

θ	ϕ_1	$S = \frac{\pi E}{2A}$	$\left(\frac{S}{\sqrt{1.04}}\right)^{1.4}$	$\frac{W-W_0}{W_0}$ in Per Cent
0°	0°	1.067	1.075	+ 4.5
± 30	∓ 3.7	1.061	1.065	+ 3.9
± 60	∓ 7.15	1.044	1.038	+ 2.3
± 90	∓ 10.0	1.018	0.997	- 0.2
± 120	∓ 11.5	0.985	0.946	- 3.2
± 150	∓ 10.0	0.951	0.895	- 6.3
± 180	0.	0.933	0.867	- 8.0

Two Harmonics.

In this case

$$E = \frac{2A}{\pi} \left[\cos \phi_1 + \frac{h_1}{m_1} \cos(m_1 \phi_1 + \theta_1) + \frac{h_2}{m_2} \cos(m_2 \phi_1 + \theta_2) \right]$$

$$\frac{A}{A_0} = \frac{1}{\sqrt{1 + h_1^2 + h_2^2}}$$

If the harmonics be not in phase with the fundamental, any special numerical case can be worked out by finding the value of ϕ_1 corresponding to definite values of θ_1 and θ_2 .

If the harmonics be assumed in phase with the fundamental, $\phi_1 = 0$ and

$$E = \frac{2A}{\pi} \left(1 + \frac{h_1}{m_1} + \frac{h_2}{m_2} \right)$$

$$\frac{B}{B_0} = \frac{1 + \frac{h_1}{m_1} + \frac{h_2}{m_2}}{\sqrt{1 + h_1^2 + h_2^2}}$$

If we let $h_1 = h_2$

$$\frac{B}{B_0} = \frac{1 + h \left(\frac{1}{m_1} + \frac{1}{m_2} \right)}{\sqrt{1 + 2h^2}}$$

and there will be some value of h which will leave the loss unchanged.

Let $m_1 = 3$, $m_2 = 5$, then

$$1 + h \left(\frac{1}{3} + \frac{1}{5} \right) = \sqrt{1 + 2h^2}$$

from which $h = 0.622$.

Again, if the fifth harmonic be reversed in phase

$$1 + h\left(\frac{1}{3} - \frac{1}{5}\right) = \sqrt{1 + 2h^2}$$

and $h = 0.1346$.

Again, we may assign a definite value to h_1 and find the value of h_2 which leaves the loss unchanged. Let $h_1 = 0.2$, $m_1 = 3$, $m_2 = 5$.

$$1 + \frac{0.2}{3} + \frac{h_2}{5} = \sqrt{1 + 0.04 + h_2^2}$$

from which $h = 0.611$ or -0.167 ,

indicating that the fifth harmonic may be introduced in reversed phase with magnitude one-sixth of the fundamental, or in same phase with magnitude 61 per cent of the fundamental.

Several Harmonics.

Going back to the general expression for E , we have

$$\frac{B}{B_0} = \frac{\left[\cos \phi_1 + \frac{h_3}{3} \cos(3\phi_1 + \theta_3) + \frac{h_5}{5} \cos(5\phi_1 + \theta_5) + \dots \right]}{\sqrt{1 + h_3^2 + h_5^2 + h_7^2 + \dots}}$$

The numerator of this expression may be either greater or less than unity, depending upon the values of θ_3 , θ_5 , etc. The denominator is independent of the phase relations and is never less than unity. We may consider the influence of the two factors separately. The denominator always tends to decrease the loss; for small values of h by an amount proportional to h^2 and for larger values by an ever decreasing proportion. Superposed upon this decrease is the effect of the numerator. When the numerator is greater than unity, it may overpower the denominator for small values of h , but can not do so for the larger values. If a number of harmonics are introduced at random, the probability is that the loss will be decreased, and the greater the number of harmonics, the greater the probable decrease. For with various phase angles some of the terms in the numerator will counteract others, while in the denominator all work together for a decrease.

In special cases the higher harmonics may be present in greater magnitude than the lower ones, but more often their amplitude is

small, and we will consider the case where h is inversely proportional to the order of the harmonic.

Let $k = 3h, 5h, 7h$, etc., $\phi_1 = 0$, $\theta = 0$ or 180° .

Then

$$\frac{B}{B_0} = \frac{1 + k \left(\pm \frac{1}{3^2} \pm \frac{1}{5^2} \pm \frac{1}{7^2} \pm \dots \right)}{\sqrt{1 + k^2 \left(\frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \dots \right)}}$$

If $k < 1$, an accuracy of one per cent will be attained by disregarding harmonics above the ninth. If each θ be zero,

$$\frac{B}{B_0} = \frac{1 + k \left(\frac{1}{9} + \frac{1}{25} + \frac{1}{49} + \dots \right)}{\sqrt{1 + k^2 \left(\frac{1}{9} + \frac{1}{25} + \frac{1}{49} + \dots \right)}}$$

If in this case we let $k = 0.5$, $q = 0.6$

$$\frac{B}{B_0} = 1.074 \quad \frac{W - W_0}{W_0} = 0.071$$

It is readily seen that the lower harmonics and especially the third will be predominant, unless it be introduced with θ near 90° , and even then its influence in the denominator is unimpaired.

It is to be noticed that the expression for $\frac{B}{B_0}$ also represents the ratio $\frac{f_0}{f}$; in other words, an increase or decrease in loss depends upon whether the form factor of the wave of voltage is less or greater than the form factor of a sine wave. In the special cases previously considered, where the loss is unchanged, the form factor has the same value as for a sine wave, namely, $\frac{\pi}{2\sqrt{2}} = 1.1107$.

We may express the percentage change in loss as

$\frac{W - W_0}{W_0} = q \left[\left(\frac{f_0}{f} \right)^{1.6} - 1 \right]$ or $1.6 q \frac{f_0 - f}{f}$ when $f_0 - f$ is small compared to f . It is not necessary, therefore, to know the components of the wave used in order to calculate the loss. *It is sufficient to determine the form factor.* This can always be done if apparatus is

available for tracing the wave³, and the more laborious process of analyzing the wave may be omitted.

If we define a "peaked" wave to be one whose form factor is greater than 1.1107 and a "flat" wave to be one whose form factor is less than 1.1107, then we may make the general statement that a peaked emf. wave reduces the iron loss of a transformer and a flat wave increases the iron loss. If we have a third harmonic combined with the fundamental, the peaked wave occurs when $\theta = 180^\circ$ or with the maxima of the two components coinciding in time. With the fifth harmonic we again have a peaked wave for $\theta = 180^\circ$, but in this case the positive maximum of the fundamental is coincident with the negative maximum of the harmonic, and from mere inspection

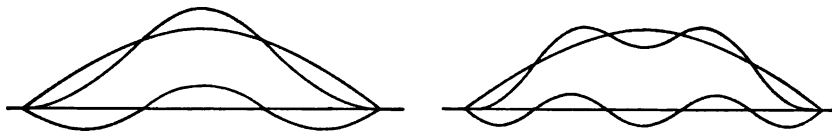


Fig. 5.

the curve would hardly be classed as peaked. These two cases are illustrated in Fig. 5. It is not desirable, consequently, to define a "peaked" wave as one having coincident maxima for fundamental and harmonic, for then "peaked" and "flat" would cease to be discriminating terms. Defining the wave by means of the form factor, its nature is determined largely by the value of θ , and if a single harmonic be present, the wave will always be peaked when $90^\circ < \theta < 180^\circ$ and will usually be flat when θ is near zero. This can usually be judged upon inspection by the steepness of the curve where it passes through zero.

2. DESCRIPTION OF APPARATUS.

The power expended in iron losses was measured by means of a dynamometer wattmeter.⁴ The fixed coils are in series connection with the primary winding of the transformer; the movable coils are connected, with a suitable multiplier, to the terminals of the

³The form factor can also be determined directly if apparatus is available for measuring the average value of the emf. Such an apparatus is described by Lloyd and Fisher, this Bulletin, p. 501, and by Rose and Kühns, E. T. Z., 24, p. 992; 1903.

⁴This instrument is of the type described in this Bulletin, 1, p. 424, by Rosa, Lloyd, and Reid.

secondary winding. If the secondary contain the same number of turns as the primary, and enclose the same magnetic flux, the wattmeter then measures the energy supplied to the core and to the secondary circuit or circuits. If the turns be not the same in number, their ratio gives the factor connecting the wattmeter reading with the true watts. As only relative values are desired in the present investigation, it is unnecessary to know this factor. This method has an advantage over the more usual method of applying the primary voltage to the potential circuit of the wattmeter, since the copper loss in the primary is not included in the measurement. Usually this must be determined and applied as a correction.

To get the iron losses alone, a correction must be applied for the energy supplied to the secondary circuits. The energy expended in a noninductive secondary is equal to the square of the effective voltage induced in that secondary divided by the resistance of the secondary circuit. In the present work this correction is small and constant, and may be neglected in considering relative losses.

The induced voltage was measured by a dynamometer voltmeter which in most cases was connected to a secondary winding. This instrument measures the effective voltage and its deflection was kept constant during a series of runs. Its inductance was known, and the effect of high frequency currents was compensated by slight changes in its noninductive multiplier. In some of the runs this voltmeter was connected across the primary terminals, notably when the secondary voltage was low and the component of high harmonic large, as in these cases the voltmeter correction would otherwise become very large. This method of connection keeps the *applied* effective voltage constant and thus more nearly fulfils the conditions of commercial operation. It does not keep the *induced* voltage so constant, however, and thus does not so nearly realize the conditions assumed in the theoretical deductions. In the cases where both methods of connection were used in turn, the difference in the results was less than one per cent.

A Thomson ammeter of the magnetic vane type was inserted in the primary circuit and readings of the magnetizing current were made. No use of these readings is made in the present work. The resistance of this ammeter is 0.4 ohm, of the field circuit of wattmeter 1.7 ohm, a total of 2.1 ohms in series with the primary.

No rheostat was used in this circuit, the applied voltage being controlled through the field excitation of the generators. As the primary current in most cases was about one ampere, the ohmic drop in the instruments was only about two volts.

The connections are shown in the diagram, Fig. 6.

When the transformer used had two secondaries, the voltmeter and potential circuit of wattmeter were connected to separate windings. When there was a single secondary, the two were connected in parallel to its terminals. The deflections of the wattmeter were taken as proportional to the watts, when the multiplier R remains constant. This is true for deflections not differing more than a few

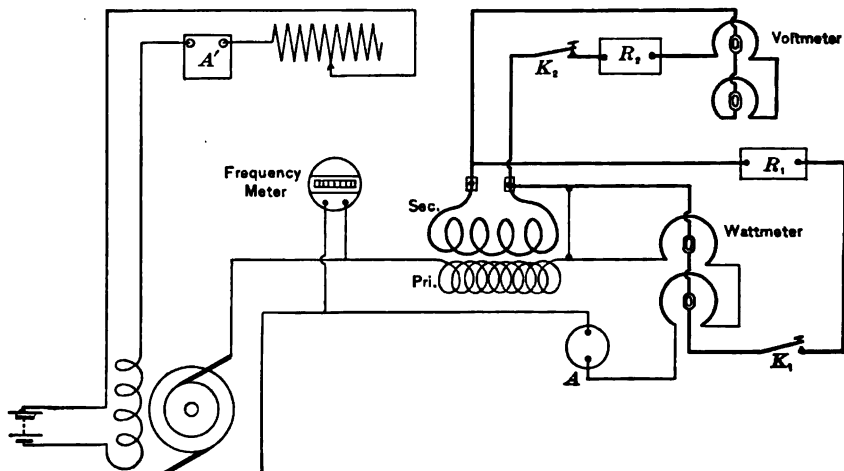


Fig. 6.—Diagram of Connections.

centimeters, as is the case in these measurements. In determining q , the multiplier R was changed to give the same wattmeter deflection on both frequencies, and the watts taken to be proportional to the resistance of the potential circuit. Calibration of the instrument was therefore unnecessary.

The frequency of the generator was measured by a Hartmann-Kempf frequency meter of the new type, containing an indicator for each half cycle. By reference to this instrument the frequency could be read with greater accuracy than $\frac{1}{4}$ cycle, which was considered sufficient. Sixty cycles was the frequency used in the measurements, and the proper speed of the generator was maintained by adjustment of the field resistance of the driving motor.

As this motor was supplied with current from a storage battery, the changes in speed were gradual, and the necessary adjustments slight and infrequent.

The transformers used were:

1. General Electric Co., Type H, No. 290645, 120/4 volts, 60 cycles, 2,000 watts, $q=0.61$.

2. Siemens and Halske No. 6700, 120/2 volts, 50 cycles, 1,600 watts, $q=0.60$.

3. General Electric Co., Type H, No. 290414, 480-240-120/30-60-120 volts, 60 cycles, 500 watts, $q=0.70$.

Transformer No. 1 was operated at 110 volts, and No. 2 at 132 volts, 60 cycles being used throughout. Otherwise the conditions were normal, and the transformers were operated under no load. In this case the ratio of transformation is not influenced by the wave-form.⁵

No. 3. has four separate primary windings and four separate secondaries, which can be connected in series or parallel, thus permitting various ratios of transformation.

3. OBSERVATIONS.

When starting a series of observations, the primary voltage was adjusted to the desired value on sine wave by reference to a calibrated portable voltmeter. The reading of the volt-dynamometer was taken at the same time, and the same deflection was maintained throughout the series by adjustment of rheostat in generator field. Readings were then taken on the wattmeter with sine wave, and followed by readings with other wave forms. At frequent intervals throughout the series the sine wave was again used, and control thus kept of the changes due to heating. If current be kept on, the iron loss continually decreases on account of the heating, and a reading on any wave form must be compared with the mean of sine wave readings before and after. Every result given is the mean of three observations taken in immediate succession, the voltmeter being set independently for each. At the beginning or end of the series observations were taken at 30 cycles, in order to determine the proportion of hysteresis to the total iron loss. For this purpose the multiplier of the voltmeter was changed so as to make the total

⁵G. Roessler, Electrician, 36, p. 151; 1895.

resistance of its circuit just one-half of the previous value, and adjustment made for the same deflection. The voltage must then also be one-half of the previous value.

In using distorted waves which involve higher frequencies the voltmeter must be corrected for its inductance error. The correction is easily made for a sine wave, and most conveniently by altering the resistance so that the impedance remains constant. Let L be the inductance of the instrument. Then, if the multiplier be noninductive and R be the total resistance, the impedance is

$$\sqrt{R^2 + 4\pi^2 n^2 L^2} = R \left(1 + \frac{4\pi^2 n^2 L^2}{R^2} \right)^{\frac{1}{2}}$$

To have the impedance the same as with direct current, the resistance must be changed to $R' = R - \frac{2\pi^2 n^2 L^2}{R}$ (L is very small compared to R , its value being 0.138 henry.) With a distorted wave the resistance required by the different components is different, but by giving proper weight to each component (i. e., in proportion to the square of the amplitude) we get

$$R - R' = \frac{2\pi^2 n^2 L^2}{R} (1 + h_1^2 m_1^2 + h_2^2 m_2^2 + \dots)$$

Hence, in changing from the sine wave to a distorted wave, we change the resistance of the multiplier by

$$\frac{2\pi^2 n^2 L^2}{R} (h_1^2 m_1^2 + h_2^2 m_2^2 + \dots)$$

To determine the components of the wave used, the magnetizing circuit is broken after adjustments have been made, and the voltage of each generator is measured with a Stanley hot wire voltmeter. This is done on no load to avoid the drop on the line. As the load is very small compared to the capacity of the generators, the armature reaction does not appreciably affect the voltage.

The procedure in taking observations is illustrated by a run taken to determine the dependence of the result upon the way of connecting the voltmeter. The readings are given in Table V.

Each of the deflections given for voltmeter and wattmeter is the mean of three observations. The calculated effects are submitted

TABLE V.
Effect of Voltmeter Connection.

Transformer No. 2. $q=0.60$, $m=3$							
h	θ	Voltmeter			Wattmeter		
		Resistance	Deflection	Connection	Deflection	Increase due to Harmonic	Increase in Per Cent
0.		25000	27.95	Primary (132 Volts)			
0.		25000	27.95	Primary	20.16		
0.23	180°	25000	27.95	"	18.25	-1.80	-9.0
0.23	0°	25000	27.95	"	21.01	+0.88	+4.4
0.		25000	27.95	"	20.12		
0.		500	19.35	{Secondary {(132 Volts on Primary)			
0.		500	19.35	Secondary	19.77		
0.23	180°	499	19.35	"	17.84	-1.91	-9.7
0.23	0°	499	19.35	"	20.61	+0.89	+4.5
0.		500	19.35	"	19.70		
0.23	180°	Calculated					-9.3
0.23	0°						+4.8

for comparison. They are readily obtained from Fig. 1. A similar set with transformer No. 1, using 21 per cent of fifth harmonic, gave -0.8 per cent and -3.0 per cent, with voltmeter on primary; -0.9 per cent and -3.4 per cent, with voltmeter on secondary.

The agreement with each other and with the calculated values are seen to be well within one per cent, and this is about the order of discrepancies throughout the work.

The value of q for the above transformer was determined from the observations given in Table VI, with the voltmeter connected to secondary.

The observations were made with the help of Mr. J. V. S. Fisher, to whom credit is also due for assistance in the computations and drawings.

4. EXPERIMENTAL RESULTS.

Table VII gives the results on two transformers of introducing a single harmonic either in phase or reversed. For comparison the calculated effects taken from Fig. 1 are added to the Table, those marked with an asterisk lying outside the limits set by the theoretical conditions.

TABLE VI.

Transformer No. 2. Separation of Hysteresis and Eddy Current Losses								
n	Voltmeter		Wattmeter		$\frac{R_p}{n}$	Hysteresis	Eddy Currents	q
	Res.	Defl.	Res.	Defl.				
60	500	19.31	900	20.05	15.00	8.94	6.06	.596
30	250	19.31	359	20.04	11.97	8.94	3.03	.747

Table VIII gives the results of shifting the phase of the harmonic, while keeping its magnitude constant, and these are plotted in the curves of Fig. 7. The calculated values in Table IV may be compared with the corresponding column of Table VIII, viz, for transformer No. 2, $m=3$.

TABLE VII.

Transformer	m	h	Percentage Increase in Loss			
			$\theta=0^\circ$		$\theta=180^\circ$	
			Exp.	Calc.	Exp.	Calc.
No. 1	3	.11	+2.8	+3.0	-4.0	-4.1
" "	5	.105	+1.7	+1.5	-2.8	-2.6
" "	7	.19	+0.7	+0.9	-3.8	-4.3 *
" "	7	.235	-0.2	+0.6	-5.2	-5.6 *
" "	7	.26	-0.4	+0.4	-5.7	-6.6 *
" "	9	.20	-0.9	+0.2	-3.3	-4.0 *
" "	11	.12	+0.1	+0.4	-1.0	-1.8 *
" "	13	.125	-0.4	+0.1	-1.1	-1.7 *
" "	15	.14	-0.9	-0.1	-1.5	-1.9 *
No. 2	3	.20	+4.0	+4.5	-8.3	-7.9
" "	5	.115	+1.2	+1.6	-3.3	-2.9
" "	7	.17	+0.5	+0.9	-3.7	-3.5 *
" "	7	.24	-0.3	+0.6	-5.2	-5.7 *
" "	9	.17	-0.3	+0.4	-3.1	-3.1 *
" "	11	.12	-0.3	+0.4	-2.0	-1.7 *
" "	13	.12	-0.4	+0.1	-1.1	-1.6 *
" "	15	.135	-0.5	0.0	-1.4	-1.7 *

The generators producing the high frequencies do not give very high voltages. In order to use a high percentage of these, the generator supplying the fundamental frequency was excited to a correspondingly low voltage, and after combination the voltage was transformed up to the proper value. Transformer No. 2 was

TABLE VIII.

Per Cent Increase in Loss							
Transformer	No. 1	No. 2	No. 2	No. 2	No. 3	No. 3	No. 3
m	3	3	5	7	3	5	15
h	.18	.20	.115	.23	.165	.115	.41
$\theta = 0^\circ$	+5.5	+4.0	+1.2	-0.3	+4.4	+1.5	-7.3
30	+5.7	+4.2		-0.1	+4.7		-7.0
45			+1.0			+1.5	
60	+4.8	+3.2		-0.6	+4.0		-7.0
90	+2.0	+0.8	-0.2	-1.4	+2.0	+0.6	-6.7
120	-1.6	-1.9		-2.3	-1.0		-7.0
135			-1.9			-1.0	
150	-5.1	-5.6		-3.7	-4.2		-7.2
180	-8.2	-8.3	-3.3	-5.2	-7.4	-2.7	-7.4
210	-8.1	-8.0		-5.1	-7.6		-7.5
225			-2.1			-3.0	
240	-5.6	-5.4		-3.6	-5.8		-7.8
270	-2.3	-2.3	-0.5	-2.5	-2.7	-1.2	-7.8
300	+1.2	+0.7		-1.4	+0.3		-7.8
315			+0.7			+0.5	
330	+3.9	+2.9		-0.8	+2.8		-7.6

used in this way with the results shown in Table IX. The voltmeter was connected to the primary side of the transformer. The calculated values for $\theta = 180^\circ$ are obtained by the method given on p. 490.

It will be noticed here that the experimental and the calculated values do not agree very well; indeed, the experimental results show very little effect from phase reversal. While seeking the reason for this, the effect of transforming the fundamental alone was tried.

The iron losses were first determined by direct connection to the generator, then after transformation. With ratio of transformation 1:2 the losses were 0.7 per cent less; with ratio 1:3 they were one per cent less than by direct connection.

TABLE IX.

Transformer No. 2						
Ratio of Previous Transformation.	m	h	Increase in Loss			
			$\theta=0^\circ$		$\theta=180^\circ$	
			Exp.	Calc.	Exp.	Calc.
1:2	13	.19	-1.9	-0.3	-2.3	-3.1*
1:3	13	.27	-4.0	-1.5	-3.5	-4.2*
1:2	15	.235	-2.9	-1.1	-3.1	-3.2*
1:3	15	.32	-5.2	-2.9*	-4.9	-4.9*
1:4	15	.44	-8.4	-6.0*	-8.3	-8.1*

Measurements were next made on Transformer No. 3, using the low-voltage winding as primary. In this way a large component of high harmonic could be used at a low voltage. Table X gives the results.

TABLE X.

Transformer No. 3							
Ratio of Transformation	m	h	Increase in Loss in Per Cent				
			$\theta=0^\circ$		$\theta=180^\circ$		$\theta=90^\circ$
			Exp.	Calc.	Exp.	Calc.	Calc.
60:120	13	.17	-1.4	-0.2	-2.3	-3.0*	-1.6
30:120	13	.36	-5.8	-3.7*	-5.9	-7.2*	-5.5*
30:120	13	.48	-10.4	-7.9*	-10.8	-11.0*	-9.5*
60:120	15	.155	-1.2	-0.2	-1.7	-2.4*	-1.3
30:120	15	.32	-4.5	-3.3*	-4.4	-5.7*	-4.5*

Table XI shows the effects of combining two harmonics with the fundamental. These harmonics were kept in the same phase relation with each other, and both shifted with respect to the fundamental, each shift amounting to 90° of the fundamental. The initial position was with the third harmonic in phase and the fifth harmonic reversed.

TABLE XI.

Transformer No. 2 Two Harmonics		$m_1=3$ $m_2=5$	$h_1=.113$ $h_2=.109$
Phase Setting		Increase in Loss in Per Cent	
θ_1	θ_2	Exp.	Calc.
0°	180°	-0.45	+0.3
180	0	-3.5	-2.6
- 90	- 90	-0.6	+0.8
+ 90	+ 90	+1.9	+0.8
+135	+ 45	-1.1	-0.8
- 45	-135	+0.15	+0.7

Table XII presents a case where all of the generators were used together. The phase of each harmonic in this case was made either zero or 180° , and the amplitudes decreased as the frequency increased, so that mh was approximately constant and had the value 0.6.

Table XIII gives the results of applying a large component of low harmonic to transformer No. 2.

5. DISCUSSION OF THE RESULTS.

Reference to Table VII discloses the fact that in the simple case of a single harmonic of small magnitude the experimental results agree with the theoretical values within one per cent. When the higher harmonics enter in large proportion the agreement is not so good; thus, with 20 per cent of the ninth harmonic the discrepancy for transformer No. 1 is 1.1 per cent. The loss with these higher harmonics in phase is always less than the calculated value; when they are reversed in phase it is nearly always greater. This is even more apparent in Tables IX and X. In the latter there is very little difference between $\theta=0^\circ$ and $\theta=180^\circ$, the losses having the value which is about the mean of the calculated values, and which would apply to the case $\theta=90^\circ$.

In Table VIII and Fig. 7 it is seen that the maximum loss does not coincide with $\theta=0^\circ$ nor the minimum with $\theta=180^\circ$, as indicated by the theory, but that the extreme values are for $\theta=20^\circ$ and $\theta=200^\circ$, approximately. This can not be due to an error in the phase settings, since in the case of different harmonics different

numbers of degrees of the fundamental correspond to 20° of the harmonic. The accuracy of setting of any generator is about 0.1° of arc, corresponding to 0.3° electrical in the fundamental, 4.5° electrical in the highest harmonic, and proportional values for the intermediate harmonics. The results point rather to a distortion of the wave between the generators and the transformer; that is to say, the wave form applied to magnetization is not the same as that supplied by the generators. This may be accounted for by the ohmic drop in leads, instruments, and primary winding. It is well known that, owing to the varying permeability of the iron, the wave of current has a different shape from the wave of emf.⁶ The ohmic drop of potential, consequently, involves other harmonics and other magnitudes than the generator emf. Since the emf. used to magnetize the iron is the difference of these two, it also must have a wave shape different from the generator emf. When any considerable resistance is in the circuit, this effect becomes so marked as to give an entirely different wave. It is for this reason that no rheostat was used in the primary circuit, the current being controlled instead through the excitation of the generator.

The fact that this effect is very noticeable is proven by the result obtained when an intermediate transformer was used, as compared with the losses obtained with direct connection to the generator. This amounted to one per cent with a sine curve for the original wave.

With a large percentage of high harmonic the effect is even more marked, as illustrated in the last column of Table VIII. Here the maximum occurs at $\theta = 270^\circ$ and the minimum at $\theta = 90^\circ$, and the range of variation is less than the theoretical value. By calculation the extreme values for this case are -5.2 per cent and -7.3 per cent as against -6.7 per cent and -7.8 per cent experimentally. In all cases of a large component of high harmonic the calculation for $\theta = 180^\circ$ is based upon the modified formula given on p. 490, where ϕ_1 is taken as $\frac{\theta}{m}$.

With a large component of lower harmonic the results given in Table XIII show an agreement with calculated values for $\theta = 180^\circ$ except in one case. Here the proper value for ϕ_1 has been found by

⁶Waves illustrating this are given by H. A. Pikler, *Electrical World*, 42, p. 218; 1903.

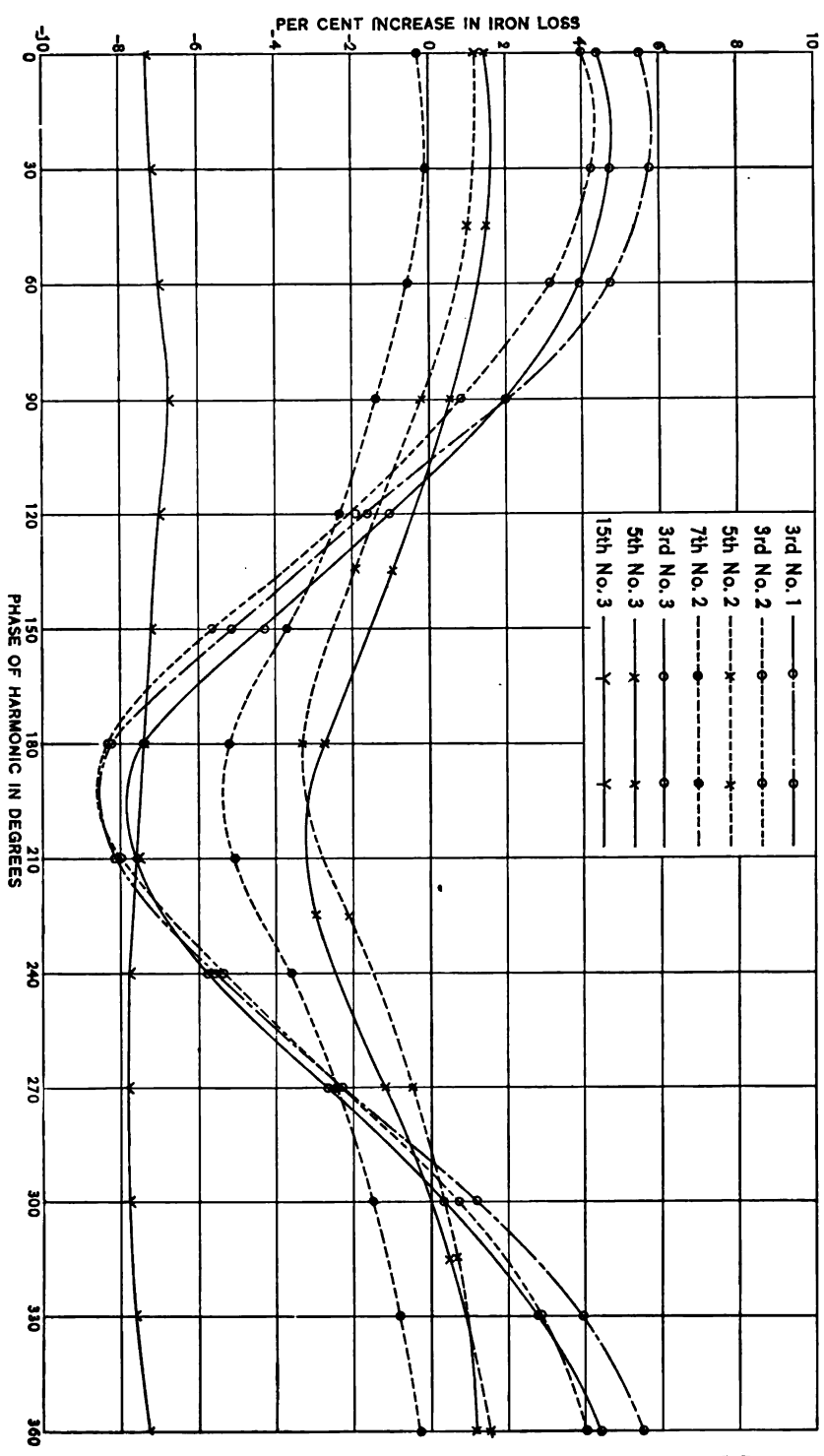


Fig. 7.—Variation of Iron Losses with Phase of Harmonic.

TABLE XII.

Transformer No. 2 Seven Harmonics $hm=k=0.6$ approximately $\theta_3=\theta_4=\theta_5=\theta_6=\theta_7$			$m_1=3$ $m_2=5$ $m_3=7$ $m_4=9$ $m_5=11$ $m_6=13$ $m_7=15$	$h_1=.20$ $h_2=.10$ $h_3=.10$ $h_4=.08$ $h_5=.065$ $h_6=.055$ $h_7=.045$
Phase Setting			Increase in Loss in Per Cent	
θ_1	θ_2	θ_3	Exp.	Calc.
180°	180°	180°	-7.8	*
0	0	0	+7.6	+7.9
0	180	180	-0.7	*
180	0	0	-5.4	-4.4
180	0	180	-7.9	*
0	180	0	+2.8	+3.5
180	180	0	-9.1	*
0	0	180	+2.2	+2.2

trial and used in the calculation. With 78.5 per cent of the third harmonic the secondary loops in the hysteresis curve become more pronounced and cover a range of induction of 14 per cent of B . This would increase the measured iron loss, and may well account for the discrepancy of 3.5 per cent.

For $\theta=0^\circ$ the discrepancies between experimental and calculated values are more marked, amounting to three per cent. They are in the usual direction, the experimental loss being less than calculated. I know of no other cause for this than the distortion of the wave due to ohmic drop of potential, already mentioned.

In the experiment with two harmonics the agreement is everywhere within one and one-half per cent; the loss is smaller than expected, with one exception.

In the experiment with seven harmonics the agreement is within one per cent in all cases where the formula can be expected to hold; that is, where $\theta=0^\circ$ for two of the lower harmonics. In the other cases ϕ_1 is not zero and would require a lengthy calculation for its determination. The discrepancy is always on the side of a lower loss than calculated. The loss is always less than with a sine curve unless the third harmonic is introduced in phase with the fundamental, in which case the wave will be flat or dimpled.

TABLE XIII.

m	h	Increase in Loss in Per Cent			
		$\theta = 0^\circ$		$\theta = 180^\circ$	
		Exp.	Calc.	Exp.	Calc.
3	.785	-3.6	-0.6	-22.1	-25.6*
5	.605	-7.1	-3.8	-14.2	-15.0*
5	.39	-1.6	+0.6	-10.1	-10.3*
5	.32	-0.4	+1.3	- 8.6	- 8.8*

It is apparent that transformer losses may be materially reduced by using an appropriate form of wave. When a generator is to be used primarily or principally to supply transformers whose load does not require a sine wave, it would be advantageous to design it with a wave form suitable for the accomplishment of this result. The desired result could be assured by the presence of a considerable component of the third harmonic in reversed phase, and suppressing as far as possible the higher harmonics, unless the phases of these can be controlled. This would give an unmistakably peaked wave.

If the generator be three phase, three wire, it is not desirable to introduce the third or integral multiple of third harmonic. For, if star connected, the harmonic does not appear in the line voltage; if delta connected, a large short-circuit current of the triple frequency may circulate in the armature, wasting power.⁷ In this case it would be desirable to choose higher harmonics to produce the desired effect. These can be introduced by a suitable choice of armature teeth, not pushed to a too high flux density. In introducing harmonics, however, it is always necessary to consider the danger from resonance.

If the generator is to be used on a high tension line, the objection must be met that a peaked wave requires a higher maximum emf. for the same effective voltage, and consequently better insulation is required. While this argument carries some weight, it is sometimes overrated. The energy lost by leakage is independent of wave form.

⁷This is discussed by C. P. Steinmetz, *Trans. A. I. E. E.*, 25, p. 780; 1906: *Electrician*, 58, p. 573; 1907.

As for the breakdown of the insulation, it has been shown⁸ that the steady voltage necessary for rupture may be greatly increased for a fraction of a cycle without breakdown. Furthermore, most breakdowns on high voltage lines can be traced to surges of abnormal voltage due to some unusual condition, such as opening or closing a switch, not dependent upon the wave form, or to accidental resonance with some high harmonic. The insulation is liable at any time to be subjected to a stress due to twice the effective voltage.⁹ It can not be taken for granted, consequently, that a peaked wave would require higher insulation than the same effective voltage in a sine wave. It would be advisable, however, to avoid high harmonics in the wave in order to lessen the danger from resonance effects.

A greater objection to a peaked wave on long transmission lines is the fact that the high maximum emf. requires a larger charging current. How serious this feature of the case may be depends upon the constants of the particular line considered and the amount of power transmitted.

If the secondary have an inductive load, so that its current and the corresponding component of primary current are not in phase with the emf., the difference in wave form due to the ohmic drop in potential will be accentuated, and the effect of the generator emf. upon the iron losses will be less certain.

CONCLUSIONS.

With a given effective electromotive force the iron losses in a transformer depend upon the form factor of the emf, and vary inversely with it.

With a given form of wave, the effect upon the iron losses may be computed approximately from the formulæ derived, providing the higher harmonics are not prominent.

By proper design of the generator supplying transformers, the iron losses may be reduced to a minimum.

WASHINGTON, Oct. 31, 1907.

⁸ C. Kinzbrunner, *Electrician*, 55, p. 809; 1905.

⁹ P. H. Thomas, *A. I. E. E. Trans.*, 24, p. 317; 1905.

THE LUMINOUS PROPERTIES OF ELECTRICALLY CONDUCTING HELIUM GAS.

By P. G. Nutting.

INTRODUCTION.

A gas conducting an electric current radiates the energy imparted to it by the current. Neglecting relatively very slight losses due to conduction and convection, the efficiency of such a gas as a transformer of energy from electrical energy into radiation should be a constant approaching unity in value. Radiation from a gas should then be expressible in terms of current and other specificable conditions with considerable precision. To express this radiation in terms of light, the spectral distribution of the radiation and the visual sensibility of the observer must be known. Upon the constancy of these latter depends the constancy of the relation of light to current.

The results here presented relate to the amount of light per unit length emitted laterally by a column of helium gas carrying a known current. The chief object of the investigation was to determine the constancy and reproducibility of such a source of light and the specifications most favorable to constancy and reproducibility. The variation of light emitted with current, potential gradient, gas density, frequency of alternation of current, orientation of tube, with diameter of capillary and with time were studied in turn.

Conducting helium emits light of a yellowish white color better for photometric comparisons with a glow lamp than that from any other of the permanent gases. Next best is, perhaps, carbon dioxide. This emits a snow-white light, but decomposes and disappears rap-

idly, with but moderate current densities. Sulphur vapor emits bluish-white light, but its density is difficult to measure and control. Helium has the further great advantage of not disappearing rapidly as other gases do when conducting a heavy current. The life of a tube of helium is at least fifty hours as compared with half an hour for a similar tube of hydrogen or nitrogen carrying the same current.

Preliminary tests with tubes of various forms led to the adoption of a simple straight form with disk electrodes. The electrodes are of aluminum 1.5 mm thick and 25 mm diameter in spherical bulbs

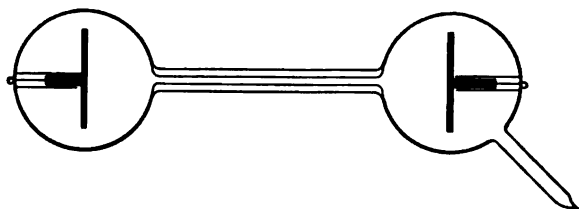


Fig. 1.

35 mm in diameter. These bulbs are connected by a straight piece of capillary tubing 50 mm in length and 2 mm in internal diameter. Foil 5 mm wide was wound about either end of the capillary to give the light emitting portion a definite measurable length. Light from the bulbs was screened off from the photometer.

2. METHOD OF PREPARING TUBES.

Ordinary methods were used in filling except that extra precautions were taken to free the electrodes from hydrogen. The tubes were cleaned with chromic acid and water and exhausted with a Geryk oil pump. After from twenty minutes to an hour of steady pumping with heavy current running through the tube, the last trace of hydrogen disappeared. Then helium was admitted, its pressure adjusted to about 5 mm and measured by means of an oil manometer of special design and the tube sealed off. The apparatus used in filling the tubes with pure helium at the proper pressure is shown in Fig. 2. Four very carefully ground stopcocks of 3 or 4 mm bore are joined in a line about 10 cm apart. The outer end of the first is fitted with a ground joint for attaching to the pump.

Between the first and second is attached a small auxiliary Plücker tube to aid in adjusting the pressure and testing the vacuum. Between the second and third cocks is attached an oil manometer of special design and a branch for attaching and sealing off the tubes to be filled. The space between the third and fourth cocks serves as an auxiliary reservoir in admitting helium and protects the open helium bulb in case the apparatus remains idle for a considerable time. Beyond the fourth cock is a piece of large tubing to which the bulb of helium is cemented.

The sealed bulb containing helium must be opened without exposing the helium to contact with rubber tubing, water, or mercury. To accomplish this a massive piece of glass rod or capillary

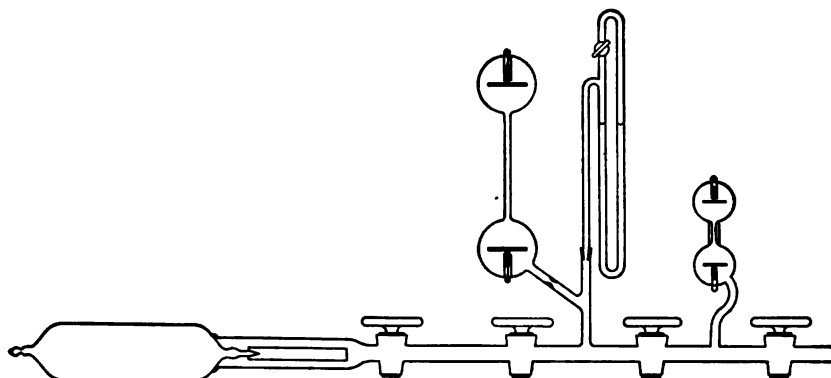


Fig. 2.

(metals contain too much absorbed gas) is cemented to the end of one tip and the neck of the tip scratched for breaking. The whole tip and weight is then inserted in the large tubing projecting beyond the fourth stopcock and the bulb back of the tip cemented, as shown, to this tubing with a fusible cement (rubber dissolved in resin and boiled in a vacuum). After the apparatus has been exhausted to a nonconducting vacuum for several days, a sharp tap will crack off the tip of the helium bulb and liberate the gas. A reservoir opened in this manner may be drawn from for months without contamination. The stopcocks should be greased with a solution of natural rubber in vaseline that has been freed from absorbed air by boiling in a

vacuum just before using; all four stopcocks should be rotated at intervals during exhaustion.

The oil manometer was filled with the oil ("standard gas engine") used in the Geryk air pump. This oil is about one-fifteenth the density of mercury, so that pressure readings are greatly enlarged. It was vacuum boiled before using. Since this oil, like so many other similar hydrocarbons, rapidly absorbs air and other gases, it was necessary to make the manometer self-exhausting as shown. The stopcock is left open except when a reading is to be taken. A mercury manometer or McLeod gauge could not be used for measuring the pressure of the helium gas without too great contamination. The vapor pressure of this oil is below that of a non-conducting vacuum except in the warmest summer weather, and then a cold-water jacket is easily applied to both manometer and pump cylinder.

In filling standard tubes with helium the greatest difficulty is in freeing the electrodes from hydrogen. This is best accomplished by steady pumping while a heavy current is passing through the tube. After but a few minutes pumping nearly all of the impurities except hydrogen will disappear, indicating, perhaps, that the interior walls are clear of gas. After about fifteen minutes further pumping the hydrogen pressure drops abruptly. This, I take it, is when the disk portions of the electrodes are depleted of included hydrogen. To completely clear the stems of the electrodes requires further pumping and current for perhaps an hour. No air or helium should be admitted during this process, as it makes the complete elimination of the hydrogen much more tedious and difficult.

After the tube refuses to conduct even a 5000-volt current, helium may be admitted. This usually shows impurities (probably atomized stopcock grease), but these are easily removed by the pump. After two or three fillings, the tube should show only the pure helium spectrum, without a trace of the red hydrogen line, and may be sealed off.

Various other methods of eliminating traces of hydrogen were tried—oxidation and removal as water vapor, absorption by an auxiliary electrode of sodium-potassium alloy, absorption by cold

palladium and hot calcium and absorption by charcoal in liquid air—but none were found entirely effective.

The purity of helium may be readily judged without a spectroscope by the color of the cathode glow. In pure helium this is of a clear magenta tint, but if there is a mere trace of impurity present, this is lost.

The tubes were operated on a 5000-volt alternating current from a transformer controlled by resistance in its primary. The light emitted was compared with that from a calibrated 4-candle glow lamp by means of a Lummer-Brodhun photometer of the equality type. The uncertainty in each determination is about two per cent, the probable error less than one per cent. In a spectrophotometric comparison, the uncertainty would have been from 10 to 100 per cent on account of the variable line width.

3. VARIATION OF LIGHT WITH CURRENT.

The variation of light with current in various forms of tube is shown in the accompanying tables and curves. The light from the whole capillary taken with the tube stationary is given in candles, but is uncorrected for mean horizontal.

Tube	Current in Milliamperes							
No.	10	15	20	25	30	35	40	45
18	0.58	0.94	1.19	1.33	1.39	1.42		
27	0.55	0.86	1.16	1.45	1.71	1.92	2.11	2.30
43	0.29	0.68	0.96	1.23	1.47	1.69	1.89	2.09
39	0.60	0.92	1.23	1.54	1.83	2.06	2.30	2.53

The dimensions of these tubes are :

No. 18.	Capillary	1.2	mm diameter,	40	mm long.
" 27.	"	1.8	"	54	" "
" 43.	"	3.12	"	50	" "
" 39.	"	2.17	"	49	" "

The plotted curves indicate the effect of size of capillary. Tube No. 18 with its small bore gives a curve much steeper at the start,

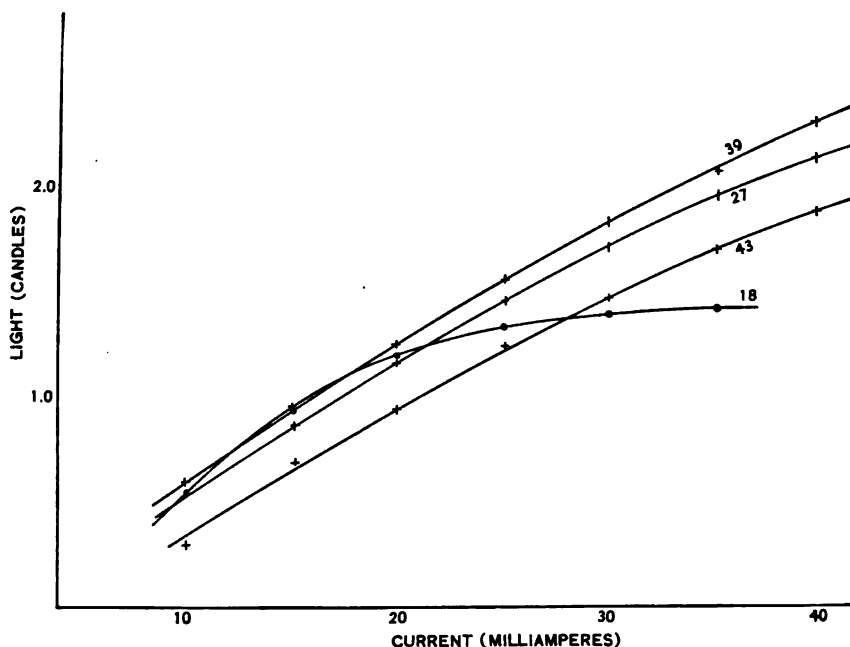


Fig. 3.

but early approaching a maximum. Tubes of larger bore give curves of less curvature.

More precise corrected values for the variation of light with current obtained later are given below.

Current (Milliamperes)	Correction to Candle Power per cm Length of Capillary
22	+0.0387
23	+0.0258
24	+0.0136
25	0.0
26	-0.0130
27	-0.0256
28	-0.0376

4. VARIATION OF LIGHT WITH POTENTIAL GRADIENT.

The variation of light with potential gradient was studied with the purpose of expressing light emitted in terms of energy absorbed. Auxiliary potential electrodes of platinum wire were inserted at each end of the capillary portion of the tubes and potential readings taken with a multicellular voltmeter. It was found that fairly constant potential readings could be obtained. It was found, however, that by playing a flame or blast of cold air upon the capillary a variation of five or ten per cent in the indicated fall of potential could be produced. This variation, however, was not accompanied by a variation in the light emitted so long as the current was held constant.

Some typical data is given in the following tables and curves:

Variation of Potential Gradient with Current

I. Tube No. 38. Capillary 2.17 mm diam., 50 mm long, Pressure 4.2 mm

II. " " 43. " 3.12 " " 50 " " " 5.0 "

Current (milliamperes)	Light (candle)	P. G. (volts)	Energy (watts)	Candle Watt	Watts Candle
(I)	10	274	2.74	0.219	4.57
	15	256	3.84	0.227	4.40
	20	245	4.90	0.246	4.07
	25	237	5.92	0.253	3.96
	30	233	6.99	0.258	3.88
	35	231	8.10	0.247	4.05
	40	230	9.20	0.244	4.11
	45	229	10.30	0.239	4.19
(II)	10	233	2.33	0.169	5.92
	15	211	3.16	0.215	4.65
	20	198	3.96	0.242	4.14
	25	190	4.75	0.259	3.88
	30	185	5.55	0.265	3.78
	35	180	6.30	0.268	3.74
	40	176	7.04	0.268	3.74
	45	173	7.79	0.268	3.74
	50	170	8.50	0.268	3.74

The two curves for fall of potential plotted against current are of the typical form for the anode column of a conducting gas. That for the larger capillary is lower and on a larger scale. These curves are of very nearly the same form for all pressures from 2 mm

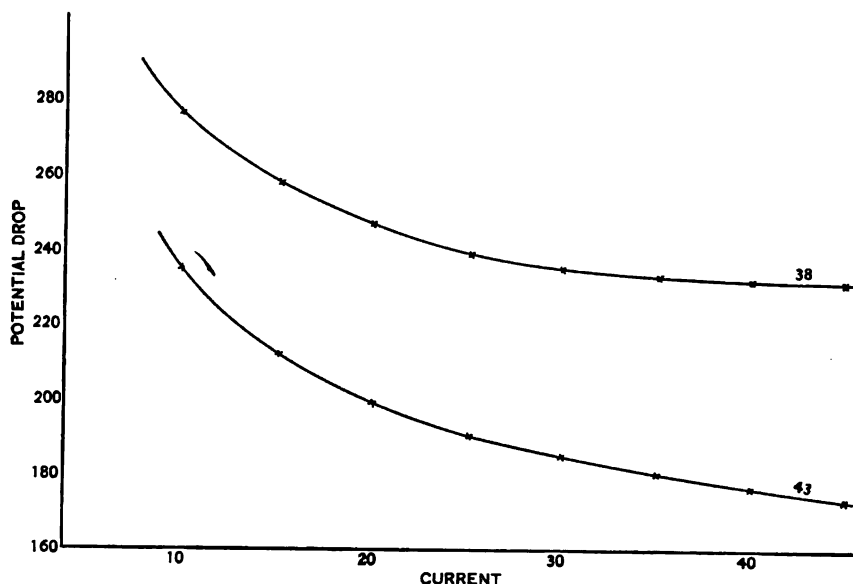


Fig. 4.

to 10 mm, but are a few per cent higher at the higher pressures. If the auxiliary electrodes took up the actual potential of the gas, the above data gives an indication of the luminous efficiency of such a tube. It passes through a maximum at a current density of about 6 milliamperes per square millimeter. The low luminous efficiency of helium is largely due to the three infra-red lines at 728, 1117, and 2040 $\mu\mu$ which represent at least 90 per cent of the radiation without contributing to the light emitted.

5. VARIATION OF LIGHT WITH GAS DENSITY.

The variation of light emitted with density of gas was studied by means of tubes of three different diameters filled with helium at pressures ranging from 2 mm to 10 mm and carrying currents of 10 to 45 m.a. Gas pressure was measured on a special manometer containing gas-free pump oil of one-fifteenth the density of mercury.

Each tube was first filled with pure gas at the highest pressure, then the pressure reduced in steps by temporarily connecting with a highly exhausted auxiliary bulb of proper size.

Variation of Light with Gas Density,

Tube No. 43. Capillary 3.12 mm diam., 50 mm long

Gas Pressure in mm Hg.					
Current	8.5	6.6	5.0	3.7	2.5
10	0.13	0.26	0.41	0.42	0.33
15	0.34	0.52	0.71	0.75	0.65
20	0.53	0.82	1.01	1.03	0.92
25	0.74	1.09	1.29	1.34	1.20
30	0.92	1.31	1.54	1.58	1.53
35	1.11	1.60	1.77	1.79	
40	1.31	1.71	1.98	1.99	
45	1.46	2.01	2.20	2.07	
50		2.12	2.40	2.17	

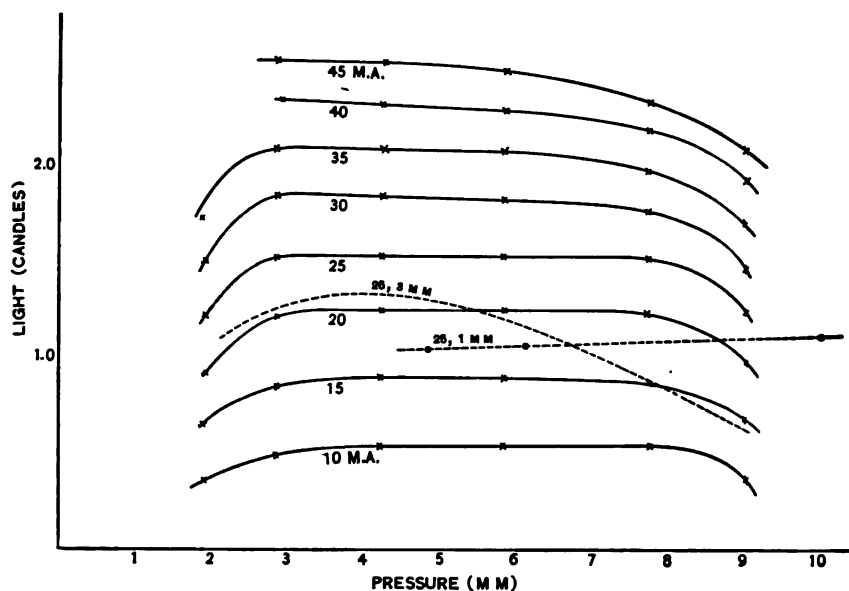


Fig. 5.

Tube No. 38. Capillary 2.17 mm diam., 50 mm long

Current	Gas Pressure					
	9.0	7.7	5.8	4.2	2.8	1.9
10	0.38	0.55	0.55	0.55	0.48	0.08
15	0.70	0.87	0.91	0.90	0.83	0.66
20	0.98	1.22	1.24	1.24	1.20	0.91
25	1.25	1.51	1.54	1.54	1.54	1.24
30	1.48	1.79	1.83	1.85	1.86	1.51
35	1.72	1.99	2.10	2.10	2.10	1.71
40	1.96	2.20	2.31	2.35	2.07	
45	2.11	2.35	2.47	2.56	2.58	

Tube No. 45. Capillary 1.03 mm diam., 50 mm long

Current	Gas Pressure		
	10.0	6.1	4.8
10	0.47	0.45	0.44
15	0.25	0.67	0.66
20	0.93	0.89	0.86
25	1.12	1.08	1.07
30	1.25	1.21	1.19
35	1.33	1.33	

The complete data on Tube No. 38 is reproduced in Fig. 5 with one curve dotted for each of Tubes 43 and 45. The curves for the 2-mm tube parallel the pressure axis over a wide range of pressure and current. For the 3-mm capillary the curves are higher at lower pressures, for the 1-mm capillary at high pressures.

6. USE OF DIRECT AND ALTERNATING CURRENT.

The form of current was varied from direct to alternating of high and low frequency and voltage without disturbing the apparatus. The direct 1000-volt current from two 500-volt generators in series could be controlled at 22.5 m.a. and 23.7 m.a. Afterwards the

more easily adjustable alternating current was brought to these values. The data taken is as follows:

Form of Current	Light Emitted	
	Current = 22.5 m.a.	23.7 m.a.
Direct current, 1000 volts	1.21	1.30
Alternating ", 5000 volts, 60 cycles	1.22	1.29
" " , 5000 volts, 900 "	1.21	1.29
" " , 2000 volts, 60 "	1.22	1.28

The slight differences observed are less than the uncertainty in the light determinations.

7. VARIATION OF LIGHT WITH ORIENTATION OF TUBE.

The light emitted by a tube varies with its orientation on account of irregularities in the capillary. Heavy-walled capillary is worst, deviations of ten per cent from the mean horizontal being not uncommon. Walls 1 mm thick are sufficiently strong for such tubes, and such thin-walled capillary, if carefully selected, is found to give deviations of but one or two per cent from the mean. Such deviations are of slight consequence when the mean horizontal is obtained.

8. VARIATION OF LIGHT WITH CURRENT DENSITY.

The diameter of the capillary affects the light emitted per cm with a given current. Tests of saturation indicated that in such a luminous column of gas practically all the radiation comes from within half a millimeter of the surface, so that light emitted would vary with the area of longitudinal section rather than with volume. Current density varies inversely as the square of the diameter, so that if the light varied directly as the diameter (at constant current density), as above indicated, it should vary approximately inversely as the diameter at constant current. Data is available on four different diameters of capillary at 25 milliamperes of current:

Diameter of capillary.....	1.029	1.949	2.168	3.12	mm
Mean horizontal candle power per cm at 25 m.a ...	0.365	0.325	0.311	0.259	

These four points lie near a smooth curve that is nearly a straight line.

In the light of the above results, the specifications most suitable for constancy and reproducibility in a helium standard of light intensity may be drawn up, namely, a current of 25 milliamperes, a capillary of 2 mm diameter, and a gas pressure of about 5 mm, which need not be accurately known. Conversely it might be stated that a tube having a capillary of 2 mm bore, filled with pure helium at 5 mm pressure, will emit N mean horizontal candles per centimeter of length when carrying a current of 25 milliamperes. The constant N has been shown to be very nearly if not quite independent of other small variations. The uncertainty in the value of N , aside from the uncertainty in its photometric determination, is very small, probably much less than one per cent.

To test reproducibility, a set of six duplicate tubes were made up with capillary 2.168 mm in mean internal diameter and filled at 5 mm pressure. These gave the following values in mean horizontal candles per cm length at 25 milliamperes:

Tube No.	Light per cm.	Deviation from mean.
37	0.325	-0.003
38	0.329	+0.001
39	0.332	+0.004
40	0.328	0.000
41	0.326	-0.002
42	0.326	-0.002
	Mean 0.328	0.002
	±0.001	

The greatest deviation is only about a per cent, the average deviation less than a per cent, and the probable error in the mean value less than one-third of one per cent.

The correction for bore of capillary is 0.009 from the curve of variation of light with bore, so that this set of tubes gives the value of the constant N , 0.337 candle per centimeter.

9. VARIATION OF LIGHT WITH TIME.

Incidentally, many tests for a time effect were made. At the end of a long series of observations with varying current or orientation, tubes would be brought back to the initial current or position and their light values checked with previous ones made half an hour before, and these with others made a week or a month previously. In every case values checked up to within the uncertainty of observation. In the few cases where apparent changes were recorded the change was large and due to an obvious change in the tube itself, a crack or the development of hydrogen or a cathode deposit due to current overload.

10. CONCLUSION.

These results are so promising that the helium tube seems to me to be worthy of consideration as a primary light standard. Its freedom from the troublesome atmospheric corrections to which the Hefner, Carcel, and pentane standards are subject is very much in its favor. The only quantity to be measured while in operation is the current, and this may easily be determined with the required precision.

The uncertainties in the value of the light constant N are of three classes, and these require further investigation before the helium tube may be considered as a competitor of existing standards. These uncertainties are:

1. Those in the separate determinations of the same observer on the same tube. These are not serious, being of the same order as the uncertainties in ordinary photometric work.

2. Uncertainties due to using different tubes. Reproducibility will require further investigation with capillaries of different kinds of glass and of different diameters. If this uncertainty is no greater than is indicated by the test of the six tubes, 37 to 42, above recorded, it is of little consequence.

3. Uncertainties due to varying the observer on account of color differences. These are the most serious of all, and require careful investigation by many different observers.

WASHINGTON, December, 1907.

FUNCTION OF A PERIODIC VARIABLE GIVEN BY THE STEADY READING OF AN INSTRUMENT; WITH A NOTE ON THE USE OF THE CAPILLARY ELECTROMETER WITH ALTERNATING VOLTAGES.

By Morton G. Lloyd.

This investigation was made in the hope of finding an instrument which would register the average (numerical) value of an alternating electromotive force, but an examination of the conditions shows that no instrument can answer this purpose unless its deflection is independent of the direction of the applied voltage and proportional to that voltage in numerical magnitude.

In instruments whose parts have relative motion, as a galvanometer, the force exerted depends upon the position of the moving part and upon the variable being measured, which for brevity will be referred to as the *voltage*. In others, such as the Siemens dynamometer or the capillary electrometer with uniform bore (or in which the hydrostatic pressure is varied to keep the meniscus stationary), the force depends upon the voltage alone.

In either case, if the moving part maintains a fixed position during the variations of the voltage, the variations of force during a period depend only upon the variations of the voltage, and the average value of that force is what determines the deflection and is measured by it. The function given by the instrument reading consequently depends only upon the law of force involved. Let us suppose that the force is such a function of the voltage that it may be expressed by a series of powers.

$$F = A_0 + A_1e + A_2e^2 + A_3e^3 + \dots$$

The average value of the force during a period, T , is

$$\frac{1}{T} \int_t^{t+T} F dt = A_0 + \frac{A_1}{T} \int_t^{t+T} e dt + \frac{A_2}{T} \int_t^{t+T} e^2 dt + \dots$$

The terms involving odd powers of e disappear, for

$$\int_t^{t+\frac{T}{2}} e^m dt = - \int_{t+\frac{T}{2}}^{t+T} e^m dt \quad \text{and hence} \quad \int_t^{t+T} e^m dt = 0$$

when m is an odd integer.

Since A_0 is the force corresponding to the zero reading, we have for the force due to the voltage

$$F - A_0 = \frac{A_2}{T} \int_t^{t+T} e^2 dt + \frac{A_4}{T} \int_t^{t+T} e^4 dt + \dots$$

and it is evidently impossible that the instrument should indicate the average value of the voltage.

Let us consider the relation between deflection and voltage to be parabolic, as is approximately the case with the capillary electrometer, so that

$$D = b - \frac{b}{a^2} (e - a)^2 = 2 \frac{b}{a} e - \frac{b}{a^2} e^2$$

where b is the maximum positive value of D , occurring when $e = a$. (See Fig. 1.) Then since the law of force is here the same as the

$$F - A_0 = - \frac{1}{T} \int_t^{t+T} \frac{b}{a^2} e^2 dt = - \frac{b}{a^2} E^2$$

where

$$E = \sqrt{\frac{1}{T} \int_t^{t+T} e^2 dt}$$

law of deflection and may be designated the *effective* value of e .

The deflection in this case is always negative and follows a parabolic law of variation with E , in approximate agreement with the experimental results (see p. 529).

An instrument can only give readings proportional to the numerical average value of e by having the coefficient A_1 change sign with e , and $A_2 = 0 = A_3 = \text{etc.}$ This can be accomplished in two ways; first, by reversal of the force at the instant e is passing through the value zero; secondly, by making the permanent factor in the deflecting force controlled as to direction by the variable factor. The first

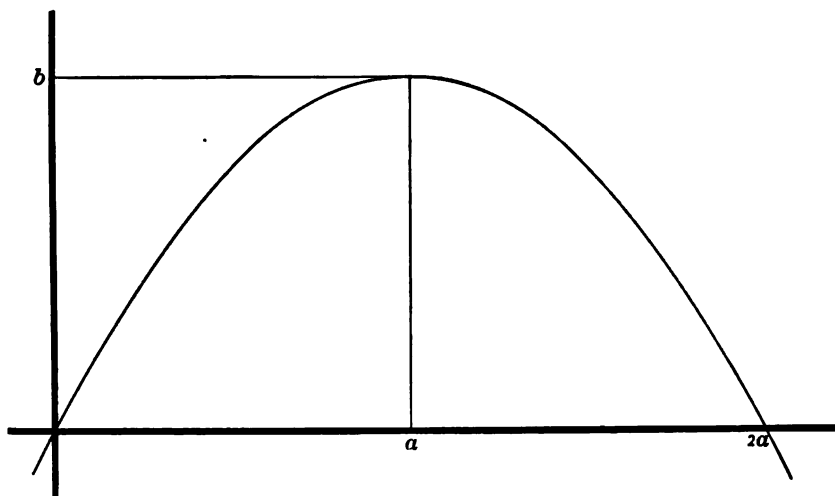


Fig. 1.

method has been applied in an apparatus already described in this *Bulletin*.¹ The second method is difficult of application, but a hypothetical case may be mentioned by way of illustration.

Let the moving element in the instrument be a soft iron needle surrounded by a coil of wire of sufficient turns, so that the smallest current to be measured—in fact, a small percentage of its maximum value—will magnetize it to saturation. Let the current to be measured traverse also a coil so placed that its field will deflect the magnetized needle. Then the torque acting on the needle will be constant in direction and proportional to the current. Besides

¹ Lloyd and Fisher, Bull. 4, p. 467; 1908, where a rotating commutator in synchronism with an A. C. generator is used.

being impracticable on account of the prohibitive inductance necessary, this scheme is open to certain theoretical objections, but it will serve as an illustration of the necessities of the case.

Dynamometer ammeters for alternating currents have been devised¹ in which the deflection is proportional to the current, since by the positions of the coils the force is made to vary inversely as the deflection.

It may seem at first sight as though the indication of such an instrument would depend upon the *average* value of the current, but it has been shown above that it would depend upon the *effective* value. The reason for this is, that it is the *force* which is integrated, and the deflection is determined by the average value of the force, and not by the average value of the deflections which would result if the variable were to proceed very slowly through a cycle.

If D be the deflection, i the instantaneous current, $F = k_1 \frac{i^2}{D}$, and since the controlling force is proportional to the deflection, we have

$$D = \frac{k_2}{D} \frac{1}{T} \int_0^T i^2 dt, \text{ or } D = \pm k_3 I$$

where I is the effective value of the current.

¹H. Bruger, *Phys. Zs.*, 4, p. 876; 1903.

ON THE USE OF THE CAPILLARY ELECTROMETER WITH ALTERNATING VOLTAGES.

The capillary electrometer, introduced by Lippmann³ in 1873, has been used extensively by physiologists and physical chemists,⁴ and more lately has received attention as a laboratory instrument for more general use.⁵

This form of electrometer has been used more generally as a null instrument, but can also be used for deflections, if the scale be calibrated. Calibration is necessary, since except for very small ranges the deflection is not proportional to the applied voltage. When used as a deflecting instrument, the mercury in the capillary is usually charged negatively and the electrolyte positively, but this is not necessary, as deflections can be obtained in either direction.

So far as is known to the author, the instrument has never heretofore been used with alternating potentials, except in the wave-tracing experiments of Burch. His instrument was designed to have short period and to be dead beat, and very low voltages were used.

If a curve be plotted between deflections and applied direct voltages, it has the general form of a parabola, as shown in Fig. 3.

It was surmised from this that the instrument would respond to an alternating voltage with a deflection, and such was found to be the case.

The electrometer was set up, as shown in Fig. 2, the beaker having a layer of mercury at the bottom covered with the electrolyte. A glass tube was drawn to a capillary and bent as shown, then partly filled with mercury and the capillary end immersed in the electrolyte. Platinum wires dipping into the mercury formed the electrodes, and were protected from contact with the electrolyte. A micrometer microscope was focussed upon the surface separating

³G. Lippmann, *Pogg. Ann.*, **149**, p. 546; 1873.

⁴W. Ostwald, *Zs. f. Phys. Chem.*, **1**, p. 404; 1887, and Ostwald and Luther's *Physiko-chemische Messungen*, 2te Auflage, p. 333.

⁵G. J. Burch, *London Electrician*, **87**, p. 380; 1896.

C. F. Burgess, *Trans. A. I. E. E.*, **15**, p. 337; 1898.

A. D. Cole, *Bull. Scientific Lab. Denison Univ.*, **11**, p. 265; 1902.

A. W. Vining, *Ann. chim. phys.*, **9**, p. 272; 1906.

the two liquids in the capillary, and any change in the position of the meniscus due to electrification could be measured.

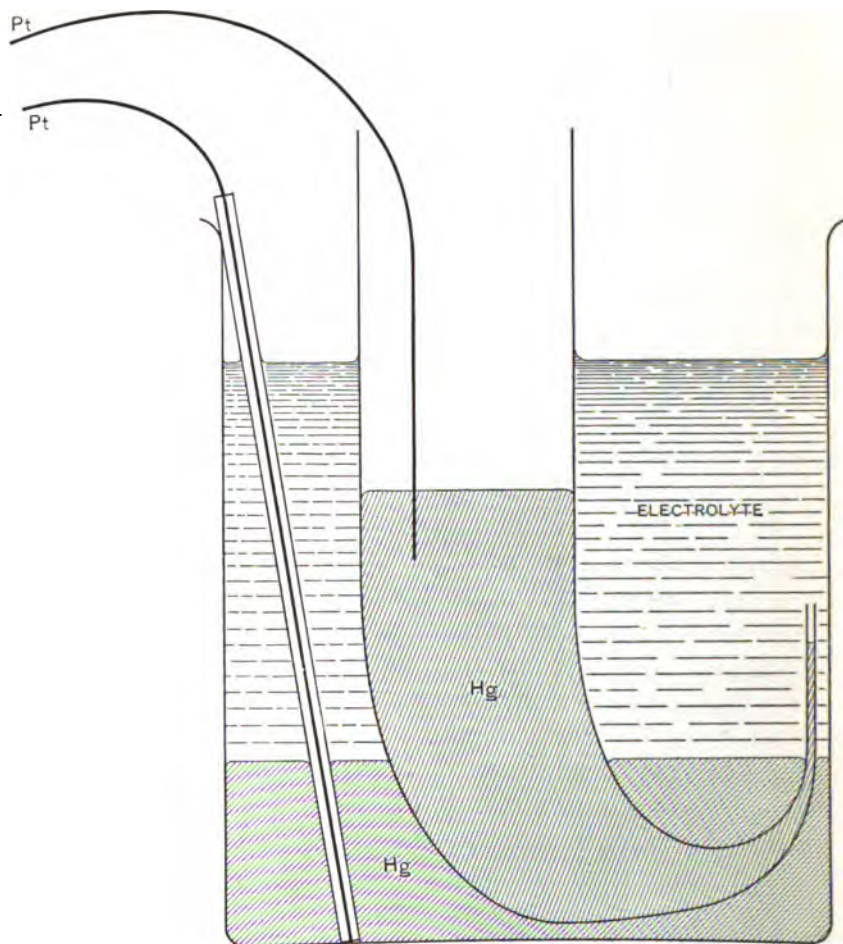


Fig. 2.

The electrolyte used was an aqueous solution of sodium chloride. This was chosen as less likely to gas at low voltages than sulphuric acid, the electrolyte usually employed.⁶ The diameter of bore at the surface of separation was about 0.7 millimeter. A finer bore was avoided in order to lessen trouble in case of gassing.

⁶ F. Paschen, Wied. Ann., **39**, p. 43; 1890.

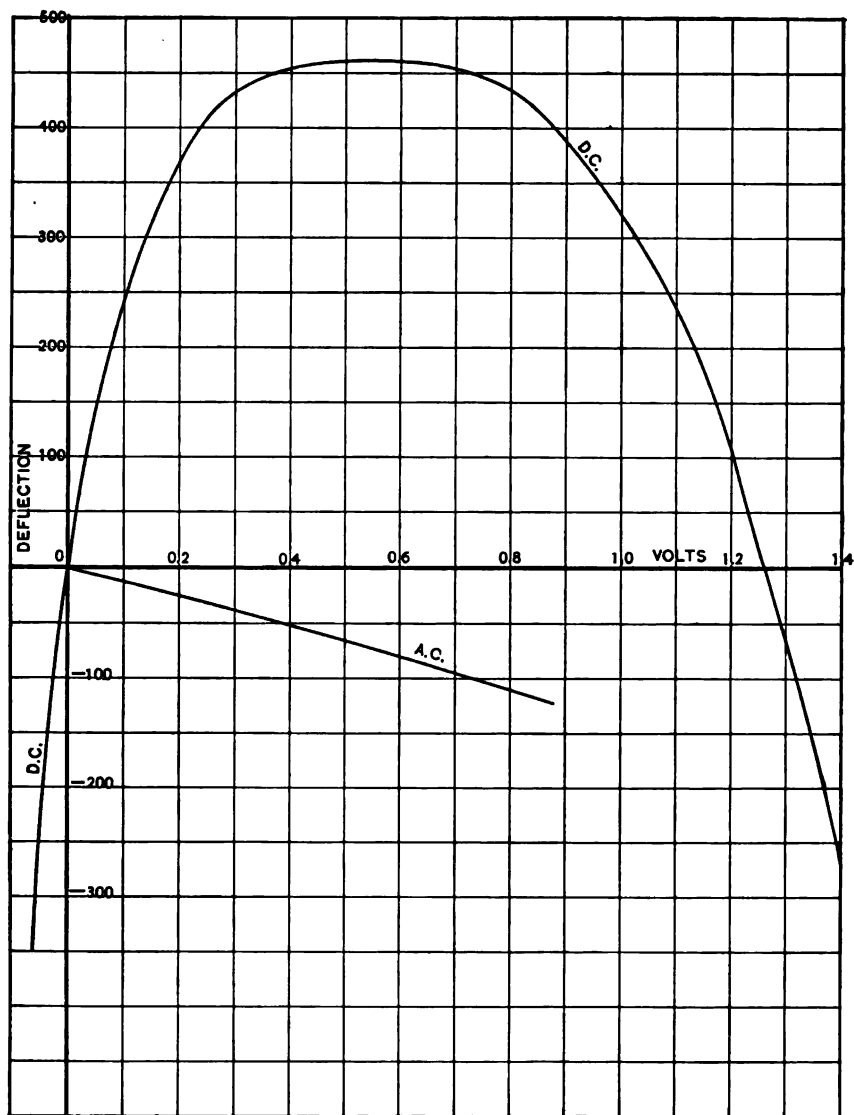


Fig. 3.

The electrometer was first calibrated with direct voltage, and the results are shown graphically in Fig. 3, curve D. C. Alternating voltages were next applied, with the results shown in curve A. C., whose abscissæ are effective volts, as read upon a dynamometer. Direct voltages are considered positive when the mercury in the capillary is connected to the negative pole of the battery. With direct voltages the position of the meniscus was perfectly definite and reproducible. With alternating voltages the readings, except for very small voltage, are not steady and are not reproducible. The set given in the figure is simply a sample of their general value. On account of this uncertainty in deflection the electrometer could not be used for an accurate measurement of alternating voltage. This difficulty is probably due to the fact that the surface never comes to rest, but a photographic record would be necessary to detect the motion.

As a *detector* of alternating voltages, it would be perfectly feasible to use the electrometer. The sensibility, however, is very much smaller for A. C. than for D. C. on the same instrument, and it does not maintain its advantages in this field of work. In an age when a telephone is a universal laboratory instrument, the electrometer can hardly be expected to attain very general use. There are two features of this instrument, however, which should be kept in mind with regard to its use for both direct and alternating voltages. These are the fact that it is an electrostatic instrument, requiring only sufficient current to charge or polarize it, and its cheapness. In many laboratories where the cost of more elaborate apparatus is prohibitive, the capillary electrometer may be made to answer many purposes.

WASHINGTON, December 30, 1907.

SELECTIVE RADIATION FROM THE NERNST GLOWER.

By W. W. Coblentz.

INTRODUCTION.

The measurement of very high temperatures is based upon an extrapolation of the laws governing the energy emitted by a body with change in temperature. Our knowledge of these laws is confined to the radiation from platinum and from a hollow inclosure (so-called "black body"), which is the nearest approach to a complete radiator. The remarkable progress that has been made in the development of processes requiring an accurate knowledge of the temperatures involved makes it imperative to study the laws of radiation of various substances with variation in temperature. In order to determine these radiation laws, it is generally necessary to study the spectral energy curves, using for the purpose a prism that is transparent to heat rays and some sort of very sensitive heat measuring device, such as, for example, a bolometer or a thermopile. It is also possible to study the total radiation emitted. The chief difficulty in establishing the so-called radiation constants of substances lies in determining the exact temperature of the radiating surface.

The distribution of radiant energy in the normal spectrum of all solid bodies thus far studied is unsymmetrical about the maximum of the energy curve, having the appearance of a probability function modified by suitable constants. The solids heretofore studied,¹ in which it was possible to determine the approximate temperature, have spectral energy curves, which are represented fairly well by the function,

$$(1) \quad E = c_1 \lambda^{-a} e^{-c_2 \lambda T}$$

¹ Paschen, *Ann. der Phys.*, (3) 58, p. 455; 60, p. 663; 1897; (4) 4, p. 277; 1901. Lummer and Pringsheim, *Verh. d. Deutsche Phys. Gesell.*, 1, p. 215; 1899.

In the case of the complete radiator, or so-called black body, the exponent $\alpha = 5$, while for platinum, $\alpha = 6$.

In order to determine the constants of the above equation from the spectral energy curves, it is necessary to know the temperature of the radiator. Fortunately the exponent, α , may also be obtained from the spectral energy curve in which the temperature, T , is constant without knowing the actual temperature, for it can be shown from eq (1) that the ratio of the emissivities (the observed bolometer-galvanometer deflections) for any two wave-lengths λ and λ_{max} , is:

$$(2) \quad \frac{E}{E_{max}} = \left[\frac{\lambda_{max}}{\lambda} e^{\frac{\lambda - \lambda_{max}}{\lambda}} \right]^{\alpha}$$

from which α may be determined. It was found by Paschen, that for carbon, platinum, etc., the value of α obtained in this way was in agreement with that obtained from a knowledge of the temperature of the radiator.

With this equation it is possible to obtain some idea of the probable total emissivity of a radiating body, as to whether it is proportional to the 4th power ($\alpha - 1 = 4$ for a black body, $\alpha - 1 = 5$ for platinum) or to some higher power of the absolute temperature.

Of course the assumption is made that the emissivity function is similar to that of platinum and of a black body. How far this assumption falls short of the observed facts is brought out in the present paper, in which it is shown that the observed radiation curve of a Nernst filament, which at high temperatures gives an apparently continuous spectrum, is in reality the composite of numerous sharp emission bands, which increase in intensity and broaden out with rise in temperature.

The constant α for various substances has not been extensively investigated, and it is purposed in this, and in subsequent reports, to state the results of a study of various substances (including incandescent filaments) together with the redetermination of the constants of a complete radiator, or so-called black body.

The study of the metal lamp filaments is of interest in connection with the speculations as to whether the great light emissivity (high

luminous efficiency) is due to an abnormal emission in the visible spectrum, with a corresponding suppression of the radiation in the infra-red, or whether the effect is due to the high temperature at which the lamp is burning. In the latter case the distribution of energy in the spectrum may be uniform (no discontinuities), but a larger portion of it will lie in the visible spectrum than at low temperatures. From a theoretical consideration of the fact that the filaments are metallic, electrical conductors with a high reflecting power in the visible, and probably reflecting uniformly high in the infra-red,² one would expect the distribution of energy in the spectrum to follow a law similar to that of platinum, but with different constants. The results obtained thus far from a study of such metals as tungsten and osmium support this hypothesis. On the other hand, in the case of oxides, which conduct electrolytically at high temperatures, there is not sufficient data from which to form even a working hypothesis. All of the oxides thus far examined have no strong absorption bands near the visible spectrum; the only exceptions being the oxides of the rare earths, such as cerium, thorium, lanthanum, didymium, erbium, etc., the compounds of which have strong, sharply defined absorption bands in the visible spectrum, and at least some of these have absorption bands in the infra-red. A low reflecting power seems to be characteristic of the oxides (like transparent media, electrical nonconductors) throughout the infra-red to about 8μ , beyond which point they have strong bands of selective reflection. In this region the emission will be suppressed in proportion to the reflecting power.³ In the rest of the spectrum the emission will be proportional to the absorbing power (general absorption), while at the point where there is a band of selective absorption in the transmission spectrum there will be an emission band in the emission spectrum, provided the radiation is a purely thermal one, following Kirchhoff's Law.

In the case of the Auer mantle the emission spectrum is a series of emission bands at 1 to 2μ , with practically no emission in the region from 4 to 7μ , while beyond 9μ the spectrum is continuous,

²Aschkinass, *Ann. d. Phys.*, (4) 17, p. 960; 1905; Einstein, *Ann. d. Phys.*, (4) 17, p. 132; 1905; 22, p. 181, 569, 800; 1907.

³Aschkinass, *Verh. d. Deutsch. Phys. Ges.* 17, p. 101; 1898; Rosenthal, *Ann. der Phys.*, (3) 68, p. 791; 1899.

and is apparently as intense as that of a complete radiator at the same temperature.⁴

In the case of the Nernst filament, which is a combination of the oxides of cerium, thorium, and zirconium, the compounds of which are noted for their strong absorption bands, one would hardly expect the emission to follow the same general law of energy distribution known for metals. This assumption, however, has been made in the past, notably by Lummer and Pringsheim⁵ and by Mendenhall and Ingersoll.⁶ The first two investigators, from a rather cursory examination of the Nernst filament, under normal power consumption, found a smooth continuous curve, with a maximum at wave-length of 1.2μ . From this and from the Wien displacement law, $\lambda_{\max} = \text{const.}$ (this constant is 2940 for a black body and 2630 for platinum), assuming that the Nernst glower belongs to the same class of radiators as platinum and a black body, Lummer and Pringsheim computed the maximum temperature and found it to be 2450° Abs. and the minimum temperature 2200° Abs. Their computed energy curve of a black body having its maximum emission at 1.2μ departs considerably from the observed curve.

Mendenhall and Ingersoll compared the emission of the Nernst glower in terms of a constant comparison lamp, for a certain wave-length in the visible spectrum, at the melting points of gold and of platinum. From this they extrapolated on a straight line (assuming that equation (1), known as Wien's Equation, holds for the glower) and found the temperature at normal or any desired power consumption. This leads to erroneous values, due to the fact that the spectrum is the composite of numerous emission bands, which rapidly increase in intensity, in the short wave-lengths, with rise in temperature. They found the normal temperature to be 2300° Abs., disagreeing with a recent determination by Hartman,⁷ who, by means of thermocouples of different thickness placed against the glower, and correcting for heat conduction by extrapolating to a temperature corresponding to an infinitely thin couple, found the temperature to be 1800° Abs. Although this method had previously

⁴Rubens., *Phys. Zs.*, 6, p. 790; 1904.

⁵Lummer and Pringsheim, *Verh. deut. Phys. Gesell.*, 8, p. 36; 1901.

⁶Mendenhall and Ingersoll, *Phys. Rev.*, 24, p. 230, 1907; 25, p. 1; 1907.

⁷Hartman, *Phy. Rev.*, 17, p. 65; 1903.

been extensively used with fair success in measuring the temperature of gas flames, it is less suited to the glower in which there is no layer of hot gas to even partially compensate for the heat lost by conduction.

That the Nernst glower emits selectively in the visible spectrum has been shown by Kurlbaum and Schulze,⁸ who found a strong emission band at $.52\mu$, which became fainter with rise in temperature and disappeared entirely at high temperatures.

To this brief review of what has been done on the Nernst glower may be added a paper by its inventor,⁹ who showed that the conductivity is electrolytic, while Kaufmann¹⁰ showed that in spite of the entirely different inner mechanism of conduction of a gas in a vacuum tube and in a Nernst glower the electrodynamic phenomena are nevertheless very similar.

METHODS AND RESULTS OF PRESENT INVESTIGATION.

In the present investigation of the Nernst glower the distribution of radiant energy in the spectrum was determined at different energy consumption, and hence at different temperatures. For several filaments the apparent black body temperature, corresponding to different values of energy consumption, was measured by Drs. Waidner and Burgess, with an optical pyrometer, for red, green, and blue light. The values given in Table I were obtained from their watt-temperature curve, extrapolated for high temperatures. These values are of interest in showing the variation in selective emission with rise in temperature.

The apparatus used in this work consisted of a spectrometer,¹¹ having mirrors 10 cm in diameter and of 50-cm focal length, a perfectly clear fluorite prism, having an angle of 60° and circular faces 33 mm in diameter, and a bolometer¹² with a hemispherical reflecting mirror. The bolometer strip and spectrometer slit were 0.6 mm. wide, or about $4'$ of arc. The upper part of the spec-

⁸ Kurlbaum and Schulze, *Verh. d. Deutsch. Phys. Gesell.*, **5**, p. 428; 1903.

⁹ Nernst *Zs. für. Electrochemie*, **6**, p. 41; 1899.

¹⁰ Kaufmann, *Ann. d. Phys.*, (4) **2**, 158 p., 1900; **5**, p. 757; 1901.

¹¹ For adjustments see "Investigations of Infra-red Spectra," Vol. 1; Carnegie Institute of Washington, 1905.

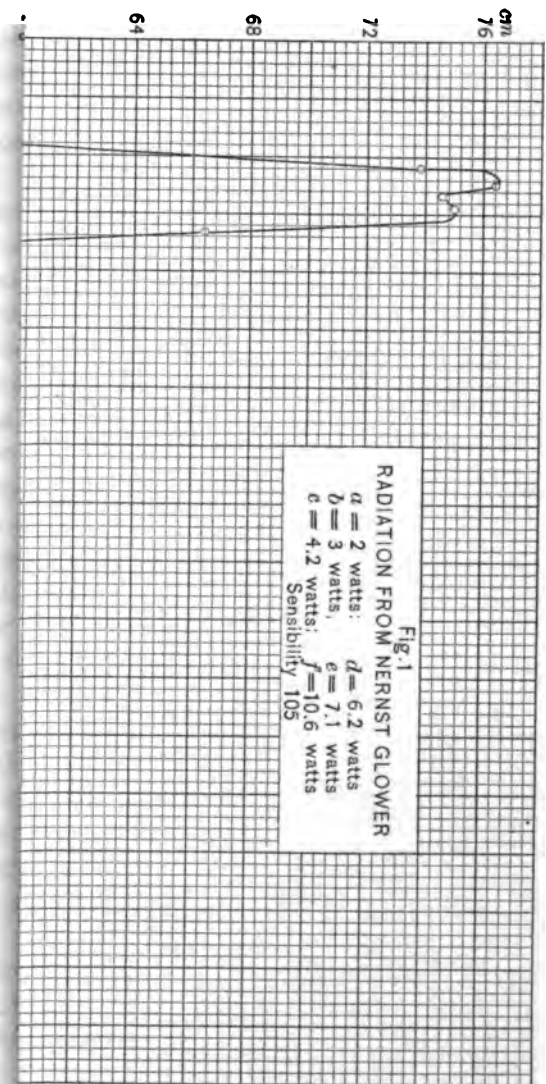
¹² Described in this Bulletin, **4**, p. 391.

trometer, containing the collimating mirrors, and the Wadsworth mirror-prism table were entirely inclosed by a thin sheet-metal box, lined with black velvet. The spectrometer slit was covered with a clear plate of fluorite. The openings in the top for adjusting the mirrors were closed with soft wax, while the hole admitting the axis for rotating the mirror-prism table was tightened with a nut and packing.

Within the box, and below the level of the mirrors, were placed vessels containing phosphorous pentoxide and sticks of potassium hydroxide, which entirely eliminated the absorption bands of CO_2 and water vapor from the emission curves. A water-cooled shutter was placed before the spectrometer slit and the Nernst glower, inclosed in an asbestos case to prevent air currents, was placed close to the shutter. Observations were also made without inclosing the glower, to prove that the effects observed are not due to stray radiation from the asbestos case.

The auxiliary galvanometer^{12a} of 5.3 ohms resistance, with a single swing of 4 to 5 seconds, had a current sensibility of $\frac{1}{2} = 1.6$ to 1.5×10^{-10} ampere. A greater sensitiveness for the same period would have been possible, by using a lighter suspension. The present suspension of 10 magnets was just heavy enough so as not to be affected by tremors. By reducing the number of magnets from 12 to 10 and by placing the mirror at the center of the suspension the sensibility was increased by 100 per cent. Using a bolometer current of .04 ampere the computed temperature sensibility was $5^\circ \times 10^{-6} \text{C.}$, which was generally far in excess of that required. The deflections were reduced to 14 to 15 cm by inserting resistance in series with the galvanometer. The individual readings varied by 1 mm, or less than 1 per cent, which is as close as the nature of the work required, since the actual deflections were as high as 2000 cm. Furthermore, at high temperatures (especially when run above normal power consumption) the filament changed in emission by that amount during the series of observations.

^{12a} Described in this Bulletin, 4, p. 432. The coils of this instrument are 32 mm in diameter. The proportionality of the deflections with current, at this sensibility, was exact up to 18 cm.



The calibration curve of the fluorite prism was constructed from the refractive indices, found by Paschen,¹³ which, after plotting all the observations made by different observers, seems to be as close to the most probable values as observation will permit. Unfortunately the dispersion curve passes through a double curvature at 1.5μ , just where the energy spectra have their maxima. In this region the correction for purity (the so-called "slit-width" correction) is a maximum.

The wave-lengths in the calibration curve were plotted to the fourth decimal place, so that there is a certainty of the values to at least the second decimal place. This, however, is of less importance than the value of the slit-width correction, which was made according to Paschen,¹⁴ the values being obtained from a curve plotted on a large scale to insure an accuracy greater than required in the work. In a few cases a correction was made for the reflecting power of the silver mirrors, but it was found negligible except in the visible spectrum.

With this apparatus a series of energy curves was obtained, varying the energy consumption from 16 watts (the lowest at which the glower would conduct without using a transformer) to 123 watts, which is far above the normal. The energy curves, which were continuous, underwent great variations in appearance with rise in temperature. At 2.5μ and 3.5μ elevations and depressions would generally appear in the curves, which could not be attributed to experimental errors. Since previous work seemed to show that the spectrum is continuous, an attempt was made to locate the cause of the disagreement in the apparatus, the calibration, or in the slit-width correction curve, but without avail until the filament was run on a 2000-volt transformer which permitted a low heating of the glower. At the lowest temperature the glower was a grayish red. The results given in Fig. 1 are for a 110-volt A. C. glower No. 118, each point being the mean of at least two observations. The ordinates are about three times the observed galvanometer deflections. These results are entirely different from anything hitherto observed in the emission of solids in the infra-red. At the

¹³ Paschen, *Ann. d. Phys.*, (4) 4, p. 299; 1901.

¹⁴ Paschen, *Ann. d. Phys.*, (3) 60, p. 714; 1897.

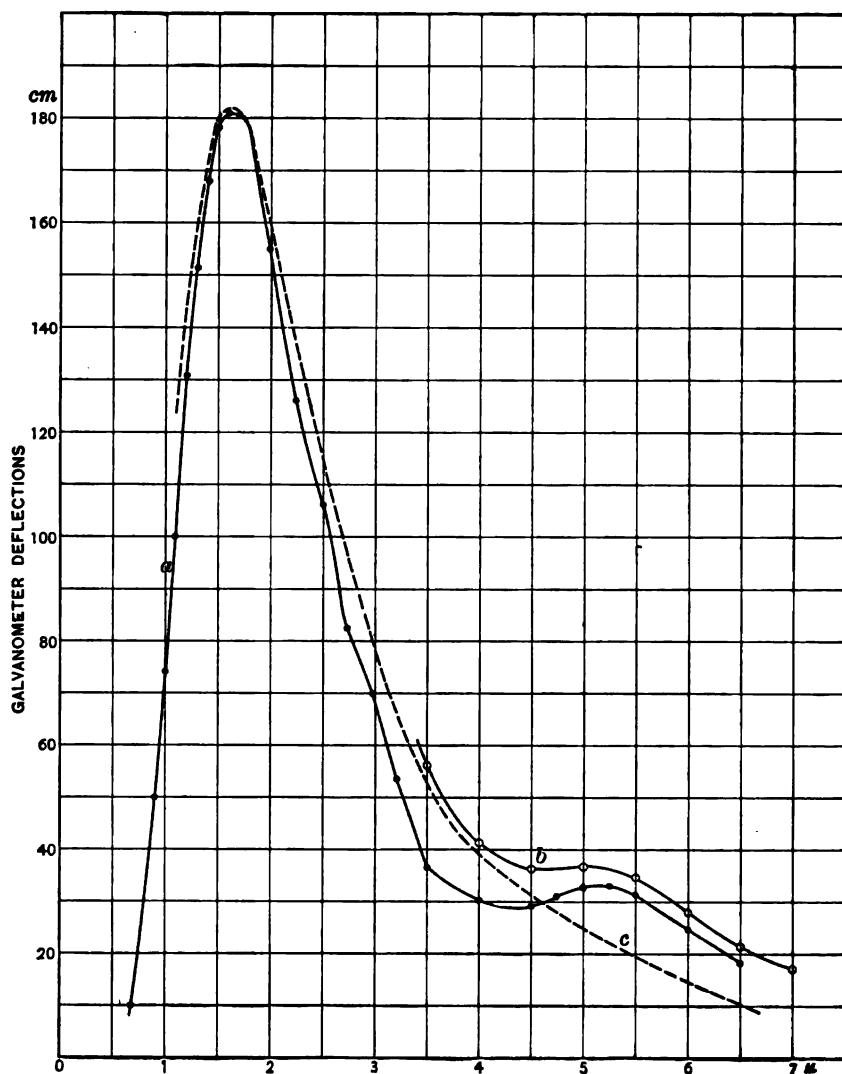


Fig. 2.—Nernst Glower, No. 118.

$a = 17$ watts, $b = 21.8$ watts, $c = \text{"black body."}$

lowest temperatures (800–900° C.) the bands in the region of long wave-lengths are the most intense. As the temperature increases, the bands in the region of 2.5μ increase very rapidly in intensity, so that by the time the temperature has increased to 1000° to 1100°, the intensity of the group of bands at 2μ is far in excess of that at 5.5μ .

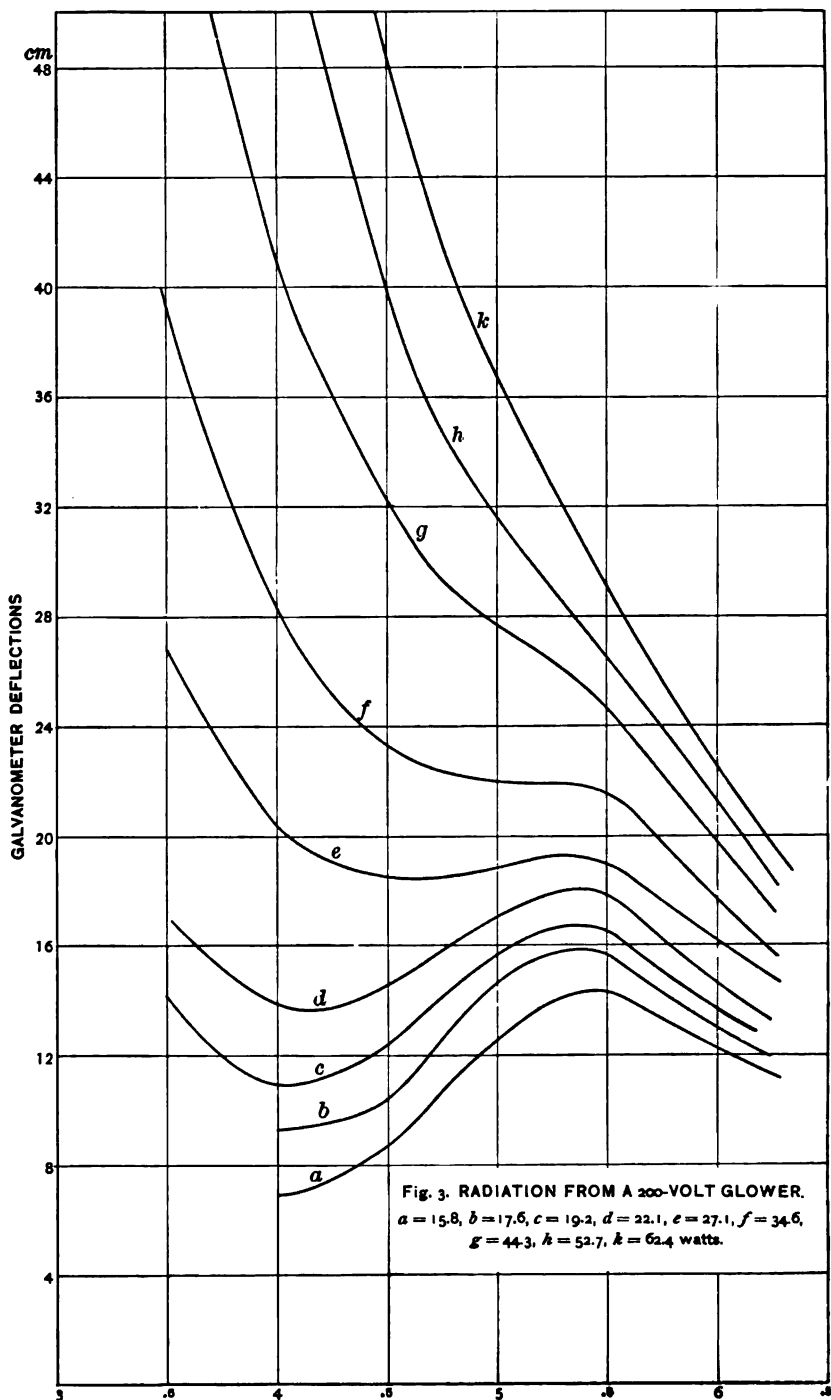
The depression at 3 to 3.5μ persists even at normal energy consumption. The curves at higher temperatures often show a slight depression at 2.5μ not attributable to experimental errors. As the temperature rises (curve *e*, Fig. 1) new emission bands appear, notably at 2.5μ and at 4μ . This shift of the maximum intensity of the bands, with increase in temperature, is to be expected, if the emission is a purely thermal one, following Kirchhoff's law, and is the most conspicuous illustration yet recorded.

In Fig. 2 the emission curves are shown for a 110-volt glower (serial number 118) at 17 and 21.8 watts respectively. It should be noticed that the emission curve has become smooth and continuous, with but two maxima, at 1.4 and 5.5μ respectively. The curve *c* for a black body, having a temperature corresponding to maximum at 1.65μ and of the same intensity as that of the filament at this point, falls far below the emission curve at 5.5μ . This shows that the maximum at 1.65μ is not as high as it should be, if the glower followed a radiation law similar to that of a complete radiator or platinum, as was heretofore assumed.

In Fig. 3 is shown a series of energy curves of a 200-volt filament, at different power consumptions, viz: $a = 15.8$, $b = 17.6$, $c = 19.2$, $d = 22.1$, $e = 27.1$, $f = 34.6$, $g = 43.3$, $h = 52.7$, $k = 62.4$ watts respectively. This illustrates very well the elimination of the emission minimum at 4μ . The curves are for the same sensibility¹⁵, 80, of the galvanometer as compared with the sensibility of 105 in Fig. 1.

The radiation of a 110-volt filament, No. 118, at a power consumption of 77.7 watts is given in Fig. 4, curve *a*. In the same figure, curve *b* represents the distribution of energy of a 220-volt filament (No. 120) at a power consumption of 102.5 watts, which is

¹⁵ For convenience, in practice the sensibility is expressed in arbitrary units which will be described elsewhere.



somewhat above the normal. The energy distribution is irregular, with an abnormal increase in intensity at 0.75μ due no doubt to the sudden increase in intensity of an emission band, as illustrated in

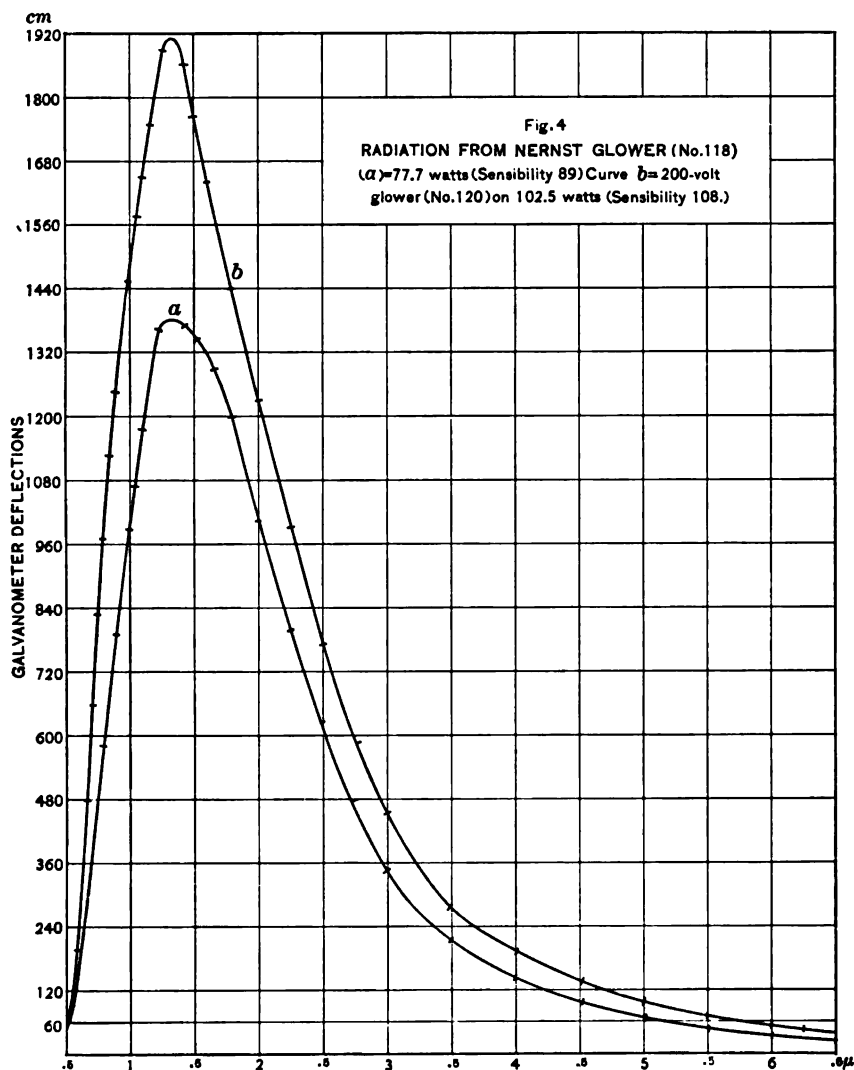
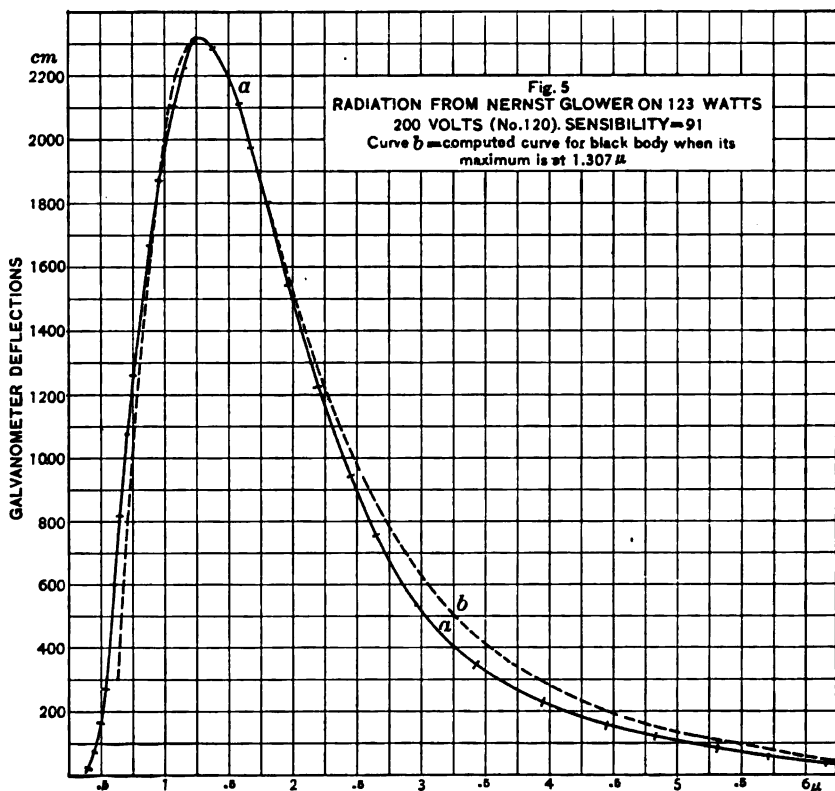


Fig. 1. Fig. 5 shows the radiation curve of the 220-volt filament (No. 120) at 123 watts, which is far above the normal. The emissivity decreased rapidly in intensity, probably due to the evapora-

tion and decrease in diameter of the filament. Within the experimental errors of observation the results indicate that the emissivity is abnormally increased in the visible. The theoretical curve of a "black body," having a maximum emission coinciding with that of the glower at 1.3μ ($t = 1975^\circ\text{C.}$) falls below the observed curve in the visible, and is higher than the observed curve at 5μ , which is



just the reverse of the results obtained when using 16.8 watts. (See Fig. 2 for a similar case.) Since the galvanometer deflections were as high as 2000 cm and since errors as great as 25 to 30 per cent would be necessary to account for this discrepancy, it is evident that the radiation from the glower can not be compared with that of a complete radiator. The data obtained at low temperatures (Fig. 1), which indicate selective emission, are a further proof of this conclusion. An attempt was made to find the transmission of a thin

section made of the material which constitutes the glower, but without success, due to the inhomogeneity of the material. As a substitute, the radiation from (and through) a 0.5 mm layer of this

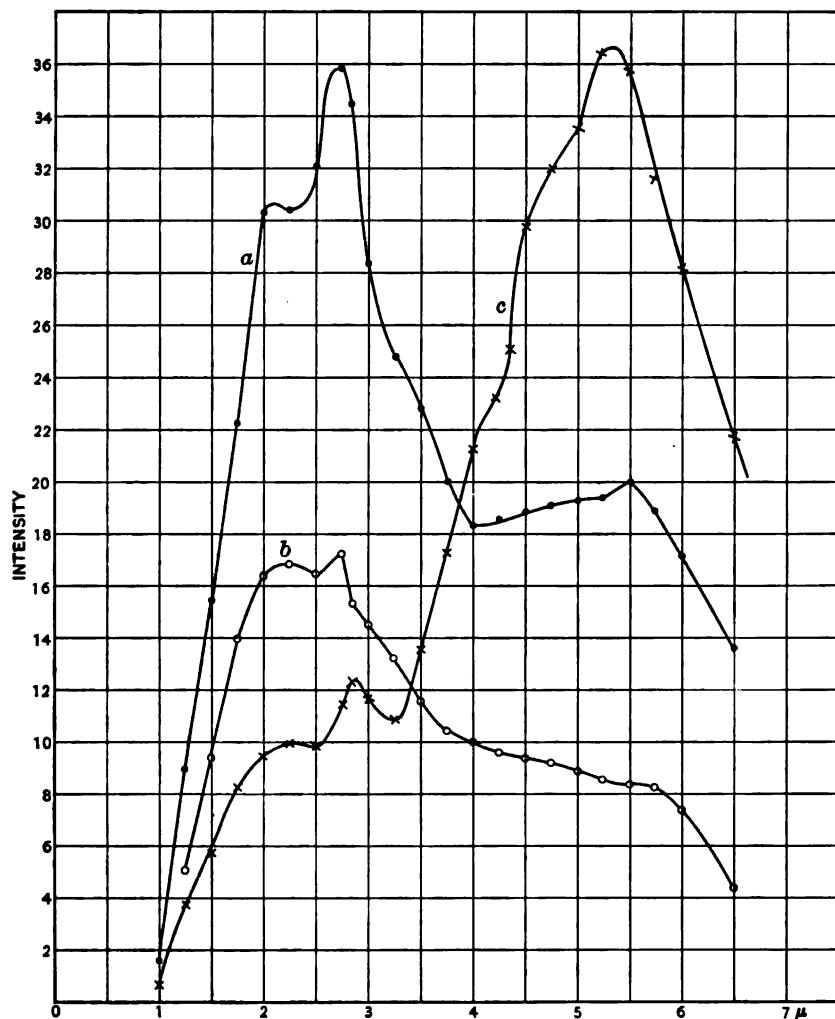


Fig. 6. Radiation from Nernst (A. C.). Material.

Curves *a* and *b*. Curve *c* = radiation from a Nernst heater tube.

material was found when placed on a strip of platinum heated electrically, or placed on a "heater tube" of a Nernst lamp from which the covering of kaolin had been removed. The surface was a deep

red, temperature about 900° C., similar to the filament in Fig. 1. The platinum was probably at a temperature of 1200° . In Fig. 6 is given the emission curves, *a* and *b*, of the glower material on the "heater tube." The emission bands at 2, 2.7, and 6μ are similar to those found in Fig. 1, but there is no great depression at 4μ , from which it would appear that this region is filled up by the radiation from the platinum wires, transmitted through the glower material. This is to be expected from the conditions indicated in Fig. 1. Curve *c*, Fig. 6, shows the radiation from a Nernst "heater tube," the covering being a refractory clay. An examination of the material separately will be necessary to show whether or not the maxima are true emission bands. The importance of such a radiator is evident in investigations in which a more uniform distribution of energy is desired than obtains in the glower, e. g. in searching for reflection bands of substances.

The radiation "constant," *a*, was found for several glowers, using equation (2). In order to do this it is necessary to accurately find the wave-length ($\lambda_{\max}$) corresponding to the point of maximum emission. This is possible from equation (1) by selecting two wave-lengths, λ_1 and λ_2 , corresponding to the points where the value of *E*, the galvanometer deflections, is the same. This gives:

$$(3) \quad \lambda_{\max} = \frac{(\log \lambda_2 - \log \lambda_1) \lambda_1 \lambda_2}{(\lambda_2 - \lambda_1) \log e}$$

In Table I are given the mean values of $\lambda_{\max}$ determined from several points on each curve. The maximum and minimum temperatures computed on the assumption that the glower follows a radiation law similar to that of a complete radiator and that of platinum, respectively (which of course is not true), are also given in this table. The apparent temperatures, determined with the optical pyrometer are included, to indicate changes in selectivity with rise in temperature. The values of *a* are plotted in Fig. 7, the wave-lengths being selected to correspond with those used in finding $\lambda_{\max}$ (eq. 3) in order to facilitate computation. For the 200-volt filament the so-called constant, *a*, drops from 7.5 at an energy consumption of 16 watts to a uniform value of 5.3 at 80 to 120 watts.¹⁸ For the

¹⁸ Excepting the curve for 102 watts, which is very irregular, showing unresolved bands of selective emission.

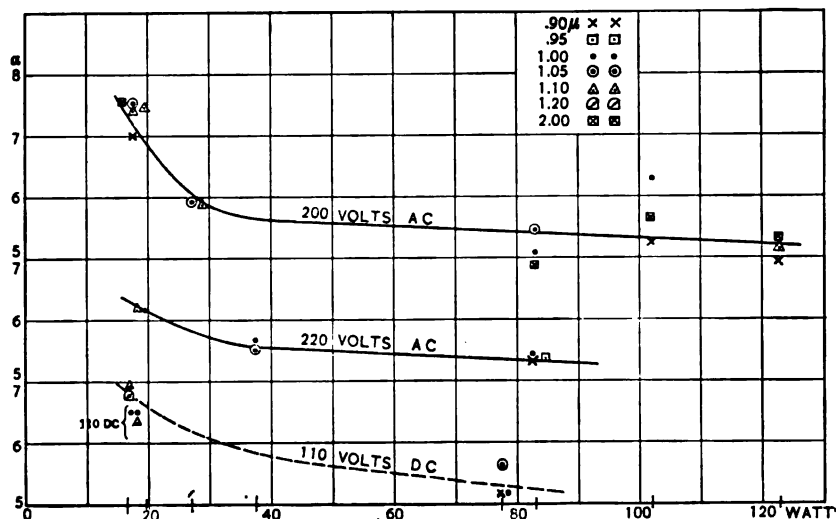


Fig. 7. Radiation constant, a , of Nernst Glower.

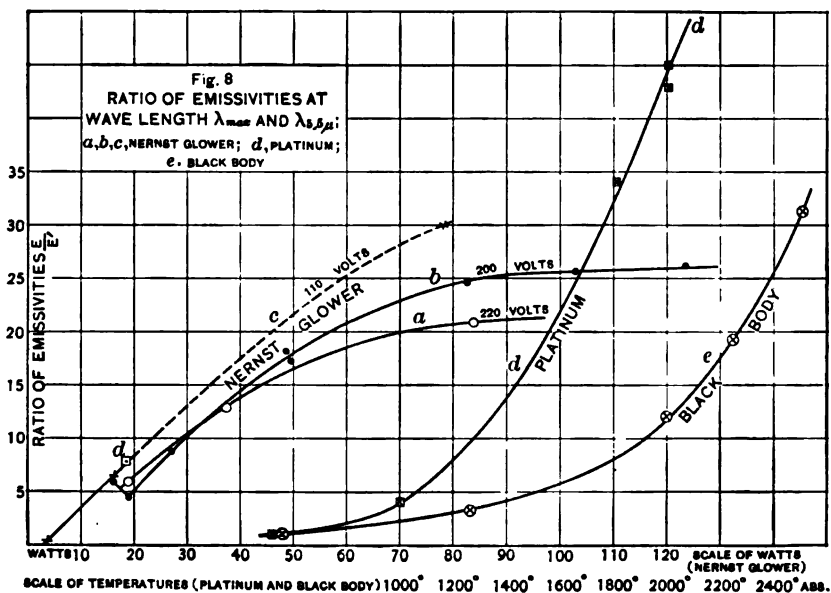
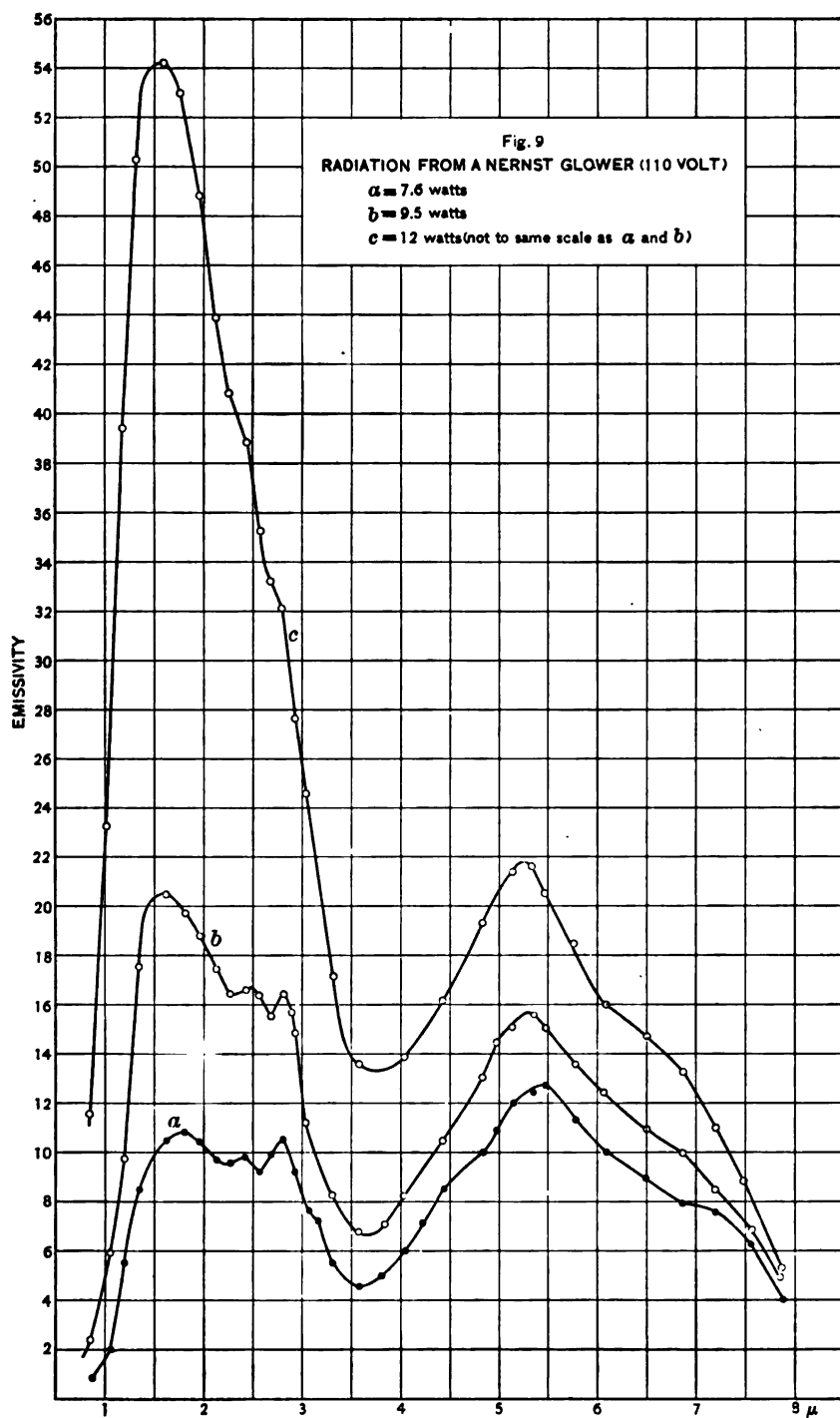


TABLE I.

Watts amp.	Temp. K. red	Temp. K. green	Temp. K. blue	$\lambda_{\max}$	$T_{\max} = \frac{2930}{\lambda_{\max}}$ — 273°	$T_{\min} = \frac{2620}{\lambda_{\max}}$ — 273°
Nernst 200 Volts, No. 120.						
{ 16.8 watts	1400° C					
{ 0.11 amp.	9452°	1490	1517	1.458 μ	1737	1517° C
{ 19.6 watts	1460			{ (1.433)		
{ 0.12 amp.	1490	1527	1550	{ 1.45 μ	1747	1532
{ 27.7 watts	1602					
{ .167 amp.	1627	1659	1677	1.445 μ	1757	1544
{ 83.2 watts	1985					
{ .412 amp.	2027	2052	2064	1.366 μ	1877	1647
{ 102.5 watts	2055			1.317	1950	1715
{ 0.5 amp.	2120	2148	2158			
{ 123. watts				1.307 μ	1972	1735
{ .6 amp.						
Nernst 110 Volts, No. 118.						
{ 17. watts	1400° C			1.621 μ		
{ .2 amp.	1438	1470	1484		1537	1342
{ 77.7 watts	2110			1.388		
{ .80 amp.	2135	2170	2988		1842	1612
Nernst 110 D. C., No. 121.						
{ 18.4 watts	1468	1507	1528	1.560	1607	1406° C
{ .2 amp.	1465					
Nernst 220 Volts.						
{ 19.6 watts				1.583	1577	1382
{ .115 amp.						
{ 37.2 watts				1.448 μ	1747	1537
{ .20 amp.						
{ 83. watts				1.360 μ	1879	1652
{ .4 amp.						



220-volt glower (No. 120) the value of a decreases from 6.3 to 5.3, while for a 110-volt D. C. glower the value changes from 6.9 to 5.30. In other words, the total emissivity drops from the 6th power to the 4.3th power of the absolute temperature. By taking the total radiation of the glower, Mendenhall and Ingersoll found the value of a to be about 9, with no consistent evidence of a variation of a with temperature. These results were obtained before the emission spectrum was resolved into separate bands, Fig. 1. From the latter it is evident that it is not permissible to apply this method of determining the "constant." In Fig. 8 is given a comparison of the ratio of the intensities of the emissivities of various glowers at various values of energy consumption for wave-lengths of 5.5μ (the position of the apparently second maximum, Fig. 2) and of λ_{max} , which, of course, varies with temperature. The thicknesses of the filaments were: $a = .63$ mm, $b = .71$ mm, $c = .98$ mm, $d = 1.05$ mm, respectively. The ratio rises from a value of 5 at 18 watts to a fairly constant value of about 25 at normal power consumption, i. e., the point on the potential-current curve where the potential is fairly constant. This ratio is a function of the thickness, and is an exact analog of what is found in the discharge of electricity through gases. In this figure this same ratio is plotted for a complete radiator and for platinum at various temperatures, from which it will be observed that the ratio is far from approaching an asymptotic value.

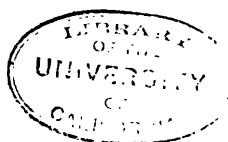
From whatever point of view we consider the data at hand, it is evident that even after the emission spectrum has become apparently continuous it does not follow so simple a law as has been established for solids emitting continuous spectra. It is also evident that any estimations of the temperature of the glower, based on these laws, will lead to erroneous results. From a commercial point of view, the efficiency of such a radiator, in which the emissivity is abnormally high at $.6$ to $.7\mu$, while the maximum at 1.2μ is abnormally low, must be much higher than that of substances having a radiation law similar to that of platinum, in which case, in order to attain a similar intensity in the visible spectrum, the maximum at 1.2μ rises to extremely high values.

By integrating the energy curve and taking the ratio of the visible to the infra-red radiation (it has been customary to take the dividing line at $.76\mu$) it is possible to obtain a rough estimate of the luminous (white light) efficiency for a given energy consumption. This gives values from 3.6 per cent at 27.7 watts, 5.5 per cent at 83 watts to 7.4 per cent at 123 watts. The total areas can not be compared, particularly for different filaments, because the cone of energy which enters the spectrometer slit does not cover more than about one-half of the prism face, and varies with the diameter of the filament and its distance from the slit. This will change the size of the galvanometer deflections uniformly throughout the spectrum. Thus in Fig. 4 the areas of the curves can not be compared, because they are for different filaments; but curve *b* might be compared with Fig. 5. This is of minor importance, for the curves are given to show the change in the relative distribution of energy with rise in temperature.

The results described in this paper are for filaments made from different lots of material, the 220-volt glower being at least 5 years old. The observations have been made at different dates, all in duplicate, some quadruplicate.

An examination of other filaments may show a variation in the minor details of the two groups of emission bands, as shown in Fig. 9, but the general results can hardly be modified without employing a much larger dispersion and a narrower bolometer. This, however, is of minor importance in considering the selective emission of the Nernst glower. The manner in which some of these emission bands are suppressed while others are intensified in different substances will be described in a subsequent paper.

WASHINGTON, February 10, 1908.



THE TESTING OF GLASS VOLUMETRIC APPARATUS.

By N. S. Osborne and B. H. Veazey.

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INTRODUCTION.

The Bureau of Standards published November 1, 1904, Bureau Circular No. 9, on "Testing of glass volumetric apparatus," containing specifications for glass volumetric apparatus, the verification of which would be undertaken, and also regulations for testing such apparatus.

As stated in the circular, the specifications with few exceptions were made to agree with those recommended by a committee of the American Chemical Society, consisting of Prof. E. W. Morley, chairman; Prof. Arthur A. Noyes; Prof. Theodore W. Richards, and Mr. E. E. Ewell.

In preparing the specifications, the regulations of the Kaiserliche Normal-Eichungs-Kommission of Germany were freely drawn upon and in many cases adopted without appreciable change. In cases, however, where the experimental work of this bureau indicated the desirability of modifications they were adopted.

Since the first publication of these regulations slight changes, suggested by the work of testing, have been introduced by revision of Circular No. 9, as "second edition," January 16, 1905, and "third edition," February 1, 1906.

From the fact that some difficulty has been experienced by manufacturers in complying with the specifications, and that users of apparatus have written in certain cases for explanations of the regulations, and also since several additional changes are deemed advisable, the occasion of the third revision of the regulations is taken to discuss in more detail the various specifications and rules for manipulation, to publish the results of experimental work having a direct bearing on construction and use, and to describe the methods employed at this bureau in testing volumetric apparatus.

A number of tables compiled for use in this work have been added for the convenience of those wishing to do their own testing.

The regulations included in this article, consisting of general specifications, special requirements, rules for manipulation, limits of error, and tests performed by this bureau, are as last revised and in the form shortly to be issued in Bureau Circular No. 9, fourth edition.

II. SPECIFICATIONS FOR GLASS VOLUMETRIC APPARATUS ACCEPTED FOR TEST.

The primary purpose of these specifications is to define the requisite qualifications for precision apparatus.

The bureau aims to encourage excellence in quality by cooperating with makers and users of apparatus, and to this end endeavors to assist manufacturers in establishing standards and perfecting methods. In order that users of standardized apparatus may fully benefit by the facilities of the bureau it is necessary for them when purchasing apparatus to be submitted for test to require that the apparatus shall comply with the specifications. By admitting for test only apparatus conforming to these standards the work of test-

ing is confined to apparatus whose utility is sufficient to justify the labor expended in the accurate calibration. Certain of the specifications, such as those regarding quality of glass and process of annealing before calibration, are for their fulfillment dependent largely on the integrity of the maker. Only by supporting conscientious makers in giving consideration, first, to quality, and, second, to cost, can users of standardized apparatus secure a high degree of excellence.

1. Types of Apparatus which will be Regularly Admitted for Test.—Measuring flasks; measuring cylinders, with or without subdivisions; transfer pipettes, i. e., without subdivisions; burettes and measuring pipettes, with partial or complete subdivisions.

2. General Specifications—

(a) *Units of capacity.*—The liter, defined as the volume occupied by a quantity of pure water at 4° C. having a mass of 1 kilogram, and the one-thousandth part of the liter, called the milliliter or cubic centimeter,¹ are employed as units of capacity.

(b) *Standard temperature.*—Twenty degrees Centigrade is regarded by the bureau as the standard temperature for glass volumetric apparatus, and an extra charge will be made for testing apparatus graduated for use at other temperatures.

(c) *Material and annealing.*—The material should be of best quality of glass, transparent and free from striæ, which adequately resists chemical action, and has small thermal hysteresis. All apparatus should be thoroughly annealed at 400° C. for 24 hours and allowed to cool slowly before being graduated.

(d) *Design and workmanship.*—The cross section must be circular and the shape must permit of complete emptying and drainage.

Instruments having a base or foot must stand solidly on a level surface, and the base must be of such size that the instruments will stand on a plane inclined at 15°. Stoppers and stopcocks must be so ground as to work easily and prevent leakage.

The parts on which graduations are placed must be cylindrical for at least 1 cm on each side of every mark, but elsewhere may be enlarged to secure the desired capacities in convenient lengths.

¹The cubic centimeter is not exactly the one-thousandth part of the liter, but the difference is of no consequence in volumetric analysis.

The graduations should be of uniform width, continuous and finely but distinctly etched, and must be perpendicular to the axis of the apparatus. All graduations must extend at least halfway around, and on subdivided apparatus every tenth mark, and on undivided apparatus all marks must extend completely around the circumference.

The space between two adjacent marks must not be less than 1 millimeter. The spacing of marks on completely subdivided apparatus must show no evident irregularities, and sufficient divisions must be numbered to readily indicate the intended capacity of any interval. Apparatus which is manifestly fragile or otherwise defective in construction will not be accepted.

(e) *Inscriptions*.—Every instrument must bear in permanent legible characters the capacity in liters or cubic centimeters, the temperature in Centigrade degrees at which it is to be used, the method of use, i. e., whether to contain or to deliver, and on instruments which deliver through an outflow nozzle the time required to empty the total nominal capacity with unrestricted outflow must be likewise indicated.

Every instrument should bear the name or trade-mark of the maker. Every instrument must bear a permanent identification number, and detachable parts, such as stoppers, stopcocks, etc., belonging thereto must bear the same number.

Figs. 1, 2, and 3 (two-fifths natural size) illustrate several arrangements of designating marks which are considered suitable.

3. Special Requirements.—(a) *Flasks*.—At the capacity mark or marks on a flask the inside diameter should be within the following limits:

Capacity of flask (in cc) up to and including	2,000	1,000	500	250	200	100	50	25
Maximum diameter (in mm)	25	20	18	15	13	12	10	8
Minimum diameter (in mm)	18	14	12	10	9	8	6	6

The neck of a flask must not be contracted above the graduation mark.

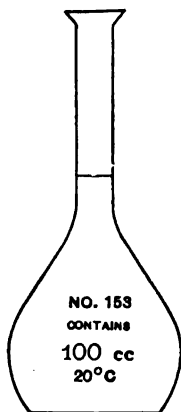


Fig. 1.

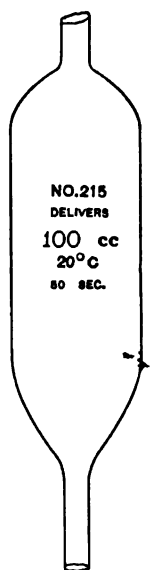


Fig. 2.

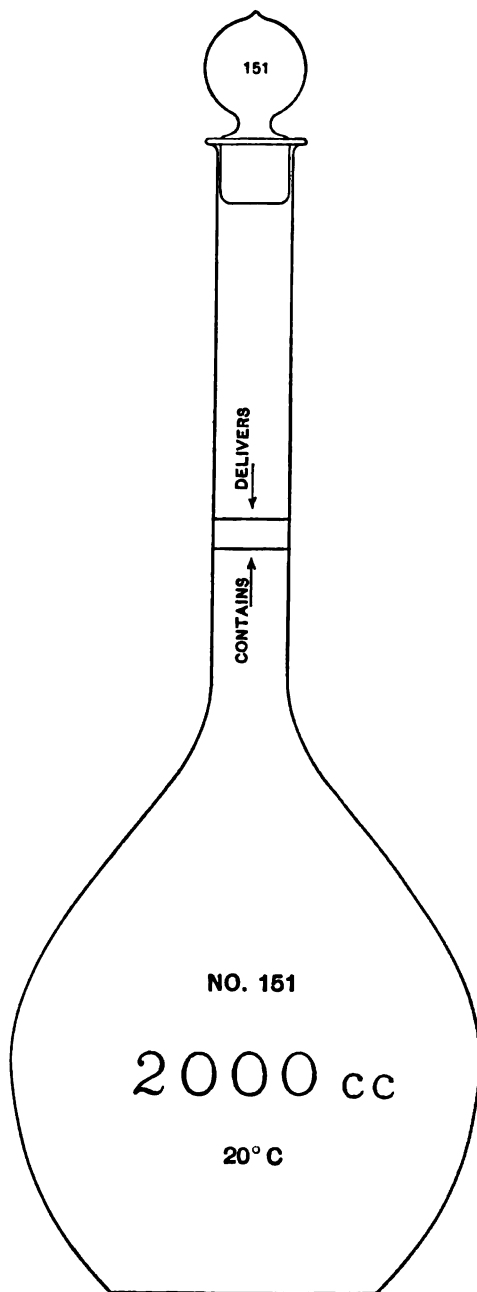


Fig. 3.

The capacity mark on any flask must not be nearer the end of the cylindrical portion of the neck than specified below:

Capacity	Distance from Upper End	Distance from Lower End
100 cc or less	3 cm	1 cm
More than 100 cc.....	6 "	2 "

Flasks of 1 liter or more but not less may be graduated both to contain and to deliver, provided the intention of the different marks is clearly indicated.

(b) *Cylinders*.—Only cylinders graduated to contain will be accepted for test.

The inside diameter of cylinders must not be more than one-fifth the graduated length.

(c) *Transfer pipettes*.—Pipettes for delivering a single volume are designated "transfer" pipettes.

The suction tube of each transfer pipette must be at least 16 cm long, and the delivery tube must not be less than 3 cm nor more than 25 cm long.

The inside diameter of any transfer pipette at the capacity mark must not be less than 2 mm and must not exceed the following limits:

Capacity of pipettes (in cc) up to and including	25	50	200
Diameter (in mm)	4	5	6

The outside diameter of the suction and delivery-tubes of transfer pipettes exclusive of the tip must not be less than 5 mm.

The capacity mark on transfer pipettes must not be more than 6 cm from the bulb.

The outlet of any transfer pipette must be of such size that the free outflow shall last not more than one minute and not less than the following for the respective sizes:

Capacity (in cc) up to and including.....	5	10	50	100	200
Outflow time (in seconds)	15	20	30	40	50

(d) *Burettes and measuring pipettes*.—Only those emptying through a nozzle permanently attached at the bottom are accepted for test.

The distance between the extreme graduations must not exceed 65 cm on burettes nor 35 cm on measuring pipettes.

The rate of outflow of burettes and measuring pipettes must be restricted by the size of the tip and for any graduated interval the time of free outflow must not be more than three minutes nor less than the following for the respective lengths:

Length Graduated	Time of Outflow	Length Graduated	Time of Outflow
65 Centimeters	140 Seconds	35 Centimeters	60 Seconds
60 "	120 "	30 "	50 "
55 "	105 "	25 "	40 "
50 "	90 "	20 "	35 "
45 "	80 "	15 "	30 "
40 "	70 "		

The upper end of any measuring pipette must be not less than 10 cm from the uppermost mark and the lower end not less than 4 cm from the lowest mark.

(e) *Burette and pipette tips*.—Burette and pipette tips should be made with a gradual taper of from 2 cm to 3 cm, the taper at the extreme end being slight.

A sudden contraction at the orifice is not permitted and the tip must be well finished.

In order to facilitate the removal of drops and to avoid splashing when the instrument is vertical, the tip should be bent slightly.

The approved form of tips for burettes, measuring pipettes, and transfer pipettes is shown in fig. 4.

4. Special Rules for Manipulation.—These rules indicate the essential points in the manipulation of volumetric apparatus which must be observed in order that the conditions necessary to obtain accurate measurements may be reproduced.

(a) *Test liquid*.—Apparatus will be tested with water and the capacity determined will, therefore, be the volume of water contained or delivered by an instrument at its standard temperature.

(b) *Method of reading.*—In all apparatus where the volume is limited by a meniscus the reading or setting is made on the lowest point of the meniscus. In order that the lowest point may be observed it is necessary to place a shade of some dark material immediately below the meniscus, which renders the profile of the meniscus dark and clearly visible against a light background. A convenient device for this purpose is a collar-shaped section of thick black rubber tubing, cut open at one side and of such size as to clasp the tube firmly.

(c) *Cleanliness of apparatus.*—Apparatus must be sufficiently clean to permit uniform wetting of the surface.

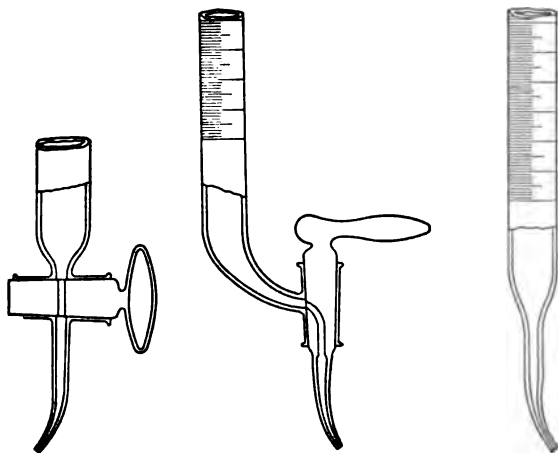


Fig. 4 (two-fifths natural size).

(d) *Flasks and cylinders.*—In filling flasks and cylinders the entire interior of the vessel will be wetted, but allowed a sufficient time to drain before reading. Before completely filling to the capacity mark flasks should be well shaken to completely mix the contents.

Flasks and cylinders when used to deliver should be emptied by gradually inclining them until when the continuous stream has ceased they are nearly vertical. After half a minute in this position the mouth is brought in contact with the wet surface of the receiving vessel to remove the adhering drop.

(e) *Pipettes and burettes*.—In filling pipettes and burettes excess liquid adhering to the tip should be removed when completing the filling.

In emptying pipettes and burettes they should be held in a vertical position, and after the continuous unrestricted outflow ceases the tip should be touched with the wet surface of the receiving vessel to complete the emptying.

Stopcocks, when used, should be completely open during emptying.

Burettes should be filled nearly to the top, and the setting to the zero mark made by slowly emptying.

While under normal usage the measurements ordinarily are from the zero mark, other initial points may be used on burettes of standard form without serious error.

5. Tolerances.

(a) Flasks.

Capacity Less Than and Including	Limit of Error	
	If to Contain	If to Deliver
25 cc	0.03 cc	0.05 cc
50 "	.05 "	.10 "
100 "	.08 "	.15 "
200 "	.10 "	.20 "
300 "	.12 "	.25 "
500 "	.15 "	.30 "
1,000 "	.30 "	.50 "
2,000 "	.50 "	1.00 "

(b) Transfer pipettes.

Capacity Less Than and Including	Limit of Error
2 cc	0.006 cc
5 "	.01 "
10 "	.02 "
30 "	.03 "
50 "	.05 "
100 "	.08 "
200 "	.12 "

(c) Burettes and measuring pipettes.

Capacity of Total Graduated Portion Less Than and Including	Limit of Error of Total or Partial Capacity	
	Burettes	Measuring Pipettes
2 cc	-----	0.01 cc
5 "	0.01 cc	.02 "
10 "	.02 "	.03 "
30 "	.03 "	.05 "
50 "	.05 "	.08 "
100 "	.10 "	.15 "

Further, the error of the indicated capacity of any ten consecutive subdivisions must not exceed one-fourth the capacity of the smallest subdivision.

(d) Cylinders.

Capacity of Total Graduated Portion Less Than and Including	Limit of Error of Total or Partial Capacity	When the Smallest Subdivisions are Not More Than	Limit of Error of any Ten Consecutive Divisions
30 cc	0.06 cc	0.2 cc	0.1 cc
50 "	.10 "	1.0 "	.2 "
100 "	.30 "	2.0 "	.4 "
200 "	.50 "	10.0 "	1.0 "
500 "	1.20 "	20.0 "	2.0 "
1,000 "	2.00 "		
2,000 "	4.00 "		

The delivery time marked on any instrument must be within the limits prescribed in the specifications and the error permitted in the marked delivery time is as follows:

(e) Delivery time.

Delivery Time Less Than and Including	Limit of Error
15 sec.	3 sec.
20 "	4 "
30 "	6 "
50 "	8 "
100 "	15 "
200 "	20 "

III. TESTS MADE BY THE BUREAU OF STANDARDS.

1. Nature of Tests.—Apparatus submitted for test is first examined as to its conformity with the specifications concerning design and marks, including test of outflow time where this is limited.

Apparatus having subdivisions is examined as to the apparent regularity of spacing.

If the apparatus complies with the specifications in other respects, a test is made of its capacity.

This test may be either to ascertain whether the capacity is correct within the prescribed limits of error or to determine the correction for use in precise measurements.

2. Precision Stamp.—If the result of examination and test of flasks, cylinders, and transfer pipettes indicates a satisfactory conformity to the specifications, the official precision stamp, consisting of the letters "U. S." and the year date, or the last two figures thereof, surrounded by a circle, is etched as shown below:



3. Certificates of Capacity.—Burettes and measuring pipettes will be tested for at least five intervals, and if found to conform to the specified requirements will be assigned a test number as shown below:

B. S. No. 1763
1908

A certificate will be furnished, giving the volumes delivered by the intervals tested.

When desired, certificates of capacities of flasks, cylinders, and transfer pipettes will also be furnished.

4. Special Tests.—Apparatus of approved design intended for special purposes, not conforming with the specifications, will be received for test only by previous arrangement, when accompanied by complete description of the intended use.

The bureau reserves the right to reject any apparatus on points affecting its accuracy or utility not covered by the regulations.

IV. DISCUSSION OF GENERAL SPECIFICATIONS.

1. **Units of Capacity.**—The liter is defined in these specifications as the volume occupied by a quantity of pure water at 4° C. having a mass of one kilogram. The one-thousandth part of the liter, the milliliter, called the cubic centimeter for convenience, is also employed as a unit of capacity. The milliliter is about 1.000029 true cubic centimeters, according to the recent determinations of the International Bureau of Weights and Measures,² but this difference is of no consequence in volumetric analysis.

The Mohr³ system of capacity units is not employed by this bureau, since it is difficult of complete definition, is not superior to the metric system in point of convenience, and introduces much confusion. The Kaiserliche Normal-Eichungs-Kommission no longer employs the Mohr unit for reasons given by W. Schloesser,⁴ who concludes that there is not sufficient justification for its use. On the other hand, the National Physical Laboratory of England recognizes the Mohr unit, and requires that vessels graduated in Mohr units be marked "gramme" or "grm.," together with the temperature at which the vessel has the specified number of such units of capacity.

2. **Standard Temperature.**—Since the capacity of glass-measuring apparatus varies with the temperature, it is necessary to specify the temperature when giving the capacity.

Another equally important reason for specifying the temperature is that in many liquids, measured volumetrically, the very properties whose relation is determined by these measurements change with the temperature, often to an extent more significant than the change in capacity. The temperature specified for use of apparatus is designated as the standard temperature.

² Procès-Verbaux, Comité Int. P. et Mes. (2), IV, p. 52.

³ The Mohr unit may be defined as the volume occupied at an arbitrary standard temperature by that mass of water which, when weighed with brass weights in air having a barometric pressure of 760 mm, the arbitrary standard temperature, and a mean content of moisture and CO₂, has an apparent weight of one gram.

⁴ Über die Einrichtung und Prüfung der Messgeräte für Massanalyse.

⁵ Zs. für Angewandte Chemie, 1903, pp. 955 to 960. Zs. für Analytische Chemie, 1907, pp. 393 to 400.

For many purposes the actual temperature chosen as standard is of minor importance provided all instruments used in conjunction have the same standard temperature, but where absolute measurements are desired, it is advantageous to employ a standard temperature which approximates the average temperature of the laboratory, or at least is readily attainable. The temperature 20° C. closely fulfills these conditions in this country and has consequently been chosen as the standard temperature for glass volumetric apparatus.

3. Material and Annealing.—Apparatus upon which especial care is bestowed to attain precision is of superior value only if it possesses permanence.

It is a well known fact that glass changes mechanically with time after manufacture into apparatus, the amount of such change depending upon the kind of glass and upon its heat treatment. If after manufacture the glass is annealed, the subsequent change in volume is usually of little importance for ordinary volumetric apparatus. Although annealing is of minor importance volumetrically for the better thermometer glasses, it should always be carried out to relieve mechanical stresses since these endanger the integrity of the apparatus.

4. Inscriptions.—These are necessary in order that the user may readily ascertain essential information, such as the capacity, standard temperature, and manner of use. Identification numbers are necessary both for the user and for the testing bureau in order to avoid confusion in identity or assembling.

Indication of the outflow time permits the user to ascertain whether the tips of burettes and pipettes have undergone alteration.

V. SPECIAL RULES FOR MANIPULATION.

The aim has been to make these rules comply as fully as possible with the most approved methods of practice. To a great extent the reasons for the individual rules are self-evident. In certain cases, however, the significance and justification of the rules have been discussed under separate headings.

1. Method of Reading.—The specifications require that all graduations extend at least halfway around. This requirement makes it possible to avoid parallax by so placing the eye that when observ-

ing the meniscus the front and back portions of the graduation coincide. The true outline of the meniscus is recognized with difficulty unless the meniscus is so shaded as to bring the meniscus and the background into contrast. This is effected by claspings around the graduated tube a collar-shaped piece of thick black rubber tubing. When this collar is placed immediately below the meniscus and a white background employed, the profile of the meniscus is very definitely outlined and is practically coincident in position with the liquid surface, while if this precaution is neglected the apparent outline of the meniscus may be considerably in error.

The exclusion of burettes with short graduations has been to some extent criticised, but experience has shown that these burettes are not capable of the accuracy required in precise work, on account of the error in reading due to parallax.

2. Cleanliness of Apparatus.—Certain contaminations, especially grease, adhering to the walls of measuring vessels prevent them from being uniformly wetted. The tendency of water to collect into drops instead of adhering to the glass surface as a continuous film indicates imperfect wetting due to uncleanness.

Imperfect wetting causes irregularities in capacity by distorting the meniscus. In instruments to deliver it also causes irregularities by affecting the residue adhering to the walls.

Since accuracy in the volume contained or delivered is possible only when the liquid will form a continuous film on the walls of the measuring vessel, this condition is prescribed as a test of cleanliness.

But even when the surface of the apparatus fulfills this condition, variations in the apparent capacity may still occur, caused by contamination of the liquid surface by minute quantities of fatty or other organic substances. This effect is due to the formation of a film on the surface which produces a change in the surface tension and a consequent variation in the shape of the meniscus.

The phenomena of surface contamination of liquids have been studied experimentally by Rayleigh⁶, Nansen⁷, Pockels⁸, and others, but since the effect upon volumetric measurements has not to our

⁶ Rayleigh, *Phil. Mag.*, 48, pp. 321-337; 1899.

⁷ F. Nansen, *Norwegian North Polar Expedition, 1893-1896, Scientific Results*, 10, p. 61; 1900.

⁸ Agnes Pockels, *Annalen der Physik*, 313 (IV)8, p. 854; 1902.

knowledge been considered, it has been made the subject of an investigation to ascertain its importance.

The questions which it was desired to answer were: (a) To what extent is the surface tension of the meniscus in volumetric apparatus changed by contamination when various methods of cleaning and drying are employed? (b) If the surface tension of the meniscus is thus changed, to what extent is the apparent capacity affected? (c) How may appreciable errors from this cause be avoided?

In the experimental portion of the investigation a one-liter flask was used. The normal capacity, deviations from which were measured was the capacity when the surface of the meniscus was uncontaminated. Previous observers have shown that surface contaminations are removed by overflowing. The flask was therefore so modified that this operation could be performed.

The flask is shown in Fig. 5. Tube (a), attached to the flask by a ground joint, carries the overflow nozzle. Tube (b) carries a stopcock, so that the meniscus can be lowered to the mark after overflow. The inside diameter at the graduation mark was 16.3 mm. The distance from the graduation to the ground joint was 65 mm.

The volume of water contained by the flask under various conditions was determined by weighing; the temperature was observed on an enclosed scale thermometer, inserted after setting the meniscus to the mark. To prevent evaporation the flask was closed during weighing by means of a rubber stopper which served to hold the thermometer. To permit comparisons the apparent capacity of the flask resulting from these determinations was reduced to 20° C., the observed temperatures being between 18° and 22° C.

The following methods of preparing and manipulating the flask were employed:

The entire interior of the flask, the overflow tube, the stopcock and thermometer were thoroughly cleaned with fuming sulphuric acid before each series of observations. The parts were thoroughly rinsed with tap water and then with distilled water. After the interior of the flask had been finally rinsed either with water, with alcohol redistilled from caustic soda, or with commercial 95 per cent alcohol, the parts were dried by evaporation. (In Series I and VIII the flask was not dried.) The air used in drying the flask was purified and dried by passing it first through sulphuric acid and then

through two tubes, the first containing glass wool and the second clean cotton. After cleaning and drying, the stopcock was slightly greased.

The filling of the flask was made in four ways, designated as A, B, C, and D.

Method A: The flask was filled from a nozzle placed about 15 mm. above the mark, the outflow tube filled to the tip, and the meniscus raised to the mark.

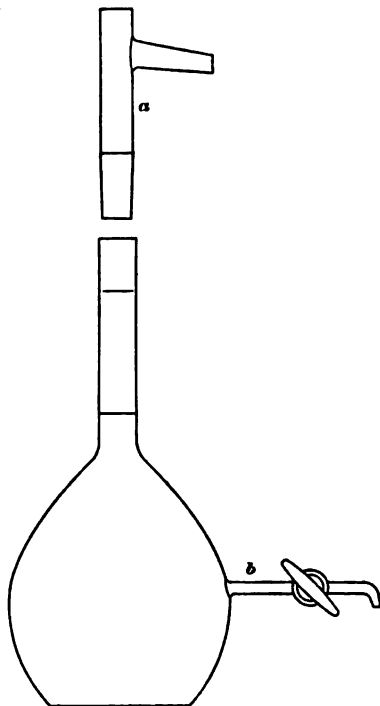


Fig. 5.

Method B: Following the weighing for A the thermometer was removed carefully and dried, the neck was filled to the ground-glass joint without disturbing the meniscus surface unnecessarily, and the setting made by means of the stopcock and outflow tube.

Method C: After B the overflow tube was inserted and water admitted until about 500 cc. had overflowed through the spout. The water was lowered below the ground joint, the overflow tube removed, and the meniscus slowly lowered to the mark as in B.

Method D: After C the neck was dried by evaporation without disturbing the meniscus, using clean dry air, after which the setting was made as in A.

The results have been referred to the mean normal capacity of the flask; that is, the capacity after overflowing. The amount of water on the neck was determined and eliminated from the results. Consequently, Tables I and II give the normal capacity minus actual capacity. This quantity we will term the *defect* in capacity.

The experimental work was performed by Miss G. C. McDermut, to whom credit it due for great care in manipulation.

TABLE I.
Observed Defect in Capacity.

Series	A	B	C	D	Remarks
I	+0.020 cc		-0.003 cc -0.004 "	-0.006 cc -0.007 "	Flask not dried except neck.
II	+0.044 cc			-0.011 cc	Rinsed with water, dried.
III	+0.061 cc +0.057 " +0.060 "	+0.006 cc +0.007 " +0.007 "	-0.004 cc 0.000 " -0.005 " +0.009 "	+0.001 cc -0.003 " +0.002 "	Rinsed with redistilled alcohol, dried.
IV	+0.094 cc +0.071 " +0.085 " +0.072 "	+0.015 cc +0.020 "	-0.006 cc +0.002 " +0.006 "	+0.013 cc +0.006 "	Rinsed with ordinary alcohol, dried.
V	+0.033 cc +0.029 " +0.036 "				As in (IV) except neck wet.
VI	+0.001 cc -0.005 " -0.002 " -0.005 "				As in (V) except agitated before completely filling.
VII	+0.064 cc +0.054 "				As in (VI) except neck dry.
VIII	+0.074 cc +0.073 " +0.055 "	+0.007 cc +0.004 " -0.001 "	-0.005 cc -0.006 " -0.007 "	+0.005 cc +0.013 " +0.009 "	As in (I) except contaminated with soap.

In all determinations except Series VIII the greatest care was exercised to avoid contamination other than that derived from the air or from the liquid last used for rinsing the flask. But since, in filling,

TABLE II.
Assembled Mean Results.

Defect in Meniscus Volume.					Remarks
Series	A	B	C	D	
I	+0.020 cc		-0.004 cc	-0.006 cc	
II	+0.044 "			+0.011 "	
III	+0.059 "	+0.007 cc	0.000 cc	0.000 "	
IV	+0.080 "	+0.018 "	+0.001 "	+0.010 "	
V	+0.033 "				
VI	-0.003 "				
VII	+0.059 "				
VIII	+0.067 "	+0.003 cc	-0.006 cc	+0.009 cc	

contaminations from the interior of the flask are collected on the relatively small surface of the meniscus, very small amounts cause sensible effects. Even when water twice distilled—the second time from alkaline potassium permanganate—was used in the final rinsing, as in Series II, a sensible diminution in the capacity indicated contamination. A greater degree of contamination was produced when purified alcohol was used for drying, and a still greater contamination resulted from the use of ordinary alcohol.

Intentional contamination was accomplished in VIII by slightly soaping a clean glass rod and touching the surface of the water. In the experiments of Miss Pockels, previously mentioned, the greatest contaminating effect observed was produced by soap, which reduced the surface tension of water to 0.49 of its normal value.

Comparison of Series II and III with Series VIII shows that despite the care taken in the former three series the surface tension was very low. It is evident that reasonable care in cleaning and drying does not insure a normal meniscus.

The effect of distributing the contamination over a larger surface is shown in Method B by the diminution of the defect in volume produced when after Method A the meniscus is raised to the joint and then lowered. Part of this effect is due to extension of the surface on withdrawal of the thermometer from the liquid. The volume is in most cases increased by Method B nearly to normal value. In Series V the neck was wet to the joint when commencing the filling

and allowed to drain until the setting was made. The defect was evidently less than with dry neck, but was not reduced so much as by Method B, suggesting that the contamination does not spread over a thin film as completely as over a free surface.

In Series VI, after wetting the entire interior of the flask from the joint down, the flask was filled about two-thirds full, closed by a stopper, and then vigorously shaken to break up the surface and distribute its parts throughout the mass of the water. It is seen that this manipulation is apparently as effective as overflowing in destroying the effect of contamination. In Series VII the water was shaken before completing the filling, keeping the neck of the flask dry. The effect is much less than in VI, probably due in part to the less vigorous agitation of the water and partly to the smaller area over which the contamination was distributed.

The greatest defect in volume (0.080 cc) was observed in Series IV, where ordinary alcohol was used in drying the flask. To estimate the reduction in the surface tension of water corresponding to this result the defect in volume of a water meniscus 16.3 mm in diameter was calculated, taking for the normal surface tension of water at 20° C 72.5 dyn/cm, and for contaminated water 39.3 dyn/cm. (This value corresponds to 0.54 of the normal tension, the value observed by Miss Pockels for water contaminated with tannin.) The tables computed by Bashforth and Adams⁹ were used. [Extrapolation was necessary, since the diameter 16.3 mm lay outside the range of the table.] A defect in volume of 0.074 cc was found, corresponding very closely with that observed in Series IV. This suggests the presence of tannin, extracted by the alcohol from the barrel in which it was kept. The surface tension, therefore, of the meniscus in this series was about 0.5 that of pure water.

The defect in capacity of apparatus caused by the contamination, which reduces the surface tension of water to 0.54 of the normal value, is shown in Table III for tubes of different diameters. The portion of the table below the broken line is extrapolated graphically. Since a maximum and minimum limit can be assigned to the slope of the graph, the extrapolation is known to be correct to at least 10 per cent.

⁹Capillary Action, Cambridge, 1883.

TABLE III.

Diameter mm	Defect in Capacity cc	Diameter mm	Defect in Capacity cc
5	0.001	15	0.058
6	0.003	16	0.067
7	0.005	17	0.076
8	0.008	18	0.085
9	0.012	19	0.095
10	0.018	20	0.105
11	0.024	21	0.115
12	0.032	22	0.125
13	0.041	23	0.135
.....		24	0.145
14	0.049	25	0.155

In Table IV is given for flasks of various capacities the limit of error, the maximum and minimum diameters of neck allowed, and the possible error from contamination for maximum and minimum diameters taken from Table III.

TABLE IV.

Capacity	Limit of Error	Allowed Diameters		Possible Error from Contamination	
		Max.	Min. °	Max. Dia.	Min. Dia.
50 cc	0.05 cc	10 mm	6 mm	0.018 cc	0.003 cc
100 "	0.08 "	12 "	8 "	0.032 "	0.008 "
200 "	0.10 "	13 "	9 "	0.041 "	0.012 "
300 "	0.12 "	15 "	10 "	0.058 "	0.018 "
500 "	0.15 "	18 "	12 "	0.085 "	0.032 "
1000 "	0.30 "	20 "	14 "	0.105 "	0.049 "
2000 "	0.50 "	25 "	18 "	0.155 "	0.085 "

Table V gives the possible error from contamination expressed as per cent of total capacity and also as per cent of total error allowed in the specifications.

TABLE V.

Capacity	Possible Error Per cent of Total Capacity		Possible Error Per cent of Allowed Error	
	Max. Dia.	Min. Dia.	Max. Dia.	Min. Dia.
50 cc	0.036	0.006	36	6
100 "	0.032	0.008	40	10
200 "	0.020	0.006	41	12
300 "	0.019	0.006	48	15
500 "	0.017	0.006	57	21
1000 "	0.011	0.005	35	16
2000 "	0.008	0.004	31	17

The following conclusions are drawn in answer to the questions stated at the outset: (a) The surface tension of the meniscus in volumetric apparatus is subject to a defect of about 0.5 the normal value due to contaminations not removed by the accepted methods of cleaning. (b) The capacity of ordinary measuring flasks may be affected by this change in surface tension by from one to four parts in 10,000. (c) Errors from this cause in capacity of flasks may be avoided by employing the method prescribed in the rules for manipulation of flasks, that is, thoroughly shaking the contents just before completing the filling.

3. Use of Liquids other than Water.—In order for the standardizing bureau to furnish corrections or affix the precision stamp to burettes, pipettes, and other vessels which are intended to measure liquids other than water, the particular liquid which the instrument is to measure must be employed in the test of the instrument until the behavior of that liquid has been studied in various apparatus. Until a demand has arisen for precise vessels delivering some particular liquid, it seems better for this bureau to test the quality of the instrument with water only, and to ascertain that the instrument complies with the specifications that have been adopted.

The determination of the corrections of the instrument for liquids other than water can be made by the individual, either by experiment or by reference to available data. If the apparatus be a vessel for delivering total volume a simple method of obtaining the correc-

tion when using a special liquid is to weigh the empty vessel, weigh again containing only the residue after delivering water, and again containing the residue after delivering the liquid. The liquid delivered is weighed roughly and the approximate knowledge thus gained of the density of the liquid suffices for a calculation of the small volume of the residue, and the difference between this volume and the volume of the water residue may be applied as a correction to the delivery capacity determined by water in the ordinary way. This disregards the difference caused by the effect of capillarity on the meniscus volume, mentioned later.

The relation between the nature of the liquid and the volume delivered by pipettes has been made the subject of a careful and extended investigation by Schloesser and Grimm.¹⁰ For the investigation transfer pipettes of 100, 25, and 10 cc were employed. The exact form and dimensions of the tips and delivery tubes are not described, and effects due to the different causes, namely, residue on the walls of the pipette bulb, residue on the walls of the delivery tube, and the residue in the tip cannot be differentiated. It cannot be concluded, therefore, that the results obtained with these pipettes would necessarily be duplicated with other pipettes of the same capacity but differing in form, although valuable information is afforded as to the general behavior of pipettes when employed for various liquids.

The concentration of the liquids of titration employed by these investigators was usually normal, but was tenth-normal where that concentration is the highest employed in titration. The adjustment of the concentrations was only approximate. The density was determined by hydrostatic weighing.

The surface tensions were for the most part determined at the Kaiserliche Normal-Eichungs-Kommission at a temperature of 17° C. The viscosities were taken from the tables of Landolt and Börnstein. The settings were made by bringing the base (center) of the meniscus to the line, except in the case of milk. The pipettes were allowed to empty themselves into a receiving vessel by free outflow, completed by touching the outflow nozzle to the wetted surface of the receiving vessel. There was no wait to allow

¹⁰ *Chemiker Zeitung*, 80, pp. 1071 to 1073; 1906.

TABLE VI.
Capacity of Pipettes with Various Liquids.

Liquid	Concentration	Density 18/4	Surface tension, mg/mm	Viscosity		Pipette 100 cc		Pipette 25 cc		Pipette 10 cc	
				t	η	Time outflow, seconds	ΔV , mm ³	Time outflow, seconds	ΔV , mm ³	Time outflow, seconds	ΔV , mm ³
Water		0.9986	7.53		1.000	39		32		20	
Alcohol and soap *		0.9263	3.24			45	- 255	38.6	- 103		
Alcohol	50.5 %	0.9175	3.23			44.8	- 229	37.4	- 88	23.0	- 59
Do	99.8 %	0.7906	2.7	20	1.186	41.6	- 142	33.5	- 67		
Alcoholic soda (Köttstorfer)		0.8320	3.04			42	- 140			23.8	- 32
Conc. sulphuric acid	96 %	1.8350	3.28	20	21.82	64	- 442			52.4	- 85
Nitric acid	n/1	1.0356	7.16	25	1.027	39	- 5	32	+ 6	20.0	0
Conc. nitric acid	30 %	1.1816	6.04			40	- 4			21.6	+ 7
Sulphuric acid	n/1	1.0308	7.34	25	1.090	39.5	+ 3	31.8	- 3		
Oxalic acid	n/1	1.0202	7.34	18	1.072	39.5	+ 13	32	+ 5		
Hydrochloric acid	n/1	1.0162	7.40	25	1.067	39	+ 17			20.7	+ 6
Fehling's solution II		1.2661	6.43			46.2	- 198			28.5	- 43
Conc. caustic potash	15 %	1.1369				40.4	- 38			21.2	- 7
Sodium carbonate	n/1	1.0318	7.05	18	1.274	43	- 35			20.5	- 3
Caustic potash	n/1	1.0482	6.23	18	1.110	39.4	- 17	32	- 5		
Caustic soda	n/1	1.0435	7.26	18	1.234	40	- 15			19.5	0
Sodium carbonate	n/2	1.0259		18	1.120	39.2	- 14			21.2	- 4
Fehling's solution I		1.0440	7.23			39.8	- 12			20.3	- 2

* Boudron & Boudet.

TABLE VI—Continued.
Capacity of Pipettes with Various Liquids—Continued.

Liquid	Concentration	Density 18°/4	Viscosity.		Pipette 100 cc		Pipette 25 cc		Pipette 10 cc	
			t	η	Time outflow, seconds	Δv mm ³	Time outflow, seconds	Δv mm ³	Time outflow, seconds	Δv mm ³
Ammonia.....	n/l	0.9916	25	1.025	39.4	+ 4			20.4	- 5
Barium chloride.....	n/l	1.0897	17.6	1.107	39.5	- 14	32.2	- 5		
Sodium thiosulphate.....	n/10	1.0119			40	- 5	31.4	- 5		
Potassium bichromate.....	n/l	1.0327			39	- 5	31.6	- 3	20.4	- 7
Sodium chloride.....	n/10	1.0031			39.2	- 4			20.8	+ 2
Ammonium sulphocyanide.....	n/l	1.0162	25	1.011	38.8	0	32.2	+ 4	20.7	- 7
Potassium permanganate.....	n/10	1.0009			40	+ 10			20.4	0
Ferric chloride.....	1 cc-0.012 g Fe	1.0237	25	1.177	39.8	- 35			20.8	0
Uranium.....	1 cc-0.005 g P ₂ O ₅	1.0281			39.4	- 6	32.0	+ 6		
Mercury nitrate.....	1 cc-0.01 g Urea	1.1006			39.2	+ 4	31.6	+ 3		
Silver nitrate.....	n/10	1.0129	25	1.008	39.2	+ 5	32.2	+ 12	21.2	+ 4
Milk.....		1.0297			41.6	-*174			20.8	-*52
Sugar.....	1%	1.0022	18.8	1.002	40	- 15			18.8	- 3
Indigo (Kubel & Tiemann).....	6.3 cc-0.001 g N ₂ O ₅	1.0375			39.3	- 3	32.0	0		
Iodine.....	n/10	1.0228			39.0	+ 25			19.5	+ 6

* Setting made with lowest point of water meniscus, highest point of milk meniscus.

† Setting made with highest points of both water and milk menisci.

after-drainage. The mass of the liquid delivered was determined by weighing and making the appropriate corrections for buoyancy of the air. The volume was then known as the quotient of mass by density. The temperature of observation varied between 16° and 19° C. The results are given in the Table VI, in which Δv signifies volume of liquid delivered minus volume of water delivered.

These results indicate that pipettes may be used interchangeably for solutions not exceeding normal concentration if the solutions have approximately the same capillary constant,¹¹ as water and the outflow time is observed to be the same as for water.

4. Effect of Temperature on Residue in Burettes and Pipettes.—The effect of temperature on the volume delivered by pipettes and burettes has been investigated by Schloesser.¹² The results of this investigation were as follows:

TABLE VII.

Pipette 10 cc. Outflow time 20 sec.		Pipette 100 cc. Outflow time 39 sec.		Burette 50 cc. Outflow time 52.5 sec.	
Temperature. ° C.	R.	Temperature. ° C.	R.	Temperature. ° C.	R.
4.3	0.067	5.5	0.260	5.3	0.233
11.5	0.067	10.8	0.224	12.6	0.205
21.3	0.061	20.6	0.216	21.5	0.191
29.9	0.062	30.6	0.197	29.8	0.191

R in the above signifies the residue in cc of water after outflow. So far as can be judged from these data the temperature effect, aside from the expansion of the glass, is appreciable, but in most work negligible if the temperature be always between 15° and 30° C. The effect in burettes whose outflow time conforms with the specifica-

¹¹ By the capillary constant or specific cohesion in this paper the quantity a^2 will be meant, which is defined by the equation

$$a^2 = \frac{2T}{g^2}$$

in which T is the surface tension in dyn/cm, s is the density, and g the gravity constant.

¹² *Zs. für Analytische Chemie*, p. 413; 1907.

tions of this bureau may be expected to be considerably less than for the burette employed by Schloesser. This will be more apparent after consideration of our own results on burette drainage.

5. Avoidance of Unnecessary Heat.—In the use of glass apparatus for measuring liquids the practice of heating to extraordinary temperatures, as in cleaning with hot water, should be avoided on account of the thermal after effect corresponding to the depression of zero in a thermometer. While for certain kinds of glass this effect may be negligible we have no assurance that it is so for all kinds of glass employed in volumetric apparatus, and it is well to avoid an unnecessary source of error.

VI. FLASKS AND CYLINDERS.

1. Difference in Volume Delivered and Contained.—Graduated cylinders accepted for test must be marked "to contain," and are tested accordingly. When flasks and cylinders are used to deliver, they should be filled as when used to contain and be emptied by gradually tilting them until, when the continuous stream has ceased, they are almost vertical. After draining in this position for half a minute the drop should be removed by bringing the mouth of the flask or cylinder lightly into contact with the wetted surface of the receiving vessel.

Cylinders correctly graduated to contain within the allowed limits of error are in general correct to deliver within twice these limits of error when the delivery is made in the above manner.

This proposition has been established by determining the difference between the amount contained and the amount delivered for various sizes of cylinders and for various intervals of each. The cylinders were of the ordinary dimensions, the ratio of diameter to length being approximately as 1 to 5.

The cylinders were emptied in the manner described, with this difference, that each cylinder was allowed to drain one minute. The dry cylinder was counterpoised, filled to the point under test, emptied, and replaced upon the scalepan of the balance. The weight added to the tare to balance the cylinder gave the weight of water remaining. This amount is obviously the difference between the amount delivered and the amount contained.

The maximum difference between the volume contained and the volume delivered occurs in every case when the interval under test is the total interval. The limits of error when the cylinder is used to contain are, however, the same for the total interval as for the sub-intervals. To show that the difference between the volume contained and the volume delivered is usually not greater than the limits of error when the cylinder is used to contain, the following comparison is sufficient:

TABLE VIII.

Capacity of cylinder.	Limits of error.	Difference contained and delivered.
10 cc	0.6 %	0.35%
25 cc	0.24 %	0.35%
50 cc	0.2 %	0.20%
100 cc	0.3 %	0.17%
250 cc	0.48 %	0.12%
500 cc	0.24 %	0.11%
1000 cc	0.2 %	0.09%

Since the period of drainage was a full minute instead of a half minute, the above results would have less weight were it not that the effect of reducing the period of drainage to a half minute is much less than the limits of error with which we are dealing. This is shown by the investigation of W. Schloesser¹⁸ on the effect of varying the period of drainage of flasks used to deliver. Although his investigation deals with flasks it is to a sufficient extent applicable to cylinders.

2. Use of Special Liquids.—In the absence of more specific knowledge as to the use of flasks and cylinders to deliver liquids other than water it is best to avoid such use in measurements requiring precision. For flasks to contain, however, it is possible to calculate, providing we know the capillary constant of a liquid and the diameter of the neck of the flask, the correction to be applied to the capacity for water in order to obtain the capacity for that liquid. This correction is obviously the difference between the two meniscus vol-

¹⁸Zs. für Analytische Chemie, p. 403, 1907.

umes. By the meniscus volume is meant the volume of the meniscus above the horizontal plane tangent to the base (center) of the meniscus.

Table IX is for use in this connection. It gives, for various values of the specific cohesion and for various tube diameters, the quantity by which the corresponding meniscus volume is less than the normal (maximum) meniscus volume of pure water at 20° C.

The specific cohesion has already been defined.

Table X gives the meniscus volume of pure water at 20° C., having the normal surface tension 72.5 dyn/cm. The meniscus volume corresponds therefore to a specific cohesion of 14.821 mm².

These tables were derived by the aid of the tables of Bashforth and Adams.¹⁴ It was found convenient to interpolate graphically in making Table IX. The maximum error in this table probably does not exceed 0.001 cc. Table X was used in making Table IX. The maximum error probably does not exceed 0.0002 cc. The table is not immediately useful in the work dealt with in this paper, so that the values are given as derived in order to preserve their accuracy.

VII. TRANSFER PIPETTES.

A slight change has been made in the regulation as to time allowed for after-drainage in transfer pipettes. Instead of waiting 15 seconds after outflow ceases, as formerly, and then removing the accumulated excess at the tip, the wait of 15 seconds is omitted and the excess at once removed by touching the wet surface of the receiving vessel. This is in accordance with the recommendations of the International Committee on Analyses, Rome, 1906.

1. **Design.**—A question which deserves some study and investigation is that of the extent to which the form and dimensions of transfer pipettes influence their behavior with various liquids. It should be possible to predict their behavior with certainty when the properties of the liquids are known.

Recommendations as to shape of tips of burettes and pipettes have been included in the specifications under the head of special requirements. These recommendations are based on observation of the performance of tips of various forms and indicate the forms consid-

¹⁴ Capillary Action, Cambridge, 1883.

TABLE IX.
Defect in Meniscus Volume.

Tube diameter	Specific Cohesion in mm ²										
	14	13	12	11	10	9	8	7	6	5	4
4 mm	.000	.000	.000	.000	.000	.000	.001	.001	.001	.001	.001
5 "	.000	.000	.000	.001	.001	.001	.001	.002	.002	.003	.003
6 "	.000	.001	.001	.001	.002	.002	.003	.004	.004	.005	.007
7 "	.000	.001	.001	.002	.003	.004	.005	.006	.007	.009	.012
8 "	.001	.001	.002	.003	.005	.006	.008	.010	.012	.015	.018
9 "	.001	.002	.004	.006	.007	.010	.012	.015	.018	.023	.027
10 "	.001	.004	.006	.008	.011	.014	.018	.022	.027	.032	
11 "	.002	.005	.008	.012	.016	.020	.024	.030	.036		
12 "	.003	.007	.011	.016	.021	.025	.032	.040			
13 "	.004	.009	.014	.020	.026	.033	.041				
14 "	.005	.011	.017	.025	.033	.042					
15 "	.006	.013	.022	.030	.041						
16 "	.007	.016	.027	.037							

TABLE X,
Meniscus Volume of Pure Water—($a^2=0.14821$ cm²).

Radius of Tube	Meniscus Volume	Radius of Tube	Meniscus Volume
0.0944 cm	0.0009 cm ³	0.6494 cm	0.1551 cm ³
0.1311 "	0.0023 "	0.6720 "	0.1669 "
0.1796 "	0.0056 "	0.6917 "	0.1774 "
0.2410 "	0.0129 "	0.7171 "	0.1912 "
0.2829 "	0.0200 "	0.7389 "	0.2033 "
0.3150 "	0.0267 "	0.7579 "	0.2139 "
0.3634 "	0.0388 "	0.7800 "	0.2265 "
0.4149 "	0.0544 "	0.7993 "	0.2377 "
0.4639 "	0.0716 "	0.8164 "	0.2476 "
0.4924 "	0.0827 "	0.8317 "	0.2566 "
0.5235 "	0.0955 "	0.8457 "	0.2648 "
0.5492 "	0.1068 "	0.8553 "	0.2705 "
0.5904 "	0.1258 "	0.8615 "	0.2742 "
0.6227 "	0.1416 "		

ered most desirable. Reasonable variation in the form of tips will be allowed, but instruments in which the construction shows manifest disregard for correct form and good workmanship will not be tested.

VIII. BURETTES AND MEASURING PIPETTES.

1. **Burette Drainage and Outflow Time.**—Drainage in burettes is considered by W. Schloesser¹⁵ in his valuable discourse on the testing of volumetric apparatus at the Kaiserliche Normal-Eichungskommission. His results clearly indicate a relation between the duration of the outflow and the amount and distribution of the drainage, but do not seem to fully indicate the relation of these variables, nor the influence of the actual length of tube emptied.

It seemed desirable to study somewhat further the characteristics of the drainage under various conditions, and for this purpose experiments were made at this bureau in the summer of 1904.

Two tubes were used of about 8 and 14.6 mm mean inside diameter, respectively. These are about the extreme diameters used for burettes of from 10 to 100 cc capacity. Using the tubes in the manner of burettes, intervals of from 5 to 60 cm in length were emptied at various rates. When the outflow was stopped at the end of any interval the meniscus was shaded and its motion observed at frequent intervals of time by means of a horizontal micrometer microscope of long focus. The first observation was made 10 seconds after stopping the outflow. The temperature of observation varied from 23° C to 27° C.

The accompanying curves, Figs. 6 to 17, exhibit the results of the observations. They show the rise of the meniscus above the first observed position for various times after stopping the outflow.

The curves indicate certain characteristics in the behavior of the water residue which have a bearing on the choice of outflow time. Thus the rate of drainage or afterflow is greater the more rapid the descent of the meniscus. A large afterflow can arise only from a heavy film of liquid left behind a falling meniscus—that is, from a heavy residue. Therefore the residue is greater the more rapid the descent of the meniscus.

¹⁵ Zs. für Angewandte Chem., 1903.

By limiting the rate of outflow the residue and the afterflow may be made negligibly small. The system of curves considered with reference to the rate of outflow enables a choice of outflow rate which shall sufficiently limit the afterflow. For the maximum or initial rate a value was sought which should render the residue and drainage so small that the volume delivered should be independent of such variations in manipulations as are often found necessary by the chemist. Reference to the curves shows that for an initial rate of 0.7 cm per second the maximum drainage from any interval emptied which occurs during the first two minutes after stopping outflow is about 0.05 mm. This initial rate was selected, and the specifications in regard to time of outflow of the total graduated length were so chosen that on burettes of customary proportions the maximum initial rate should not exceed 0.7 cm/sec.

Reducing the initial rate of outflow decreases the effect on the volume delivered of stops at intermediate points. The effect of stops can best be determined by trial in each instance, since it seems likely that stops sometimes increase and sometimes decrease the volume delivered. The tendency for decrease in volume delivered is due to heavier residue at the points where the descent of the meniscus has been retarded.

The prescribed minimum time of outflow increases more rapidly the greater the length of graduated interval. Consequently, burettes should be made as short as the minimum length of a single subdivision will permit, in order to economize time without loss in accuracy.

The danger of clogging the tip, which has been mentioned as an objection to the small size of the orifice required by these specifications, is one which should not cause a careful observer much concern.

As an example of the necessity for restricted time of outflow a curve, Fig. 18, is given showing the apparent capacity corresponding to various outflow times of a 50-cc burette whose graduated length is 63 cm. It is readily seen that such variations in the time of outflow, as often occur in practice, have a large effect if the outflow time is much less than the specifications permit. If the outflow time of the burette were 65 seconds, increasing from this minimum by one minute the time consumed in making a titration would





cause an increase of about 0.05 cc in the volume delivered, as much as the permissible error for this size of burette.

2. Use of Special Liquids.—The use of burettes to measure liquids other than water differs from the similar use of transfer pipettes, since the uniform diameter of the burette renders the measurements less dependent on the capillary properties of the liquids used. The effect of viscosity, however, is quite as apparent as in pipettes, since

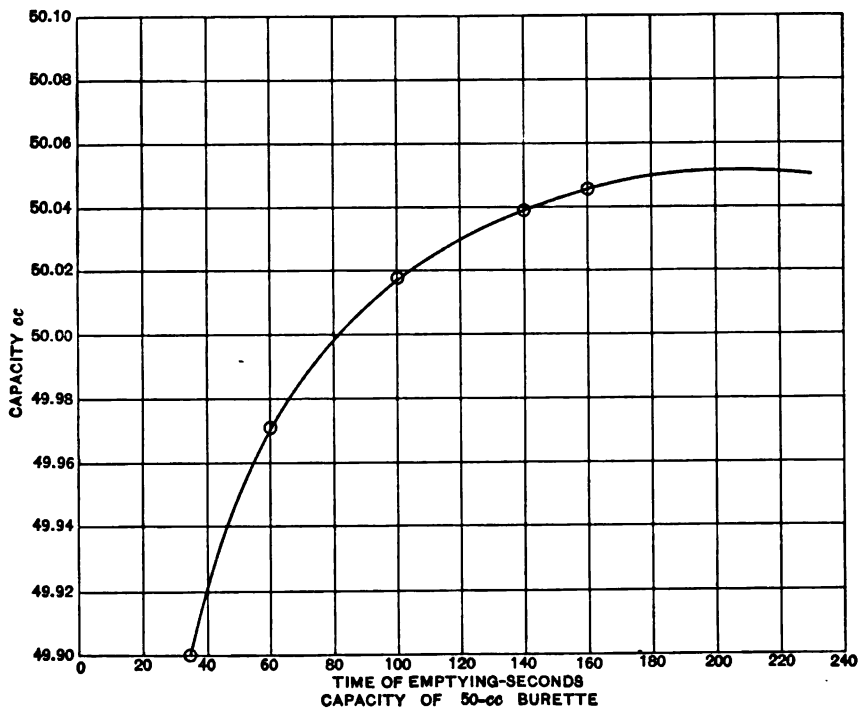


Fig. 18.

this property is one of the most important factors in determining the amount and movement of the residue when emptying a burette. It is apparent that liquids of approximately the same viscosity and density as water may be measured without error. Hence burettes may be used with confidence for liquids of about the density of water and giving a normal outflow time.

If with any liquid the outflow time is observed to differ appreciably from the normal it is not to be assumed that the capacity of the

burette to measure that liquid is the same as for water and it is best to retest the burette with the liquid at several points.

3. Calibration.—In using burettes for precise work the chemist probably more often uses the intervals beginning at the zero graduation than those starting at intermediate points. There is a difference in the effective capacity of the intervals depending on how they are used, although if the outflow is made sufficiently slow this difference is generally quite small.

The calibration may be made using either successive intervals or those beginning at zero. For the purpose of determining the quality of the instrument the latter is preferable since it avoids cumulative error due to summation, as pointed out by Schloesser.¹⁶

In the manufacture of burettes it is important that the subdivision as well as the calibration be carefully performed. Several examples of faulty subdivision have been observed, among which several from the same maker (not domestic) showed such similar defects as to leave no doubt of systematic error in its production. One such example is shown on Fig. 19, which by a curve represents the cor-

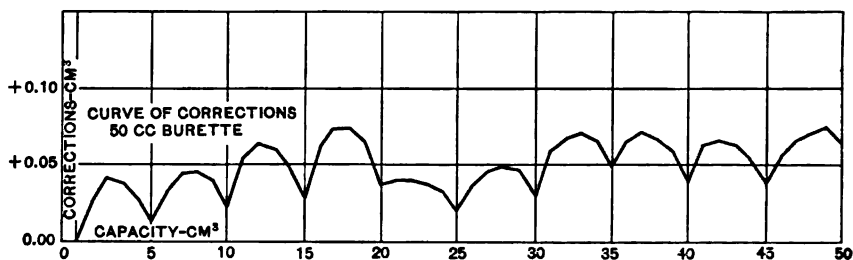


Fig. 19.

rections to the indicated capacities of a burette determined at every cubic centimeter. It is apparent that a burette with graduation of this kind is unfit for precise use. Furthermore, too much dependence must not be placed on calibrations at a few points. If the subdivisions are of uniform or uniformly varying length, intermediate corrections may be obtained by interpolation. The most preferable burettes are those in which the caliber is sufficiently uniform to permit uniformity of the subdivisions.

In regard to the arrangement of the division and subdivision of the intervals, attention may be called to advantages of interrupted

¹⁶ Zs. Angew. Chem., p. 985; 1903.

or partial subdivision with portions enlarged into bulbs for use in cases where subdivision of the entire interval is not required. The advantages of this form of construction are gain in compactness, decrease in drainage, permitting rapid outflow, and economy of labor in calibration.

In regard to the arrangement for control of outflow, the use of delivery tips attached by rubber tubing and control of outflow by pinchcock (Mohr burette) is to be avoided in accurate work on account of the errors caused by the stretch of the rubber. The specifications exclude this type of burette. It is suggested that where lubricated glass stopcocks are undesirable measuring pipettes be employed in a manner similar to that described and illustrated under the head of methods of testing transfer pipettes.

4. Measuring Pipettes.—The principles which have been discussed in the preceding pages in reference to burettes are equally applicable to measuring pipettes. Since, however, the same precision is not ordinarily attainable in their use as with burettes, owing to the less complete control of the outflow, limits of error have been permitted twice as great as for burettes. The permissible maximum length graduated has been limited to 35 cm, and also the permitted outflow time has been changed to correspond with that for burettes of similar graduated length instead of that for transfer pipettes of the same capacity. These regulations do not permit so frequent subdivisions on the larger pipettes as on burettes of the same capacity, but greater length; but the greater convenience of the shorter pipettes and the advantage of reduced drainage should quite compensate for whatever loss in precision is caused by the less frequent subdivisions.

5. Limits of Error.—The limits of error are the same for both total and partial capacities of any burette or measuring pipette. For example, on a 50-cc burette the error must not exceed 0.05 cc for any interval starting at the zero point for continuous emptying. Further, the errors of intermediate intervals, as determined by differences of errors found in intervals beginning at zero, must not exceed 0.05 cc. Thus, if the error of the interval 0 to 20 cc is +0.02 cc and the error of the interval 0 to 40 cc is -0.05 cc, the error of the interval 20 cc to 40 cc is reckoned -0.07 cc and the instrument is rejected.

IX. METHODS OF TESTING.

1. Preliminary Examination.—Apparatus submitted for test is first examined as to its conformity with the specifications. This examination determines the following items: general quality and workmanship, correctness of inscriptions, correctness of design and proportions, regularity in spacing of subdivisions, outflow time, etc.

If the apparatus complies with the specifications in other respects a determination of the capacity is made either to determine its fitness for the precision stamp or to obtain data for a certificate of capacity.

2. Methods of Cleaning Apparatus.—The liquids usually employed in preparing apparatus for test are concentrated solution of caustic soda in 95 per cent commercial alcohol, concentrated or fuming sulphuric acid, water, and alcohol. The caustic soda solution usually removes contaminations if left in the apparatus several hours, and is ordinarily employed for flasks. For badly contaminated apparatus the fuming sulphuric acid is more effective, and for small instruments, pipettes, and burettes, and all cases where haste is necessary the acid is found most expeditious.

After the use of the soda or acid the apparatus is thoroughly washed in clean water. If for immediate use it is rinsed with alcohol and dried by an air blast. No method has been found for cleaning apparatus such that drying leaves the apparatus free from contamination. If the apparatus is to deliver it is not dried, and in other cases the standard of cleanliness sought is the condition which will permit wetting of the entire interior of the apparatus by a continuous layer of water.

3. Measurement of Capacity.—The capacity of instruments to deliver is at present generally tested at this bureau by weighing the water delivered. Instruments to contain are sometimes tested by weighing their contents, but usually by filling them from a special measuring pipette.

(a) *Testing Flasks by Direct Measurement.*—Flasks are tested by filling them from a standard pipette whose lower stem is graduated to enable a direct reading of the volume of liquid delivered. The arrangement used in testing 100 cc flasks is shown in fig. 20. It permits the testing of long-necked flasks and graduated cylinders without placing the standard pipette at an inconvenient height.

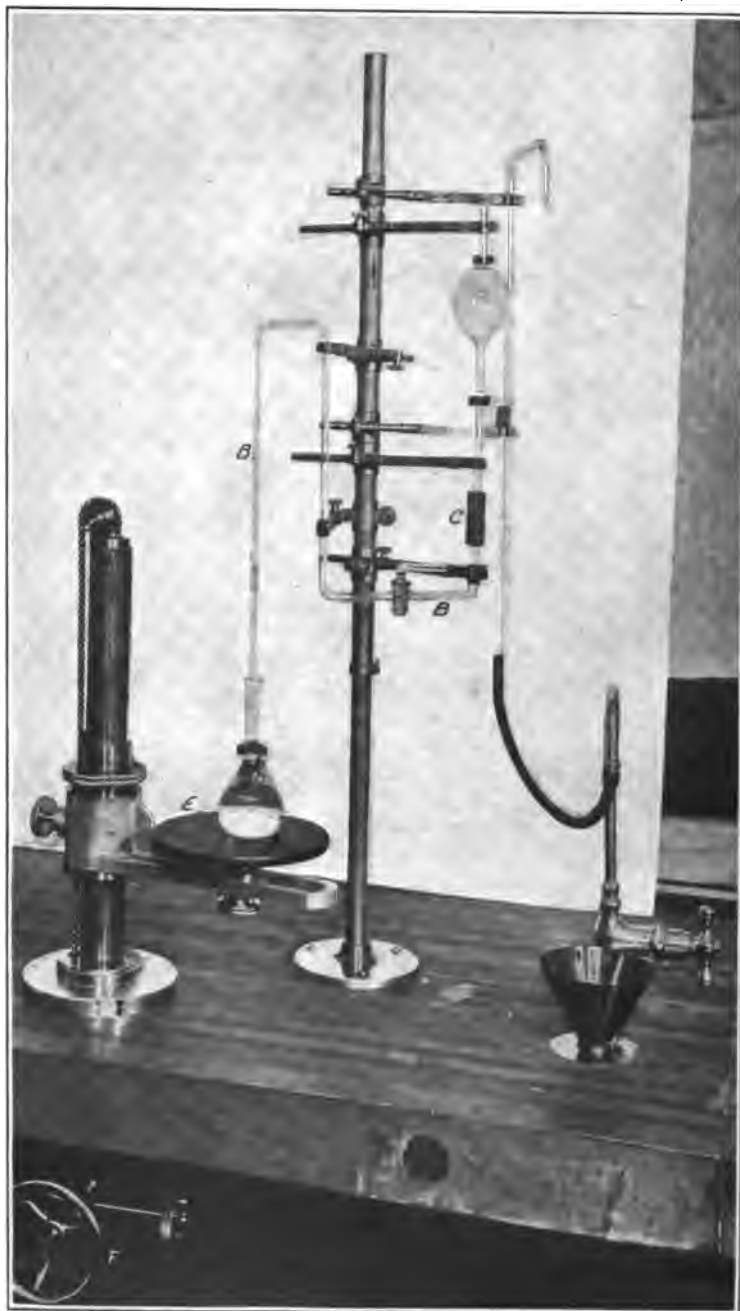


Fig. 20.

The standard pipette is inserted into a heavy rubber connection C. The delivery nozzle meets the connecting tube B in a ground joint.

The pipette is filled through a glass nozzle, which is connected to a water faucet as shown in the illustration and is directed into the top of the pipette. The pipette is filled to the top, the nozzle is swung aside, and by opening the stopcock the meniscus is slowly lowered to the zero mark. Excess of water is then removed from the outflow nozzle, and the flask to be tested is placed on the platform E immediately under the outflow nozzle. The platform is raised by turning the wheel F until the outflow nozzle is just inside the neck of the flask. The stopcock is opened wide and the flask rotated to wet the entire neck, and the flask is then raised until the nozzle is from 1 to 2 cm above the mark. Before completing the filling of the flask it is removed and the contents shaken as directed in the rules for manipulation. The filling is completed with the tip in contact with the wetted wall 1 to 2 cm above the mark. The meniscus in the flask is finally brought to the mark by breaking contact of the tip with the wetted surface.

The standard pipette is read at the end of its normal outflow time, plus 15 seconds. The pipette reading plus the instrumental correction is the capacity of the flask at the standard temperature of the pipette—that is, 20° C. The object of shaking the water is to disperse the contaminations and thus produce a meniscus of normal volume. This manipulation reproduces the conditions of ordinary use. If the test is to merely ascertain whether the capacity is within the allowed limits of error, this detail is omitted unless the error is too near the limit to allow discrimination, in which case a retest is made. The magnitude of the possible error due to contamination has already been indicated in the previous discussion of variation in capacity of flasks.

This method of testing flasks obviates observations of temperature and weight and the calculations required in their reduction to equivalent volume, while it affords equal accuracy if care is observed. The method assumes that the temperature of the water is the same in the pipette and in the flask and that the coefficients of expansion of the two vessels are nearly equal. No large systematic error is peculiar to the volumetric method except, under very unfavorable conditions, this error of temperature.

The pipettes used for this purpose at this bureau are interchangeable, it being only necessary to employ a nozzle of the proper size in order to use any pipette with the holder and outflow tube.

The use of standard graduated pipettes for determining the capacity of flasks has been described by Morse and Blalock,¹⁸ and the method described above merely introduces certain details of refinement adapting it to the immediate requirements of the testing laboratory.

(b) *Testing Flasks, Pipettes, and Burettes by Weighing.*—Although to economize time, flasks are usually tested at this bureau by direct measurement, as described above, they may also be tested, with equal accuracy, by weighing. This test consists in filling and observing the temperature and weight of the water contained. The manner of weighing is described later.

In testing either transfer pipettes or measuring pipettes the pipette is clamped in a vertical position. The pipette is filled by suction from a beaker of water. The pipette is first filled and emptied back into the beaker to bring the pipette to the temperature of the water and also to observe the time of outflow. The temperature of the water in the beaker is then observed. The pipette is again filled and is emptied into the weighing flask. The method of weighing is described later.

The manner of holding and manipulating pipettes at this bureau will be explained with the aid of the accompanying photograph, Fig. 21. The end of the suction tube is inserted in the rubber connection C and is held in position by a clamp lower on the tube. The filling device consists of a bent capillary of about 2 mm diameter, the end of which is also inserted in the rubber connection C. This capillary terminates in a well-ground stopcock B, which is fitted with a long handle. To permit nice control of the opening, the movable part of the cock is provided with tapering nicks extending from the edge of the perforating hole for several millimeters of the circumference on opposite ends and opposite sides of the hole. The stopcock is lubricated with graphite from a soft pencil, since the use of grease would prevent nice control of the air flow. The flask F is connected by the tube V with a vacuum pipe.

¹⁸ Amer. Chem. Jour. 16; 1894.

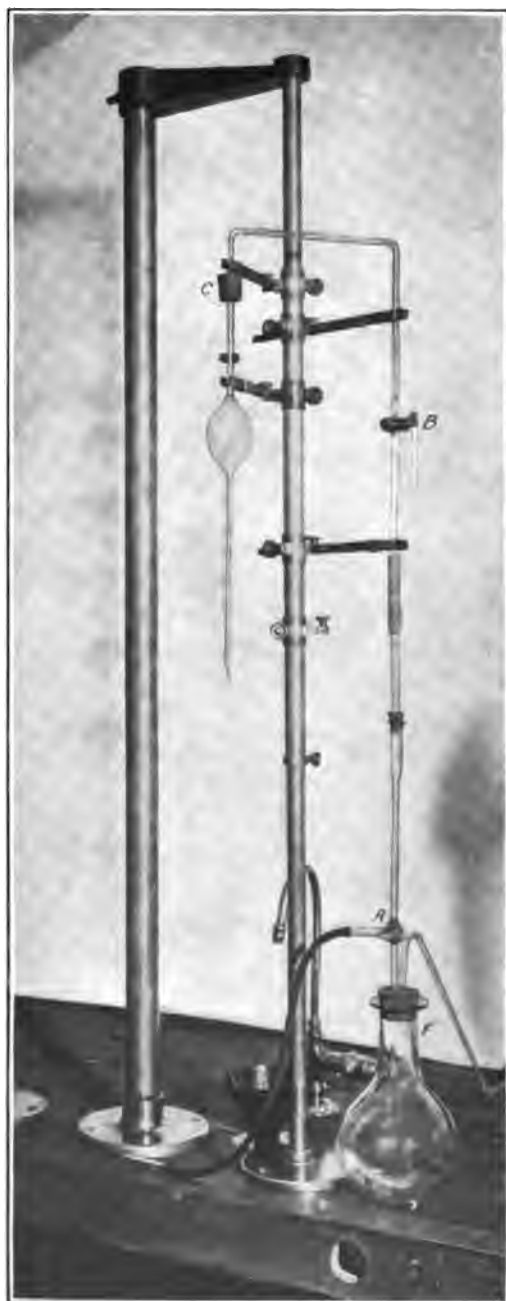


Fig. 21.



To fill the pipette, the two-way cock A is turned to connect the evacuated flask F with the stopcock B. A beaker of water is so held that the tip of the pipette is submerged, and by partially opening the stopcock B the pipette is rapidly filled. The two-way cock A is then turned to shut off the vacuum and admit atmospheric pressure. The outside of the tip is wiped with clean filter paper if necessary to remove adhering drops. A clean beaker is held touching the tip and the cock B again carefully opened until the meniscus descends slowly to the mark. The cock is closed and the beaker removed. The emptying is accomplished by fully opening the cock B.

This arrangement is used for both transfer and measuring pipettes. It has been found that settings can be made with almost the same precision as with burettes if the tip is separated from the receiving vessel immediately after closing the cock. The advantage of filling by automatic suction is greatest in the larger pipettes, where it avoids fatigue and waste of time.

Burettes are mounted for test with clamps on firm uprights. They are filled through a glass nozzle which is connected to a distilled water faucet and is directed into the top of the burette. The burette is filled and emptied into a beaker. The temperature of the water delivered is observed; the burette is again filled, the nozzle swung aside, and the interval to be tested is emptied into a weighing flask. The method of weighing is described later.

Burettes and measuring pipettes are tested in intervals beginning with the zero mark.

In observing the apparent weight in air of the water contained by a flask, two weighings of the flask are made, the first before receiving the water whose volume is required, and the second after the addition of the water. The rubber stopper for closing the flask is included in both weighings, and when the temperature is observed immediately before weighing, the thermometer is also included, being held by inserting its upper end into the stopper.

The flask is hung from the left balance hook, leaving the pan free to receive weights. On the right pan is placed a weight heavier than the filled flask. This weight is not changed during any determination. The sensibility of the balance is thus unchanged, and is usually so adjusted that the milligrams are indicated directly on the

scale by the pointer. Sufficient weights are placed on the left pan to secure equilibrium and their amount observed. The difference between two successive observed weights is the apparent weight of the water added.

When several volumes of water are to be weighed, as in calibrating a burette or testing transfer pipettes, a flask sufficiently large is used to permit these volumes to be added to the flask successively, the weight being observed between each addition. The balances used are provided with special mechanism designated as the Rueprecht system. This mechanism furnishes three different sensibilities such that the pointer indicates on the scale, in division units of 100, 10, or 1 mg as desired, the defect in counterpoise, and so avoids the usual approximation by trial. The small weights are manipulated without opening the balance case.

The buoyancy constant; that is, the difference between the mass and apparent weight of unit volume of water, is determined by observations of temperature, pressure, and humidity, and the use of suitable tables. At this laboratory the buoyancy constant is determined directly by weighing in air a hollow sealed glass bulb whose volume is about 900 cc. The mass of this bulb is such that when the bulb and a brass weight of equal mass are upon opposite sides of a balance, the mass of the additional weights on the bulb side necessary to secure balance; that is, the difference between the mass and apparent weight of the bulb, equals the buoyancy constant for one liter as defined above.

(c) *Calculation of Capacity.*—In measurement of capacity by weighing water the following quantities are obtained by observation:

Nominal capacity; that is, volume indicated by instrument being tested.

Temperature of water.

Apparent weight of water in air; that is, mass of weights required to counterpoise the water in air.

Buoyancy constant.

The degree of precision required determines whether or not observation of the buoyancy constant is necessary.

The quantity sought is the actual capacity of the instrument at the standard temperature; that is, 20° C. The capacity at the

observed temperature; that is, volume of water weighed is first calculated.

For calculating the volume of water weighed three methods are here given and the special application of each indicated.

The symbols employed are as follows:

Quantities Observed.

V = nominal capacity.

t = temperature of water.

a = apparent weight in air of water.

b = buoyancy constant.

Quantities Obtained from Tables.

d_t = density of water at $t^\circ \text{C}$.

M = mass of water having a volume V at $t^\circ \text{C}$.

A = apparent weight in air of water having volume V at $t^\circ \text{C}$.

Quantities Calculated.

v = volume of water at $t^\circ \text{C}$.

C = capacity of instrument at standard temperature, 20°C .

In all cases where precision is desired the following formula serves,

$$v = \frac{a}{d_t - b} \quad (1)$$

d_t is obtained from Table XII, using temperature of water. b may be observed directly or obtained from Table XI, using temperature and barometer observations.

Where V is a round number, use of the following approximate formula simplifies the calculation.

$$v = V + a + Vb - M \quad (2)$$

M is obtained from Tables XIII to XVIII, using temperature of water. b may be observed directly or obtained from Table XI, using temperature and barometer observations. Where $v - V$ is numerically less than $\frac{1}{1000}$, v the approximation employed in this formula

causes no error greater than $\frac{1}{200000} v$.

Where V is a round number and the buoyancy constant is not observed the following formula is used:

$$v = V + a - A \quad (3)$$

A is obtained from Table XIX, using temperature of water. Formula (3) is equal in accuracy to formula (2).

After calculating v by one of the above methods, C , the capacity of the instrument at 20°C is calculated by the formula (4), $C = v + va(20 - t)$.

Calculated values of $va(20 - t)$ are given in Table XX.

(d) *Tables Used in Calculating Capacity.*—For use in making measurements of capacity or volume by weighing, a number of tables are inserted to supply data.

Table XI gives the difference in milligrams between the mass and the apparent weight of 1 cubic centimeter of water weighed with brass weights ($d = 8.4$) in air at various temperatures and barometer readings (unreduced). A humidity of 50 per cent is assumed. This quantity is b , the buoyancy constant used in formulas (1) and (2).

TABLE XI.—Buoyancy Constant mg/cm^3 .

Pressure	Temperature in Degrees Centigrade			
	15	20	25	30
720	1.017	0.998	0.979	0.960
725	1.024	1.004	0.985	0.967
730	1.031	1.011	0.992	0.973
735	1.038	1.018	0.999	0.980
740	1.045	1.025	1.006	0.987
745	1.052	1.032	1.013	0.994
750	1.059	1.039	1.020	1.000
755	1.067	1.046	1.027	1.007
760	1.074	1.053	1.034	1.014
765	1.081	1.060	1.040	1.020
770	1.088	1.067	1.047	1.027
775	1.095	1.074	1.054	1.034
780	1.102	1.081	1.061	1.041

Table XII is a copy of the table of density of water by P. Chapuis (Vol. XIII, Travaux et Memoires, Bureau International des Poids et Mesures, 1907). This quantity is d_t , used in formula (1).

Density of pure water free from air, by tenths of degrees from 0° to 40° and under standard pressure.

Standard Degrees.	Tenths of Degrees.										Mean Differences.
	0	1	2	3	4	5	6	7	8	9	
0											
0	0.999 8681	8747	8812	8875	8936	8996	9053	9109	9163	9216	+ 59
1	9267	9315	9363	9408	9452	9494	9534	9573	9610	9645	+ 41
2	9679	9711	9741	9769	9796	9821	9844	9866	9887	9905	+ 24
3	9922	9937	9951	9962	9973	9981	9988	9994	9998	*0000	+ 8
4	1.000 0000	*9999	*9996	*9992	*9986	*9979	*9970	*9960	*9947	*9934	— 8
5	0.999 9919	9902	9884	9864	9842	9819	9795	9769	9742	9713	— 24
6	9682	9650	9617	9582	9545	9507	9468	9427	9385	9341	— 39
7	9296	9249	9201	9151	9100	9048	8994	8938	8881	8823	— 53
8	8764	8703	8641	8577	8512	8445	8377	8308	8237	8165	— 67
9	8091	8017	7940	7864	7784	7704	7622	7539	7455	7369	— 81
10	7282	7194	7105	7014	6921	6826	6729	6632	6533	6432	— 95
11	6331	6228	6124	6020	5913	5805	5696	5586	5474	5362	— 108
12	5248	5132	5016	4896	4780	4660	4538	4415	4291	4166	— 121
13	4040	3912	3784	3654	3523	3391	3257	3122	2986	2850	— 133
14	2712	2572	2431	2289	2147	2003	1858	1711	1564	1416	— 145
15	1266	1114	0962	0809	0655	0499	0343	0185	0026	*9865	— 156
16	0.998 9705	9542	9378	9214	9048	8881	8713	8544	8373	8202	— 168
17	8029	7856	7681	7505	7328	7150	6971	6791	6610	6427	— 178
18	6244	6058	5873	5686	5498	5309	5119	4927	4735	4541	— 190
19	4347	4152	3955	3757	3558	3358	3158	2955	2752	2549	— 200
20	2343	2137	1930	1722	1511	1301	1090	0878	0663	0449	— 211
21	0233	0016	*9799	*9580	*9359	*9139	*8917	*8694	*8470	*8245	— 221
22	0.997 8019	7792	7564	7335	7104	6873	6641	6408	6173	5938	— 232
23	5702	5466	5227	4988	4747	4506	4264	4021	3777	3531	— 242
24	3286	3039	2790	2541	2291	2040	1788	1535	1280	1026	— 252
25	0770	0513	0255	*9997	*9736	*9476	*9214	*8951	*8688	*8423	— 261
26	0.996 8158	7892	7624	7356	7087	6817	6545	6273	6000	5726	— 271
27	5451	5176	4898	4620	4342	4062	3782	3500	3218	2935	— 280
28	2652	2366	2080	1793	1505	1217	0928	0637	0346	0053	— 289
29	0.995 9761	9466	9171	8876	8579	8282	7983	7684	7383	7083	— 298
30	6780	6478	6174	5869	5564	5258	4950	4642	4334	4024	— 307
31	3714	3401	3089	2776	2462	2147	1832	1515	1198	0880	— 315
32	0561	0241	*9920	*9599	*9276	*8954	*8630	*8304	*7979	*7653	— 324
33	0.994 7325	6997	6668	6338	6007	5676	5345	5011	4678	4343	— 332
34	4007	3671	3335	2997	2659	2318	1978	1638	1296	0953	— 340
35	0610	0267	*9922	*9576	*9230	*8883	*8534	*8186	*7837	*7486	— 347
36	0.993 7136	6784	6432	6078	5725	5369	5014	4658	4301	3943	— 355
37	3585	3226	2866	2505	2144	1782	1419	1055	0691	0326	— 362
38	0.992 9960	9593	9227	8859	8490	8120	7751	7380	7008	6636	— 370
39	6263	5890	5516	5140	4765	4389	4011	3634	3255	2876	— 377
40	2497	2116	1734	1352	0971	0587	0203	*9818	*9433	*9047	— 384
41	0.991 8661										

Tables XIII to XVIII give the mass of certain volumes of water for temperatures between 15° and 30°. This quantity is M used in formula (2). These tables are calculated using the densities of water given in Table XII.

Table XIX gives for temperatures between 15° C and 30° C, the apparent weight in air, humidity 50 per cent, uncorrected barometer reading 760 mm, of certain volumes of water weighed with brass weights. This quantity is A , used in formula (3). This table is based on the data given in Table XI and Tables XIII to XVIII, and may be conveniently employed to obtain desired volumes of water for calibrating instruments. The table assumes the air to be at the same temperature as the water.

Table XX gives the correction to be added to capacities of glass instruments determined at various temperatures to give the capacity at 20° C. By subtracting the corrections from the indicated capacity of an instrument standard at 20° C the capacity at other temperatures is obtained. This quantity is $va(20-t)$ in formula (4). The table assumes for the cubical coefficient of expansion of glass 0.000025 per degree centigrade.

Table XXI gives the density of dry air containing .04 per cent of CO_2 at various temperatures and pressures. This table is computed from the formula:

$$\rho = \frac{1.293052}{1 + 0.00367} \times \frac{h}{760}$$

where h is pressure in mm of mercury at 0° C, and standard gravity. This table may be used in precise work, for reducing to vacua weighings made in air.

TABLE XIII.—Mass of 2000 cc of Water.

Standard Degrees.	Tenths of Degrees.									
	0	1	2	3	4	5	6	7	8	9
15	1998.253	223	192	162	131	100	069	037	005	*973
16	1997.941	908	876	843	810	776	743	709	675	640
17	1997.606	571	536	501	466	430	394	358	322	285
18	1997.249	212	175	137	100	062	024	*985	*947	*908
19	1996.869	830	791	751	712	672	632	591	550	510
20	1996.469	427	386	344	302	260	218	176	133	090
21	1996.047	003	*960	*916	*872	*828	*783	*739	*694	*649
22	1995.604	558	513	467	421	375	328	282	235	188
23	1995.140	093	045	*998	*949	*901	*853	*804	*755	*706
24	1994.657	608	558	508	458	408	358	307	256	205
25	1994.154	103	051	*999	*947	*895	*843	*790	*738	*685
26	1993.632	578	525	471	417	363	309	255	200	145
27	1993.090	035	*980	*924	*868	*812	*756	*700	*644	*587
28	1992.530	473	416	359	301	243	186	127	069	011
29	1991.952	893	834	775	716	656	597	537	477	417

TABLE XIV.—Mass of 1000 cc of Water.

Standard Degrees.	Tenths of Degrees.									
	0	1	2	3	4	5	6	7	8	9
15	999.127	111	*096	*081	*066	*050	*034	*018	*003	*986
16	998.970	954	938	921	905	888	871	854	837	820
17	998.803	786	768	750	733	715	697	679	661	643
18	998.624	606	587	569	550	531	512	493	474	454
19	998.435	415	396	376	356	336	316	296	275	255
20	998.234	214	193	172	151	130	109	088	066	045
21	998.023	002	*980	*958	*936	*914	*892	*869	*847	*824
22	997.802	779	756	734	710	687	664	641	617	594
23	997.570	547	523	499	475	451	426	402	378	353
24	997.329	304	279	254	229	204	179	154	128	103
25	997.077	051	026	000	*974	*948	*921	*895	*869	*842
26	996.816	789	762	736	709	682	654	627	600	573
27	996.545	518	490	462	434	406	378	350	322	294
28	996.265	237	208	179	150	122	093	064	035	005
29	995.976	947	917	888	858	828	798	768	738	708

TABLE XV.—Mass of 500 cc of Water.

Standard Degrees.	Tenths of Degrees.									
	0	1	2	3	4	5	6	7	8	9
15	499.563	556	548	540	533	525	517	509	501	493
16	499.485	477	469	461	452	444	436	427	419	410
17	499.401	393	384	375	366	358	349	340	330	321
18	499.312	303	294	284	275	265	256	246	237	227
19	499.217	208	198	188	178	168	158	148	138	127
20	499.117	107	096	086	076	065	054	044	033	022
21	499.012	001	*990	*979	*968	*957	*946	*935	*924	*912
22	498.901	890	878	867	855	844	832	820	809	797
23	498.785	773	761	749	737	725	713	701	689	677
24	498.664	652	640	627	615	602	589	577	564	551
25	498.538	526	513	500	487	474	461	448	434	421
26	498.408	395	381	368	354	341	327	314	300	286
27	498.273	259	245	231	217	203	189	175	161	147
28	498.133	118	104	090	075	061	046	032	017	003
29	497.988	973	959	944	929	914	899	884	869	854

TABLE XVI.—Mass of 400 cc of Water.

Standard Degrees.	Tenths of Degrees.									
	0	1	2	3	4	5	6	7	8	9
15	399.651	645	638	632	626	620	614	607	601	595
16	399.588	582	575	569	562	555	549	542	535	528
17	399.521	514	507	500	493	486	479	472	464	457
18	399.450	442	435	427	420	412	405	397	389	382
19	399.374	366	358	350	342	334	326	318	310	302
20	399.294	285	277	269	260	252	244	235	227	218
21	339.209	201	192	183	174	166	157	148	139	130
22	399.121	112	103	093	084	075	066	056	047	038
23	399.028	019	009	000	*990	*980	*971	*961	*951	*941
24	398.931	922	912	902	892	882	872	861	851	841
25	398.831	821	810	800	789	779	769	758	748	737
26	398.726	716	705	694	683	673	662	651	640	629
27	398.618	607	596	585	574	562	551	540	529	517
28	398.506	495	483	472	460	449	437	425	414	402
29	398.390	379	367	355	343	331	319	307	295	283

TABLE XVII.—Mass of 300 cc of Water.

Standard Degrees.	Tenths of Degrees.									
	0	1	2	3	4	5	6	7	8	9
15	299.738	733	729	724	720	715	710	705	701	696
16	299.691	686	681	676	671	666	661	656	651	646
17	299.641	636	630	625	620	614	609	603	598	593
18	299.587	582	576	570	565	559	554	548	542	536
19	299.530	524	519	513	507	501	495	489	483	476
20	299.470	464	458	452	445	439	433	426	420	413
21	399.407	400	394	387	381	374	367	361	354	347
22	299.341	334	327	320	313	306	299	292	285	278
23	299.271	264	257	250	242	235	228	221	213	206
24	299.199	191	184	176	169	161	154	146	138	131
25	299.123	115	108	100	092	084	076	069	061	053
26	299.045	037	029	021	013	005	*996	*988	*980	*972
27	298.964	955	947	939	930	922	913	905	897	888
28	298.880	871	862	854	845	837	828	819	810	802
29	298.793	784	775	766	757	748	739	731	721	712

TABLE XVIII.—Mass of 250 cc of Water.

Standard Degrees.	Tenths of Degrees.									
	0	1	2	3	4	5	6	7	8	9
15	249.782	778	774	770	766	762	759	755	751	747
16	249.743	739	734	730	726	722	718	714	709	705
17	249.701	696	692	688	683	679	674	670	665	661
18	249.656	651	647	642	637	633	628	623	618	614
19	249.609	604	599	594	589	584	579	574	569	564
20	249.559	553	548	543	538	533	527	522	517	511
21	249.506	500	495	490	484	478	473	467	462	456
22	249.450	445	439	433	428	422	416	410	404	398
23	249.393	387	381	375	369	363	357	351	344	338
24	249.332	326	320	314	307	301	295	288	282	276
25	249.269	263	256	250	243	237	230	224	217	211
26	249.204	197	191	184	177	170	164	157	150	143
27	249.136	129	122	116	109	102	095	088	080	073
28	249.066	059	052	045	038	030	023	016	009	001
29	248.994	987	979	972	964	957	950	942	935	927

TABLE XIX.—Apparent Weight in Air of Water.

Standard Degrees.	2000 cc	1000 cc	500 cc	400 cc	300 cc	250 cc	150 cc
15	1996.11	998.05	499.03	399.22	299.42	249.51	149.71
16	1995.80	997.90	498.95	399.16	299.37	249.48	149.68
17	1995.48	997.74	498.87	399.10	299.32	249.43	149.66
18	1995.13	997.56	498.78	399.03	299.27	249.39	149.63
19	1994.76	997.38	498.69	398.95	299.21	249.34	149.61
20	1994.36	997.18	498.59	398.87	299.15	249.30	149.58
21	1993.95	996.97	498.49	398.79	299.09	249.24	149.55
22	1993.51	996.76	498.38	398.70	299.03	249.19	149.51
23	1993.06	996.53	498.26	398.61	298.96	249.13	149.48
24	1992.58	996.29	498.15	398.52	298.89	249.07	149.44
25	1992.09	996.04	498.02	398.42	298.81	249.01	149.41
26	1991.57	995.79	497.89	398.31	298.74	248.95	149.37
27	1991.04	995.52	497.76	398.21	298.66	248.88	149.33
28	1990.49	995.24	497.62	398.10	298.57	248.81	149.29
29	1989.92	994.96	497.48	397.98	298.49	248.74	149.24
30	1989.33	994.66	497.33	397.87	298.40	248.67	149.20

TABLE XX.—Temperature Correction.

Temperature.	2000 cc	1000 cc	500 cc	400 cc	300 cc	250 cc
15	+0.25	+0.12	+0.06	+0.05	+0.04	+0.031
16	+ .20	+ .10	+ .05	+ .04	+ .03	+ .025
17	+ .15	+ .08	+ .04	+ .03	+ .02	+ .019
18	+ .10	+ .05	+ .02	+ .02	+ .02	+ .012
19	+ .05	+ .02	+ .01	+ .01	+ .01	+ .006
21	—0.05	—0.02	—0.01	—0.01	—0.01	—0.006
22	— .10	— .05	— .02	— .02	— .02	— .012
23	— .15	— .08	— .04	— .03	— .02	— .019
24	— .20	— .10	— .05	— .04	— .03	— .025
25	— .25	— .12	— .06	— .05	— .04	— .031
26	—0.30	—0.15	—0.08	—0.06	—0.04	—0.038
27	— .35	— .18	— .09	— .07	— .05	— .044
28	— .40	— .20	— .10	— .08	— .06	— .050
29	— .45	— .22	— .11	— .09	— .07	— .056
30	— .50	— .25	— .12	— .10	— .08	— .062

TABLE XXI.

Weight in Grams of 1 Liter of Dry Air Containing 0.04 % of CO₂.

Temperature Degrees C.	Pressure in mm of Hg (0° C., standard gravity).											
	730	735	740	745	750	755	760	765	770	775		
15	1.1611	1.1691	1.1772	1.1853	1.1933	1.2014	1.2095	1.2175	1.2256	1.2336	1.2417	1.2498
16	1.1571	1.1651	1.1731	1.1812	1.1892	1.1972	1.2053	1.2133	1.2213	1.2294	1.2374	1.2454
17	1.1531	1.1611	1.1691	1.1771	1.1851	1.1931	1.2011	1.2091	1.2171	1.2251	1.2331	1.2411
18	1.1491	1.1571	1.1650	1.1730	1.1810	1.1890	1.1970	1.2049	1.2129	1.2209	1.2289	1.2369
19	1.1451	1.1531	1.1611	1.1690	1.1770	1.1849	1.1929	1.2008	1.2088	1.2167	1.2247	1.2326
20	1.1412	1.1492	1.1571	1.1650	1.1729	1.1809	1.1888	1.1967	1.2046	1.2126	1.2205	1.2284
21	1.1373	1.1452	1.1531	1.1610	1.1689	1.1768	1.1847	1.1926	1.2005	1.2084	1.2163	1.2242
22	1.1335	1.1414	1.1492	1.1571	1.1650	1.1728	1.1807	1.1886	1.1965	1.2043	1.2122	1.2201
23	1.1296	1.1375	1.1453	1.1532	1.1610	1.1689	1.1767	1.1846	1.1924	1.2002	1.2081	1.2159
24	1.1258	1.1337	1.1415	1.1493	1.1571	1.1649	1.1727	1.1806	1.1884	1.1962	1.2040	1.2118
25	1.1220	1.1298	1.1376	1.1454	1.1532	1.1610	1.1688	1.1766	1.1844	1.1922	1.2000	1.2078
26	1.1183	1.1261	1.1338	1.1416	1.1494	1.1571	1.1649	1.1727	1.1804	1.1882	1.1959	1.2037
27	1.1146	1.1223	1.1300	1.1378	1.1455	1.1533	1.1610	1.1687	1.1765	1.1842	1.1920	1.1997
28	1.1108	1.1186	1.1263	1.1340	1.1417	1.1494	1.1571	1.1648	1.1726	1.1803	1.1880	1.1957
29	1.1072	1.1149	1.1225	1.1302	1.1379	1.1456	1.1533	1.1610	1.1687	1.1764	1.1840	1.1917
30	1.1035	1.1112	1.1188	1.1265	1.1342	1.1418	1.1495	1.1571	1.1648	1.1725	1.1801	1.1878
31	1.0999	1.1075	1.1151	1.1228	1.1304	1.1381	1.1457	1.1533	1.1610	1.1686	1.1762	1.1839

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